



Hochschule für Angewandte Wissenschaften Hamburg  
*Hamburg University of Applied Sciences*

## Project

Department of Automotive and Aeronautical Engineering

### **Aircraft Design Studies Based on the ATR 72**

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## Abstract

This project gives a practical description of a preliminary aircraft design sequence. The sequence starts with a preliminary sizing method. The design sequence is illustrated with a redesign study of the ATR 72 turboprop aircraft. The requirements for the redesign aircraft are those of the ATR 72. The ATR 72 serves also as the reference during the redesign. The Preliminary sizing method was available (at the university) only for jet-powered aircraft. Therefore the method was adapted to work also with propeller driven aircrafts. The sizing method ensures that all requirements are met: take-off and landing field length, 2nd segment and missed approach gradients as well as cruise Mach number. The sizing method yields the best (low) power/weight ratio and the best wing loading. The redesign process covers all the aircraft components: fuselage, wing, empennage and landing gear. The aircraft design sequence defines the cabin layout, the wing parameters, the type of high lift system, the configuration and surface of the empennage. A mass distribution analysis is made, the position of the CG is calculated and the wing position determined. Finally the Direct Operating Costs (DOC) are calculated. DOCs are calculated applying the method from the Association of European Airlines (AEA). The DOCs serve for an aircraft evaluation. In order to meet requirements, the redesigned ATR 72 had to be slightly modified compared to the original ATR. E.g. the redesigned high lift system shows added slats. In general, the resulting parameters of the redesigned aircraft came out similar with the original ATR 72. Since data of the original ATR 72 is not completely available in public, one of the challenges was to discover the driving factors and secret parameters from the original design.





**DEPARTMENT FAHRZEUGTECHNIK UND FLUGZEUGBAU**

# **Aircraft Design Studies Based on the ATR 72**

Project work towards a thesis at Universitatea Politehnica din Bucuresti (PUB)

## **Background**

Aircraft design studies at universities seem to concentrate quite often at civil transport jets. In this respect, Hamburg University of Applied Sciences is no exception. In order to cover a little bit of new ground in aircraft design teaching, it was decided to pay more attention to propeller driven aircraft, starting with passenger aircraft (certified based on CS-25) and subsequently considering the whole field of propeller aircraft certified based on CS-23, CS-VLA and to ultra light aircraft. Emphasis should be given to the respective methods for preliminary sizing of the respective category of aircraft. Furthermore, effects that have to be handled differently from jets should be considered carefully and in depth.

## **Task**

An ATR 72 should be redesigned. If time allows, preliminary sizing according to CS-VLA should be considered. ATR 72 redesign should comprise of:

- Parameter studies: power, propeller efficiency and glide ratio as a function of operational and geometric parameters.
- Data collection and evaluation for the ATR 72.
- Preliminary sizing of turboprop aircraft for CS-25 – general approach and application to ATR 72.
- Sizing of cabin / fuselage, wing (including high lift devices) empennage and landing gear.
- Calculation of mass, drag polar and DOC.

The report has to be written in English based on German or international standards on report writing.

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## List of Symbols

$a$	Speed of sound
$a, b$	Parameters for describing a linear equation
$A$	Aspect ratio, constant
$b$	Wing span
$c$	Chord, coefficient
$B$	Breguet factor
$C$	Constant
$D$	Drag, constant
$c_D, C_D$	Drag coefficient
$c_L, C_L$	Lift coefficient
$d$	Diameter
$e$	Oswald-Factor
$E$	Glide ratio, engine
$f_{th}$	Floor thickness
$FF$	Form factor
$g$	Earth acceleration
$h$	Altitude
$i$	Incidence angle, flight segment
$k, K$	Constants
$l$	Length or lever arm
$L$	Lift, Lower surface
$L_v$	Transverse force caused by the vertical tailplane
$L/D$	Aerodynamic efficiency
$M$	Constant
$M$	Mass
$M$	Mach number or Moment around the lateral axis
$M_{ff}$	Mission fuel fraction
$m / S_w$	Wing loading
$n$	Indicates the number of elements, constant
$n_c$	Number of compressor stages
$N$	Indicates the number of elements or the moment around the vertical axis
$P$	Power
$P/(mg)$	Power to weight ratio
$q$	Dynamic pressure
$Q$	Interference factor
$R$	Range
$Re$	Reynolds number
$s$	Distance

$S$	Surface
$t$	Thickness, time
$t/c$	Relative thickness
$T$	Thrust
$T/(mg)$	Thrust to weight ratio
$U$	Upper surface
$V$	Velocity
$x$	Distance between a reference point parallel to the cabin floor in the direction of the tail
$y$	Distance from the plane of symmetry of the aircraft in the direction of the span
$\Delta y$	Leading-edge sharpness parameter
$z$	Distance from a reference point situated in the $(x,y)$ plane in upward direction

## Greek Symbols

$\alpha$	Angle of attack
$\beta$	is equal to $\sqrt{1-M^2}$ , or sideslip angle
$\delta$	Angle of a control surface
$\varepsilon$	Wing twist
$\varphi$	Sweep
$\phi_{TE}$	Trailing edge angle
$\gamma$	Glide slope, adiabatic coefficient
$\eta$	Relative span or propeller efficiency
$\lambda$	Taper ratio
$\mu$	Dynamic viscosity
$\nu$	Cinematic viscosity
$\rho$	Air density
$\sigma$	Relative density
$\theta$	Opening angle
$\psi$	Turn over angle

## Indices

### Indices for flight phases

$( )_{TO}$	Take-off
$( )_{CLB}$	Climb
$( )_{CR}$	Cruise
$( )_{DES}$	Descend
$( )_{APP}$	Approach
$( )_L$	Landing
$( )_{ES}$	Start up
$( )_{SO}$	Switch off
$( )_T$	Taxi

### Indices for Aircraft Components

$( )_{AISLE}$	Aisle
$( )_{BUG}$	Fuselage nose length
$( )_{CABIN}$	Cabin
$( )_{DOOR}$	Door
$( )_E$	Engine
$( )_f$	Flap, friction
$( )_F$	Fuselage
$( )_{GALLEY}$	Galley
$( )_{HECK}$	Rear part of the fuselage
$( )_H$	Horizontal tailplane
$( )_L$	Landing
$( )_{LG}$	Landing gear
$( )_{LAVATORY}$	Lavatory

$( )_{LG,M}$	Main landing gear
$( )_{LG,N}$	Nose landing gear
$( )_N$	Nacelle
$( )_{SA}$	Seats abreast
$( )_{SYS}$	System
$( )_s$	Slat, stall or referring to the range factor
$( )_{TAIL}$	Tail
$( )_V$	Vertical tailplane
$( )_{WARDROBE}$	Wardrobe
$( )_W$	Wing

### Indices of the Wing

$( )_{clean}$	Referring to the wing, without flap deflection
$( )_{DD}$	Drag divergence
$( )_i$	Inner
$( )_k$	Kink
$( )_o$	Outer
$( )_{MAC}$	Related to mean aerodynamic chord
$( )_r$	Root
$( )_t$	Tip

### Other Indices

$( )_a$	Approximate
$( )_{AF}$	Airframe
$( )_{av}$	Average

$( )_{ADDITIONAL}$	Additional
$( )_{APP}$	Approach
$( )_{CARGO}$	Cargo
$( )_{CONE}$	Cone
$( )_C$	Crew
$( )_{CYL}$	Cylinder
$( )_{D,flap}$	Drag induced by the flaps
$( )_{D,slat}$	Drag induced by the slats
$( )_{D,0}$	Zero lift drag
$( )_{D,P}$	Profile drag
$( )_{CG}$	Center of gravity
$( )_{disc}$	Disc
$( )_{eff}$	Effective value
$( )_{eq}$	Equivalent
$( )_{EQUIP}$	Equipment
$( )_{exposed}$	Exposed surface
$( )_f$	Friction, final
$( )_F$	Fuel
$( )_{gear}$	Drag induced by the landing gear
$( )_{INF}$	Inflation
$( )_{INT}$	Interest
$( )_{INS}$	Insurance
$( )_{inst}$	Installed
$( )_L$	Labor
$( )_{LEMAC}$	Leading edge mean aerodynamic chord
$( )_{LFL}$	Landing field length
$( )_{LOI}$	Loiter
$( )_M$	Material
$( )_{max}$	Maximum

$( )_{md}$	Minimum drag
$( )_{ML}$	Maximum landing
$( )_{MTO}$	Maximum take-off
$( )_0$	Sea level
$( )_{OE}$	Operating empty
$( )_{OPT}$	Optimum
$( )_P$	Propeller
$( )_{PAX}$	Passenger, pax
$( )_{PL}$	Payload
$( )_{ref}$	Reference
$( )_{RES}$	Reserve
$( )_S$	Stall, range
$( )_{SL}$	Sea level
$( )_{SEGMENT}$	Referring to a segment
$( )_{SA}$	Seats abreast
$( )_{seat}$	Seat
$( )_t$	Time
$( )_{t,a}$	Flights per year
$( )_{tank}$	Fuel tank
$( )_{TOT}$	Total
$( )_{TOFL}$	Take-off field length
$( )_{WATER}$	Referring to water line
$( )_{wet}$	Wetted
$( \bar{ } )$	Medium value

## List of Abbreviations

AC	Aerodynamic center or Advisory Circular
A/C	Aircraft
AEA	Association of European Airlines
AEO	All Engines Operating
APU	Auxiliary Power Unit
ATR	Avions de Transport Régional or Aerei da Trasporto Regionale
EAS	Equivalent Air Speed
EADS	European Aeronautic Defense and Space Company
EFIS	Electronic Flight Instrument System
FAA	Federal Aviation Agency
FAR	Federal Aviation Regulations
F/C	Flight control
CG	Center of gravity
ICAO	International Civil Aviation Organization
JAR	Joint Aviation Requirements
LER	Leading-edge radius
MAC	Mean aerodynamic chord
MF	Margin Factor
NACA	National Advisory Committee for Aeronautics
NM	Nautical Miles
P&W	Pratt & Whitney, engine manufacturer
PW	Pratt & Whitney, engine manufacturer
SFC	Specific Fuel Consumption
TAS	True Air Speed
THS	Trimmable horizontal stabilizer
WWW	World Wide Web

# 1 Introduction

## 1.1 Motivation

The subject approached in this thesis is related to Aircraft Design. The preliminary design procedure was chosen to be applied on a propeller driven aircraft, due to the lack of examples in the frame of HAW on this type of aircrafts. The method needed several investigations in order to achieve proper results and adapt the method from jets to regional turboprops. Since usually the main objective of a thesis is to bring the subject as close as possible to the industry interest, a passenger aircraft was chosen, the ATR 72-500.

Also, the Aircraft Design as a subject, represents in itself a practical approach, and is not studied in the frame of PUB Bucharest, Faculty of Aerospace Engineering. The subject is therefore of interest.

The preliminary investigation of the most important parameters is the starting point of any aircraft project, and decides whether or not the detailed approach would give good results or not. This is also what this exercise intends to achieve.

## 1.2 Definitions

The key words in this thesis are already mentioned in the title: *aircraft design studies based on an existing aircraft*.

“The starting point of any new aircraft is to clearly identify its purpose” (**Corke 2002**)

“(…)The aircraft design process is a compromise of all the engineering disciplines. An effective design is the integration of aerodynamics, propulsion, flight control, structures and materials, avionics and subsystems; blended in just the right way to give a synergistic effect.” (**Nicolai 1975**)

“There are equally important aspects of aircraft design: *design layout* and *design analysis*. (...) The designer’s product is a drawing. (...)A good aircraft design seems to miraculously glide through subsequent evaluations by specialists without major changes being required. (...) Design is not just the actual layout, but also the analytical process used to determine what should be designed and how the design should be modified to better meet the requirements” (**Raymer 2006**)

In a conceptual design phase, according to **Raymer 2006**, the configuration of the aircraft is set, the size, the weight and also the performance of the aircraft.

If the starting point is an existing aircraft the conceptual design becomes an exercise and the main challenge is to achieve the same results and understand the solutions of the design engineers, assuming that the existing aircraft has been already optimized. In the same time, the purpose is to understand and study how the design can be improved, based on the requirements.

### **1.3 Task**

The design case is a regional turboprop, the ATR 72-500. The redesign of the airplane has to include a parametric study in order to understand and overcome the differences between the turbojets and turboprops methods of preliminary design. This means a research is required in order to estimate the propeller efficiency, the variation of power with altitude or the glide ratio.

The design phases have to begin with the preliminary sizing in order to obtain the major parameters of the design, like: wing loading, power to weight ratio, wing surface and span or fuel mass.

Next the work should comprise of fuselage layout definition, wing layout and parameters, high lift systems, landing gear configuration. The drag polar calculation allows a new estimation for parameters like lift-to-drag ratio.

The mass and center of gravity location is also part of the task, and helps with the sizing of the empennage.

The estimation of the Direct Operating Costs is another aspect that has to be considered.

In order to compare the results with the original ATR a collection of data has to be gathered.

The preliminary design procedure has to start from the requirements identification and the results have to finally produce a three views drawing with the specifications of the new configuration.

## 1.4 Literature

The method used for the design process was developed by Prof. Dr. Dieter Scholz, University of Applied Sciences Hamburg, Department of Automotive and Aeronautical Engineering, having as background publications written by Daniel P. Raymer, Laurence K. Loftin Jr. , Jan Roskam or Egbert Torenbeek.

The thesis uses as reference a large number of specialized books, written either by professors or engineers in the aircraft design field. Another important reference was the database of Projects and Theses, which form the “Digitale Bibliothek” of the University of Applied Sciences, Hamburg, (URL : <http://www.fzt.haw-hamburg.de/pers/Scholz/ArbeitenAbgegeben.html>).

## 1.5 Structure of Work

The work comprises of eleven chapters.

The *Preliminary Design* chapter represents the basis of the work, providing the main parameters, for defining the aircraft. The next chapters give a more detailed approach regarding the *fuselage* and the *wing* parameters, always having as starting point the associated requirements.

The design of the *high lift systems* gives a background to the first chapter, and proves that the assumptions regarding the lift coefficients can be practically fulfilled.

An *empennage general design*, based on other aircraft experience, gives the starting point for estimating the *CG location*. The center of gravity influences the stability and control of the aircraft, and therefore allows a final positioning of the wing in relation to the fuselage and the *sizing of the empennage* according to stability and control requirements. The *landing gear design* is also influenced by the CG; the landing gear has to insure that the aircraft does not tip over while being maneuvered on ground and also must face an engine failure during take-off and prevent the aircraft from being damaged.

The *drag estimation* allows the graphical representation of the polar, and the estimation of the *L/D*, a parameter which is to be compared with the values from the preliminary design phase. The *DOC* calculation is also of great importance, due to the fact that every new project is meant to fail if the product cannot be sold; the *DOC* estimation gives a starting point for the new potential clients and proves that the design is successful.

In the end the parameters obtained with this method are compared with the data of the real ATR 72 and conclusions are drawn.

## 2 ATR 72 Parameters

The redesign case is developed on the example of a regional turboprop, the ATR 72-500. The aircraft design methods were intensively applied on turbojets transport aircrafts, therefore when applying it to propeller driven aircrafts a more deep approach is required and special attention has to be given to the propulsion system integration.

The ATR 72 is part of the ATR family of regional turboprops; it is a high wing aircraft equipped with two P&W 127F engines, designed for efficiency and operational flexibility. One of the major objectives for the manufacturer was related to reducing operating costs, by including new technologies with the declared aim of obtaining reduced fuel consumption, a high maintainability level and a maximum commonality, as part of a family aircraft.



**Fig.2.1** The ATR 72-500 (ATR 2008)

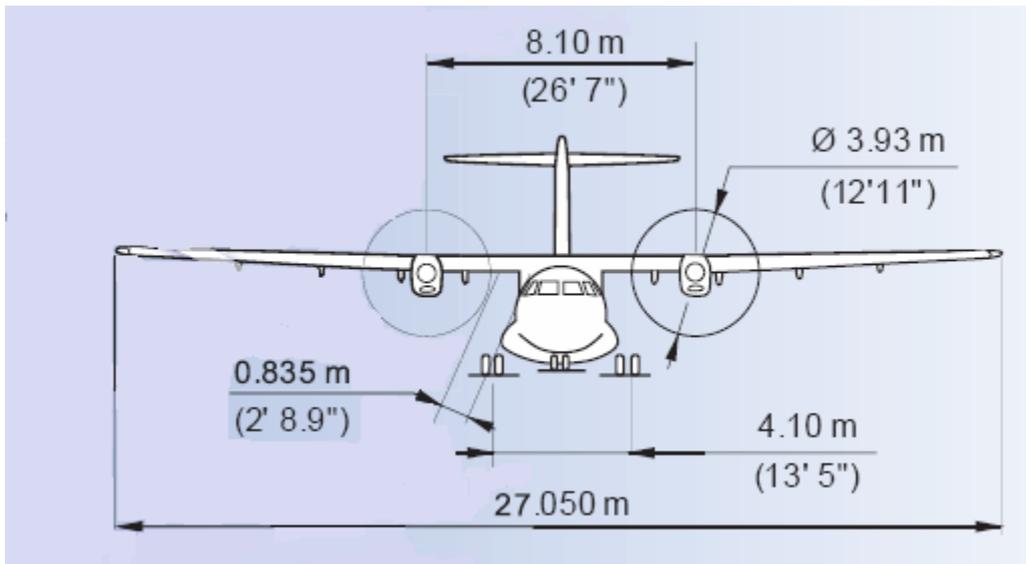
The ATR42/72 is representative for the regional transportation segment; the main competitors are Bombardier Dash 8-Q300 and -Q400, which have a smaller market segment by almost 40%.

The payload range diagram of the ATR-72, as it is defined by the manufacturer is shown in Fig. 2.2:

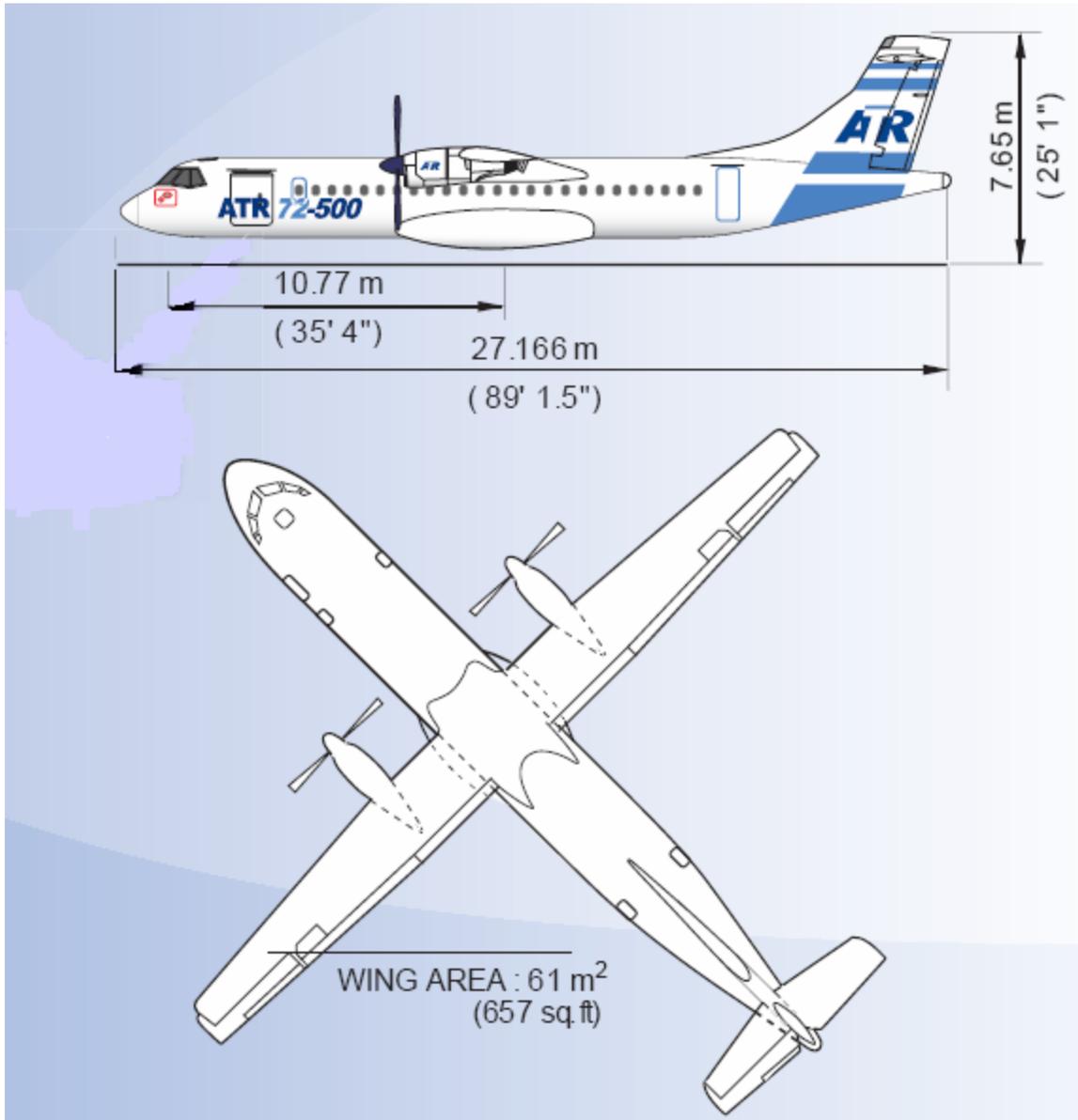


**Fig. 2.2** Payload-range diagram for the ATR 72-500 (ATR 2008)

In our design we will define a payload range diagram for a passenger weight of 95kg; the design range will be 800NM, for 70 passengers (so a payload of 6650kg)



**Fig.2.3a** The main parameters of the ATR 72 (ATR 2008)



**Fig. 2.3b** The main parameters of the ATR 72 (ATR 2008)

### 3 Preliminary sizing

The preliminary sizing of an aircraft has to take into account the requirements and constraints imposed by market, passengers and payload or weight. The procedure proposed by **Loftin 1980** is one which begins the sizing from requirements related to landing distance, take-off distance, second segment, cruise and missed approach. Each requirement gives back output data about power to weight ratio and wing loading. In the end a design point is defined in a matching chart; this design point should return the values which meet all the requirements in the most economical manner. The process can be optimized by modifying the input data, which differ in each requirement, so the preliminary sizing becomes an iterative process. The aim of the optimization is achieving the smallest possible power to weight ratio and the highest possible wing loading.

For the ATR 72-500 aircraft, the manufacturer data are listed in the next table. At the end of the sizing, the obtained results are to be compared with the original model.

**Table 3.1** ATR 72-500 Data

Wing surface	61 m <sup>2</sup>
Wing span	27.05 m
Aspect ratio	12
Maximum take-off weight	22800 kg
Maximum landing weight	22350 kg
Basic operating weight	12950 kg
Maximum payload	7850 kg
Maximum Fuel	5000 kg
Landing Field Length	1067 m
Take-off field Length	1290 m
Range	890 NM
Cruise speed	141.94 m/s

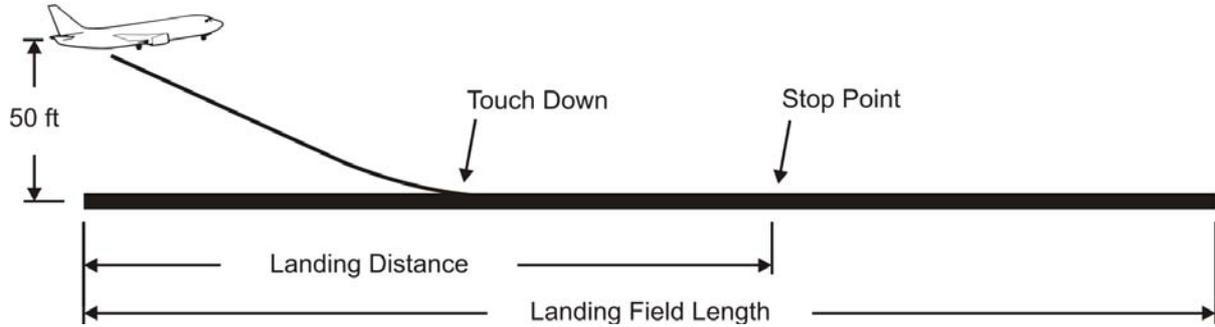
#### 3.1 Landing distance

The basis for analyzing the landing distance is the aviation regulations. (see Fig. 3.1).

According to **Loftin 1980** a statistical relation between landing field length and the approach speed can be defined:

$$V_{APP} = k_{APP} \cdot \sqrt{S_{LFL}} \quad (3.1.1.)$$

The  $k_{APP}$  factor is equal to  $1.85\sqrt{m/s^2}$ .



**Fig. 3.1** Definition of the landing field length according to CS 25 and FAR (**Scholz 2008**)

This requirement returns a maximum value of the wing loading when the input data are the maximum lift coefficient in landing configuration and the landing field distance. This comes out from the equation  $lift = maximum\ landing\ weight$ :

$$\frac{m_{MTO}}{S_W} \leq \frac{k_L \cdot \sigma \cdot C_{L,max,L} \cdot S_{LFL}}{m_{ML} / m_{MTO}} \quad (3.1.2)$$

where  $k_L = 0.141 kg/m^3$  is a factor that comes from statistics. This wing loading must not be exceeded if the aircraft is to meet the requirements.

For the lift coefficient the value must be chosen from empirical data or statistics. For propeller driven aircrafts, statistics according to **Roskam I 1997** show that the lift coefficient lies between 1.6-2.5. The coefficient can also be calculated from the  $lift = weight$  equation.

The value of the maximum landing mass over maximum take-off mass comes also from statistics; for regional aircrafts the values lies between 0.9 and 1.

For the ATR 72, the following data are input data for sizing after landing distance:

$$S_{LFL} = 1067m \quad (3.1.3)$$

$$m_{MTO} = 22800kg \quad (3.1.4)$$

$$m_{ML} = 22350kg \quad (3.1.5)$$

$$C_{L,max,L} = 2.44 \quad (3.1.6)$$

$$\sigma = 1 \text{ (sea level)} \quad (3.1.7)$$

$$m_{ML} / m_{MTO} = 0.98 \quad (3.1.8)$$

When applying the formula we obtain the following results:

$$\frac{m_{MTO}}{S_w} \leq \frac{k_L \cdot \sigma \cdot C_{L,max,L} \cdot S_{LFL}}{m_{ML} / m_{MTO}} = \frac{0.141 \cdot 1 \cdot 2.44 \cdot 1067}{0.98} = 374.32 \text{ kg} / \text{m}^2 \quad (3.1.9)$$

Having as input data the landing field length, we can also obtain the minimum landing speed and the approach speed:

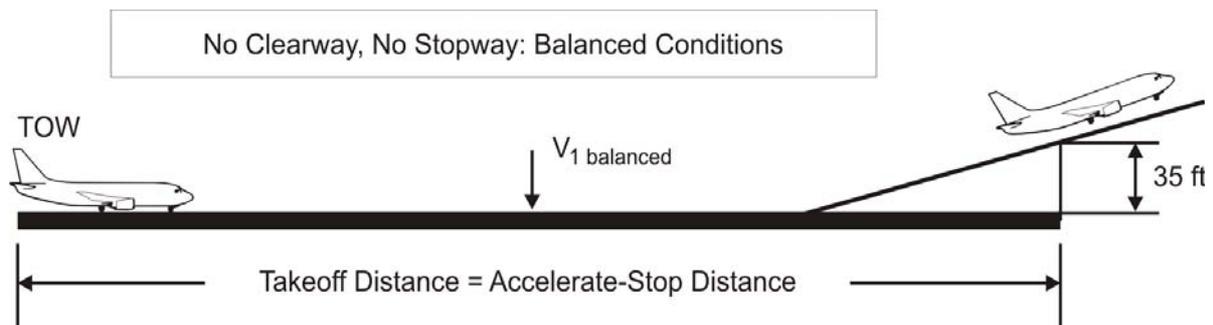
$$V_{APP} = 1.85 \cdot \sqrt{1067} = 60.4 \text{ m} / \text{s} \quad (3.1.10)$$

$$V_{s,0} = \frac{V_{APP}}{1.3} = 46.48 \text{ m} / \text{s} \quad (3.1.11)$$

The factor  $1.3$  comes from **JAR-25.125** /Change 14 27 May 1994, regarding landing requirements. At the moment, this requirement has changed from  $1.3$  to  $1.23$ ; yet we agree to use the same certification requirements which were used when the original ATR-72 was designed.

## 3.2 Take-off Distance

The parameters analyzed in this paragraph have to respect also the aviation regulations. According to **CS 25.113 (a)(2)** the take-off distance AEO (all engines operating) is  $115\%$  of the distance required to fly over an obstacle of  $35\text{ft}$ .



**Fig. 3.2** Definition of the balanced field length according to CS and FAR (engine failure after  $V_1$ ) (Scholz 2008)

The sizing after take-off distance will provide a minimum value for power to weight ratio as a function of wing loading, when the take-off field length and maximum lift coefficient in take-off configuration represent the input data.

In a statistical evaluation, for jet engines the following (Loftin 1980) equation is valid:

$$a = \frac{T_{TO} / m_{MTO} \cdot g}{m_{MTO} / S_W} \geq \frac{k_{TO}}{S_{TOFL} \cdot \sigma \cdot C_{L,max,TO}} \quad (3.2.1)$$

The ratio from thrust to weight ratio and wing loading must not be undershot if the aircraft is to meet requirements.

The same equation is valid for the turboprops, but the thrust has to be replaced by the power, as follows:

$$T = \frac{\eta_P \cdot P_S}{V} \Rightarrow a = \frac{P_S / m_{MTO}}{m_{MTO} / S_W} \geq \frac{k_{TO} \cdot V \cdot g}{S_{TOFL} \cdot \sigma \cdot C_{L,max,TO} \cdot \eta_P} \quad (3.2.2)$$

In this equation the factor  $k_{TO} = 1.805 m^3 / kg$  comes from statistics.

The lift coefficient is obtained in take-off configuration and it can be chosen from the literature in the domain, or from similar aircrafts data. Still, during take-off, the lift coefficient can be approximated with the formula:

$$C_{L,max,TO} = 0.8 \cdot C_{L,max,L} \quad (3.2.3)$$

During take-off, the speed grows from 0 until  $V_2 = 1.2 \cdot V_{s,1}$ ; that means we have to use an average speed which comes from:

$$V = \bar{V} = V_2 / \sqrt{2} m/s \quad (3.2.4)$$

The  $\sqrt{2}$  factor appears because, for obtaining the average speed we need to find in fact an average dynamic pressure, between the initial configuration and the required configuration in take-off. When we extract the speed, the  $\sqrt{2}$  appears, while density is simplified.

$$\bar{q} = \frac{q_0 + q_2}{2} \Leftrightarrow 0 + \sqrt{\frac{1}{2} \rho V_2^2} = 2 \sqrt{\frac{1}{2} \rho \bar{V}^2} \Leftrightarrow \bar{V} = \frac{1}{\sqrt{2}} V_2 \quad (3.2.5)$$

The efficiency of the propeller can also be selected, according to engine characteristics, from the literature. In the next subparagraphs an approach about this subject will be delivered (see paragraph 3.5.5).

In order to proceed with the calculations, we need the following input data:

$$S_{TOFL} = 1290 m \quad (3.2.6)$$

$$C_{L,max,TO} = 0.8 \cdot 2.44 = 1.952 \quad (3.2.7)$$

It is the moment to make the following comment: the 2.44 lift coefficient is a value that fits with the statistics. If we make the calculations for the real ATR, the value obtained is rather high (approximately 3.07), according to the equation  $lift=weight$ . In order to get realistic values for the lift coefficient, the  $k_L$  value has been increased from a value of  $0.107kg/m^2$  (suitable in generally for jets) to  $0.1409kg/m^2$ . With these corrections the  $C_{L,max,TO}$  has the value calculated with the formula (3.2.7). If we round this value to a value of 1.8, the speed modifies, according to (3.2.8); as a consequence the power needed for take-off is larger than if we assume that the aircraft is able to produce a take-off lift coefficient of 1.952. Since we assume that for our redesign we use the same type of engines, then the reference value for the take-off power that we need is the same one with the PW 127, which is 4102kW. If the 1.8 lift coefficient is used, the power-to-weight in take-off becomes 203.094W/kg and we would need an engine that produces 4731.321kW (these results were previously calculated in an *EXCEL* worksheet). We are now in the point where we have to decide how far we are going to deviate from the original model. We can take a decision if we analyze the matching chart, which is to be presented in the next paragraphs. Still we can already specify, that in order to make our design optimum, in the matching chart the lines representing the flight phases should come as close together as possible; this would be the case of a proper design, when the airplane is not over sized or sub sized for a specific flight phase, in our case take-off. We correspondingly choose the smaller value of the lift coefficient, which is  $C_{L,max,TO} = 1.8$ ; in the next chapters we will have to design the wing and the high lift devices in order to match the required lift coefficient, necessary to safely carry the payload.

$$V_{s,1} = V_{s,0} \cdot \sqrt{\frac{C_{L,max,L}}{C_{L,max,TO}}} = 54.121m/s \quad (3.2.8)$$

$$\eta_{P,TO} = 0.64545 \quad (3.2.9)$$

We can now obtain the power to weight ratio at wing loading calculated in landing configuration:

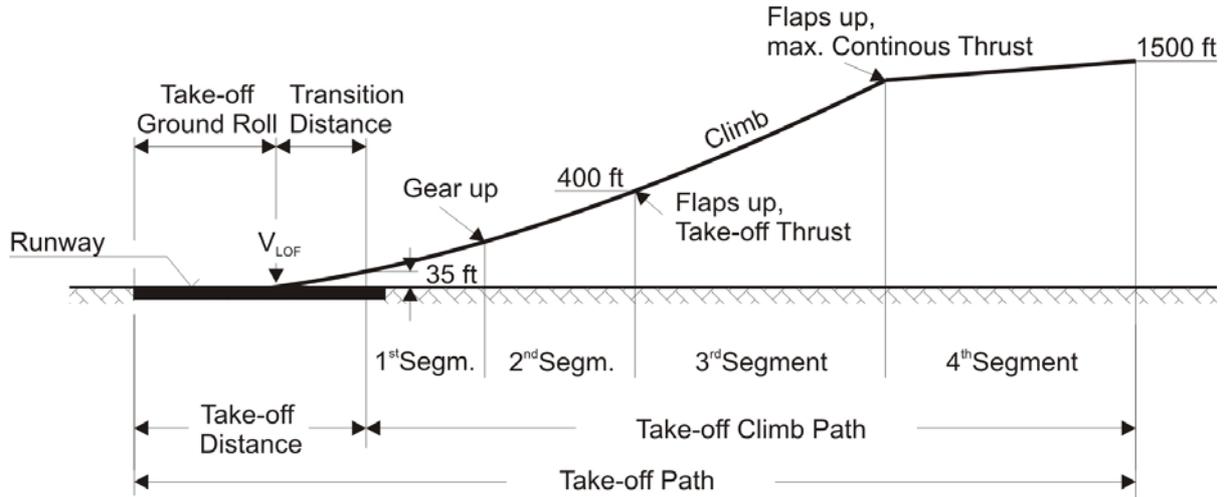
$$a = \frac{P_S / m_{MTO}}{m_{MTO} / S_W} \geq \frac{k_{TO} \cdot 1.2 \cdot V_{s,1} \cdot g}{S_{TOFL} \cdot \sigma \cdot C_{L,max,TO} \cdot \eta_P \cdot \sqrt{2}} = \frac{1.805 \frac{m^3}{kg} \cdot 1.2 \cdot 54.121 \frac{m}{s} \cdot 9.81 \frac{m}{s^2}}{1290m \cdot 1.1952 \cdot 0.64545 \cdot \sqrt{2}} = 0.5425714 \frac{W \cdot m^2}{kg^2} \quad (3.2.10)$$

$$P_S / m_{MTO} = m_{MTO} / S_W \cdot a = 373 \cdot 0.54257 = 203.094 \frac{W}{kg} \quad (3.2.11)$$

### 3.3 Climb Rate during 2<sup>nd</sup> Segment

The take-off path of an aircraft is defined in several paragraphs of the certification regulations. The climb path is shown clearly in Fig. 3.3:

According to **CS-25.121**, “ in take –off configuration existing at the point of the flight path at which the landing gear is fully retracted, ... the steady gradient of climb may not be less than 2-4% for two engine aeroplanes...”.



**Fig. 3.3** Take-off path, definitions and nomenclature (Scholz 2008, based on Brüning 1993)

During climb, the thrust  $T$  must overcome the drag and also a fraction of the weight, due to climb angle  $\gamma$ . The sum of the forces affecting the aircraft in this case becomes:

$$T = D + m \cdot g \cdot \sin \gamma \quad (3.3.1)$$

The vertical force balance is (taking into account the small climb angle):

$$L = m \cdot g \cdot \cos \gamma \approx m \cdot g \quad (3.3.2)$$

When dividing by  $m \cdot g$ , equation (3.3.1) becomes:

$$\frac{T}{m \cdot g} = \frac{1}{E} + \sin \gamma \quad (3.3.3)$$

The thrust to weight ratio must be greater when we take in to account the number of engines, as follows:

$$\frac{T_{TO}}{m_{MTO} \cdot g} = \frac{n_E}{n_E - 1} \cdot \left( \frac{1}{E} + \sin \gamma \right) \quad (3.3.4)$$

These formulas change when the aircraft is propeller driven into the following equations:

$$\frac{P_{S,TO}}{m_{MTO}} \geq \frac{n_E}{n_E - 1} \cdot \left( \frac{1}{E} + \sin \gamma \right) \cdot \left( \frac{V_2 \cdot g}{\eta_{P,CL}} \right) \quad (3.3.5)$$

In (3.3.5) formula,  $n_E$  represents the number of engines and  $E$  is the lift to drag ratio.

For the speed we use the same formula from take-off paragraph, which is:

$$V_2 = 1.2 \cdot V_{s,1} = 64.95 \text{ m/s} \quad (3.3.6)$$

The climb rate is  $\sin \gamma = 0.024$ .

An equation for the lift to drag ratio with extended landing gear and flaps can be calculated with an approximate procedure which takes into account the expression of profile drag and induced drag. The induced drag depends on the lift coefficient, wing aspect ratio and Oswald efficiency factor; the equation becomes as follows:

$$E = \frac{C_L}{C_D} = \frac{C_L}{C_{D,P} + \frac{C_L^2}{\pi \cdot A \cdot e}} \quad (3.3.7)$$

An evaluation of the profile drag considering the influence of high lift system and landing gear is also needed. According to **Loftin 1980**, the following formula can be applied:

$$C_{D,P} = C_{D,0} + \square C_{D,flap} + \square C_{D,slat} + \square C_{D,gear} \quad (3.3.8)$$

According to Fig. 3.3, at the beginning of the 2<sup>nd</sup> Segment, the landing gear comes up, so  $\square C_{D,gear} = 0$ . Also the  $\square C_{D,slat}$  is neglectable. For  $C_{D,0}$  a value of 0.02 is applied for normal passengers aircraft.

For the evaluation of these coefficients, **Loftin 1980** uses the following assumptions:

$$\square C_{D,flap} = 0.05 C_L - 0.055 \quad (3.3.9)$$

The lift coefficient is evaluated with:

$$C_L = C_{L,max} \left( \frac{V_S}{V} \right)^2 \quad (3.3.10)$$

$$C_{L,max,TO} = 1.2^2 \cdot C_L \quad (3.3.11)$$

$$C_{L,max,L} = 1.3^2 \cdot C_L \quad (3.3.12)$$

We obtain the following results:

$$C_{L,max,TO} = 1.25 \quad (3.3.13)$$

$$C_{L,max,L} = 1.444 \quad (3.3.14)$$

$$\square C_{D,flap} = 0.008 \quad (3.3.15)$$

With these results, the profile drag has the value of  $C_{D,P} = 0.028$ .

For the Oswald factor we choose the value of  $e=0.75$ .

We are now able to calculate the lift to drag ratio; the result is  $E=14.42$ .

Using the formula (3.3.5) the power to weight ratio now becomes:

$$P_{S,TO} / m_{MTO} = 162.909W / kg \quad (3.3.16)$$

### 3.4 Climb Rate during Missed Approach

When the aircraft finds itself in the procedure of making the final approach for landing, and for some reason a decision is taken not to land, the take-off thrust has to be applied and the aircraft has to climb in order to start a new approach, according to a predefined procedure.

The procedure takes place in the landing configuration, with the landing gear extracted and the flaps in the landing position. The consequence is a higher drag.

The CS regulations require sufficient thrust to carry out this maneuver safely. According to **CS 25.121**, when one engine is inoperative and a discontinued approach takes place, then the “steady gradient may not be less than 2-1% for two engine aeroplanes”.

The calculations are similar to the ones used for the 2<sup>nd</sup> segment, but the aircraft finds itself in landing configuration, so we have a different value for the weight of the airplane:

$$\frac{P_{S,TO}}{m_{ML}} \geq \frac{n_E}{n_E - 1} \cdot \left( \frac{1}{E} + \sin \gamma \right) \cdot \left( \frac{V_2 \cdot g}{\eta_{P,CL}} \right) \quad (3.4.1)$$

Still, we should convert the power to weight ratio to the maximum take-off mass instead of landing mass, in order to be able to compare the results:

$$\frac{P_{S,TO}}{m_{MTO}} \geq \frac{n_E}{n_E - 1} \cdot \left( \frac{1}{E} + \sin \gamma \right) \cdot \left( \frac{V_2 \cdot g}{\eta_{P,CL}} \right) \cdot \left( \frac{m_{ML}}{m_{MTO}} \right) \quad (3.4.2)$$

According **FAR Part 25** the landing gear remains extended and we have to take into account the drag produced  $\square C_{D,gear}$  which is no longer 0, but 0.015 (in the **CS 25** regulations, the gear is retracted). The coefficient  $\square C_{D,flap}$  becomes 0.040 and  $C_{D,P}$  will have the value of 0.052. That means the glide ratio in landing configuration is:  $E = 11.01$ .

For  $\sin \gamma$  in this configuration we have the value 0.021.

With these values we easily obtain with (3.4.2) the requested value of power to weight ratio:

$$P_{S,TO} / m_{MTO} = 191.268W / kg .$$

## 3.5 Cruise

In order to proceed with the calculations, we assume a stationary straight flight at cruise altitude. We need to make a statement about the same parameters: power to weight ratio and wing loading. In order to achieve this, we can use the two equilibrium equations: *Lift=Weight* and *Drag=Thrust*, while we of course replace thrust by power. Both parameters are a function of altitude and are calculated separately.

### 3.5.1 Power to Weight Ratio

For the thrust to weight ratio the next formula is valid:

$$T_{CR} = D_{CR} = \frac{m_{MTO} \cdot g}{E} \quad (3.5.1)$$

If we divide this by take-off thrust, this gives the following result:

$$\frac{T_{TO}}{m_{MTO} \cdot g} = \frac{1}{(T_{CR} / T_{TO}) \cdot E} \quad (3.5.2)$$

Using (3.2.2) formula, the equation changes for propeller driven aircrafts into:

$$\frac{P_{S,TO}}{m_{MTO}} = \frac{V_{CR} \cdot g}{P_{CR} / P_{S,TO} \cdot E \cdot \eta_{P,CR}} \quad (3.5.3)$$

We are now again able to estimate a value for power to weight ratio during cruise flight. However we first need to approximate the terms of the equation, which are  $P_{CR} / P_{S,TO}$  and also  $E = \frac{L}{D}$ . This implies a research concerning the variation of power with speed and altitude, for turboprop engines. For the value of E, a research is also necessary.

### 3.5.2 Estimation of Lift to Drag Ratio During Cruise

The lift to drag ratio depends directly on wing aspect ratio (a bigger value for A gives back a bigger value for E) and is also influenced by the wetted area relative to wing area,  $S_{wet} / S_W$ .

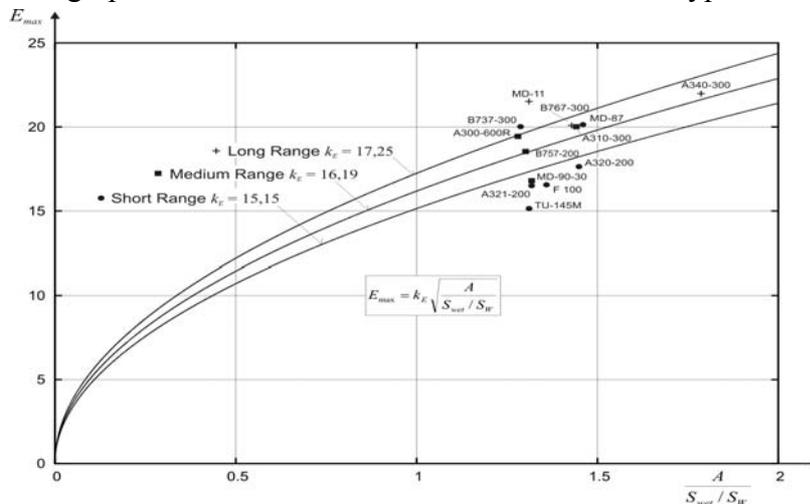
According to theory, the next formula is valid, but we need an approximation for  $k_E$  factor:

$$E_{max} = k_E \cdot \sqrt{\frac{A}{S_{wet} / S_W}} \quad (3.5.4)$$

The literature in the domain of Aircraft Design (**Torenbeek 1988** or **Raymer 2006**), gives us some approximate values for this factor. The values can also be obtained from:

$$k_E = \frac{1}{2} \cdot \sqrt{\frac{\pi \cdot e}{c_f}} \quad (3.5.5)$$

The graph below shows this influence for different types of aircrafts.



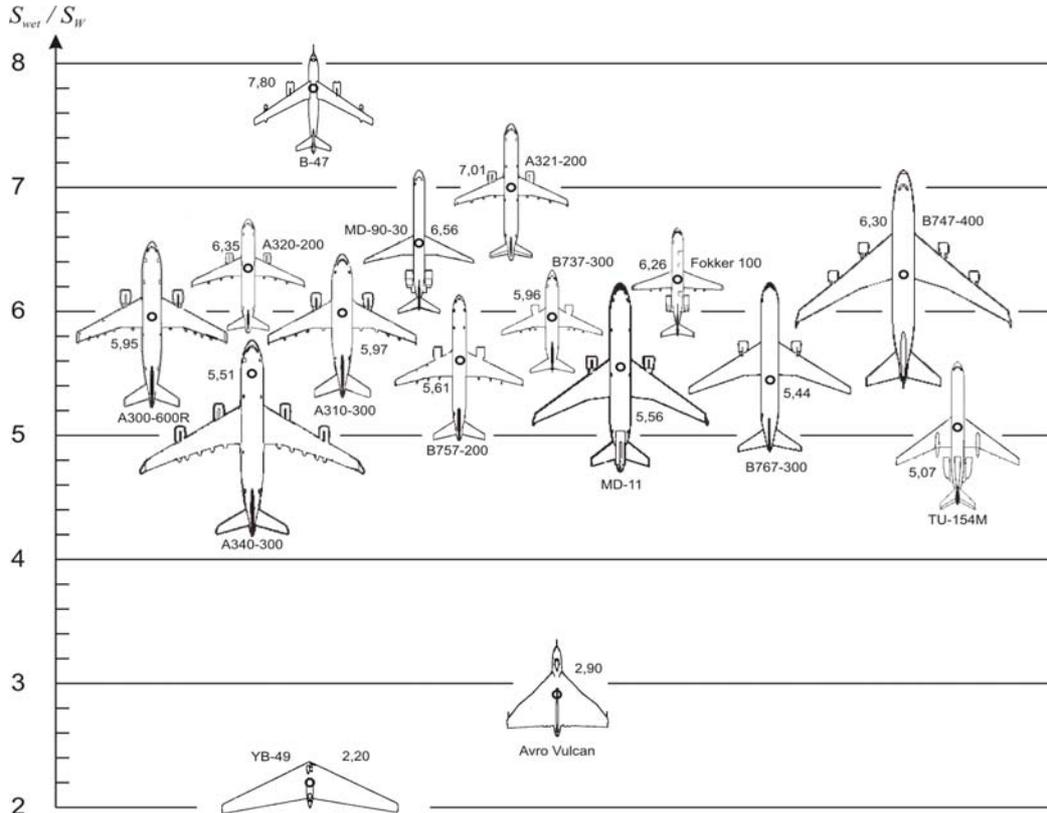
**Fig. 3.4** Estimation of glide ratio, wetted area and wing area (**Raymer 2006**)

The formula (3.5.5) comes from the derivation of the equation (3.5.4), having the form  $y = \sqrt{x}$ . With this formula an estimation of  $k_E$  is possible if we consider a value of  $e = 0.75$  for the Oswald factor and  $\bar{c}_f = 0.03$  for the friction coefficient. The value is:  $k_E = 14.012$ .

For the wetted area to wing area **Raymer 2006** provides a diagram for different concepts of aircrafts (see Fig. 3.5).

This diagram shows the right ratio  $S_{wet} / S_W$  for different aircraft categories. In our case a value of  $S_{wet} / S_W = 6.0 - 6.2$  is recommended. According to **Raymer 2006** the value for  $k_E = 15.8$  is given. For a chosen value of  $k_E = 14$ , the maximum L/D becomes:  $E_{max} = 19.64$ .

We choose a final value of  $19.7$  for the  $L/D_{max}$  estimation.



**Fig. 3.5** Aircraft plan forms and their relative wetted area  $S_{wet} / S_W$  (**Raymer 2006**)

The lift coefficient during cruise, when the drag is minimum, can be obtained by derivation  $E$  to  $C_L$  and by making this relation equal to zero. The result is this one:

$$C_{L,md} = \frac{\pi A e}{2 E_{max}} \quad (3.5.6)$$

We can obtain a relation between actual lift coefficient to minimum drag coefficient and velocity for minimum drag flight:

$$C_L / C_{L,md} = 1 / (V / V_{md})^2 \tag{3.5.7}$$

and therefore:

$$C_L = \frac{C_{L,md}}{(V / V_{md})^2} \tag{3.5.8}$$

In this way we can finally approximate the actual lift to drag ratio, during cruise, depending on  $E_{max}$  :

$$E = \frac{2E_{max}}{\left(\frac{C_L}{C_{L,md}}\right) + \left(\frac{C_L}{C_{L,md}}\right)} \tag{3.5.9}$$

For propeller driven aircrafts the value of  $V / V_{md}$  is proven to be  $\approx 1$ , so that  $E \approx E_{max} = 19.71$ . Another possible estimation for  $L/D$  comes from statistics. According to Fig. 3.6, for the ATR 72 turboprop,  $L/D$  has the value of 17.143.

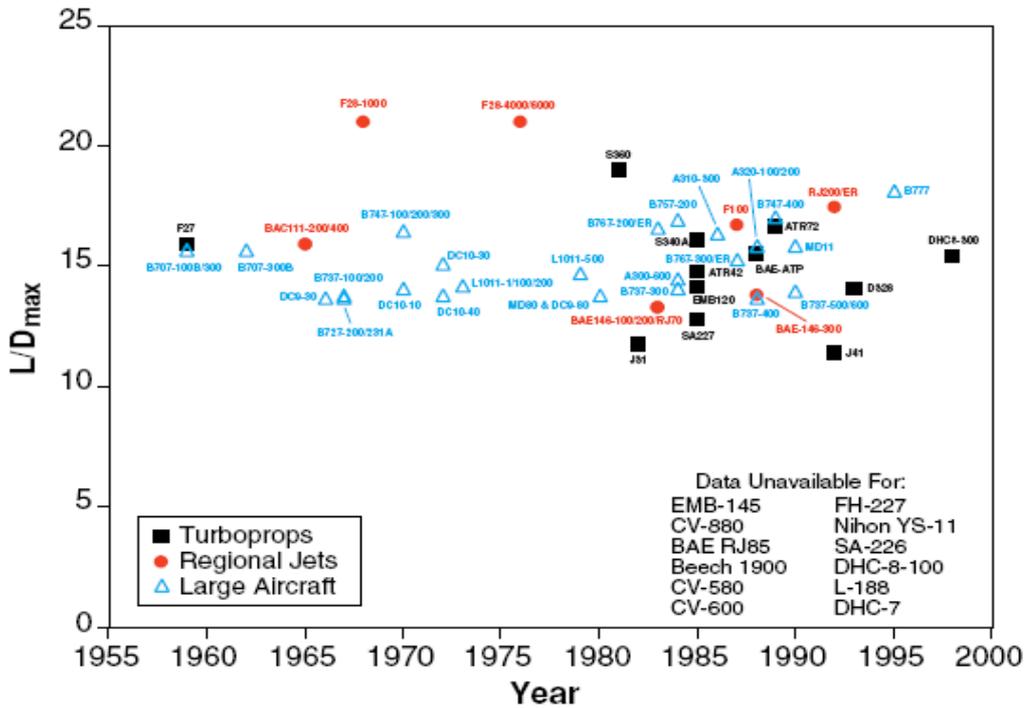


Fig.3.6 Aerodynamic data for commercial aircraft:  $L/D$  for cruise (Babikian 2001)

A third way to estimate this parameter is by using an Excel Sheet (courtesy of **Raymer**, [www.aircraftdesign.com](http://www.aircraftdesign.com), Simplified Aircraft Design Spreadsheet for Homebuilders). When sizing the aircraft with this method, the estimation of  $L/D$  brings us to the calculated value of 19.72. This value is closer to the one obtained with our estimations.

### 3.5.3 Estimation of Power Variation with Height

In order to proceed with the calculations according to (3.5.3) formula, we need to find out how the power of a turboprop varies with altitude, during cruise flight.

It is now a good time to mention some of the turboprops characteristics, in order to have a better understanding of the reason for choosing one.

A turboprop is a gas turbine driving a propeller. This gives a relatively high weight per unit of cruise thrust, caused by the weight of the propeller and especially of the reduction gear. However, when comparing to a turbofan, for example, the disadvantage of low cruise speed and low thrust to weight ratio must be balanced against the advantage of good fuel economy. The propulsive efficiency for a turboprop remains nearly constant during cruise. Also its performance is much superior to that of a turbojet or turbofan engine during take-off and climb. An aspect concerning costs is also an advantage, because the maintenance cost is much less and the turboprop engine is more reliable.

The estimations for  $P = P(H)$  from literature are only approximate equations, depending on coefficients which have specific values for different types of engines. Therefore, the characteristics (especially the control system) of the turboprop engine are highly important in the analysis of this variation. A general equation can only be mentioned by means of unknown coefficients (precisely, known only by the manufacturer of the engine). In the next table different equations are shown for comparison; eventually we have to choose the best estimation possible for our design.

**Table 3.2** Different equations for estimating power with height variation for turboprop engines

No.	Equation	Observations	Author
1.	$P_S / P_{S,0} = A \cdot M^n$	$n$ is always between 0 and 1, depending on engine characteristics; $A$ is also a constant	W. Austyn Mair David L. Birdsall ( <b>Birdsall 1992</b> )
2.	$\frac{P_{eq}}{P_{eq,0}} = \left( \frac{\rho}{\rho_0} \right)^n = \sigma^n$	$n=0.75$ -approximately	G.J.J. Ruijgrok ( <b>Ruijgrok 1996</b> )

3.	$\frac{P_A}{P_{A,0}} = \left(\frac{\rho}{\rho_0}\right)^n = \sigma^n$	n=0.7-approximately	John D. Anderson Jr. ( <b>Anderson 2005</b> )
4.	$P/P_0 = A\sigma^n$	n=0.5-approximately A-constant	Trevor Young ( <b>Young 2001</b> )
5.	$P/(P_0)_{EQ} = \sigma + 0.75\sigma^{0.85}M_N$	this is the climb/cruise continuous power rating; $M_N$ is the Mach number	Denis Howe ( <b>Howe 2000</b> )
6.	$P/P_0 = AM^m\sigma^n$	m-between 0 and 1 A, n-constants (can be greater than 1)	Mihaela Niță

In order to have a good approximation for the  $A$ ,  $m$  and  $n$  coefficients we should compare measured results for different typical turboprop engines, from different sources, with the results obtained when the equations are applied. We choose to approach equations 4 (**Young 2001**) and 6. The references we use for the measured results and also the values of the coefficients are shown in the Table 3.3. These measured results are given for typical turboprop engines and the coefficients are obtained by minimizing errors when comparing experimental results with the theoretical equations. This represents the analysis of a regression, which we consider to be linear and it is done using the *Excel Solver*. The dependent variable in the regression equation is modeled as a function of the independent variables, corresponding parameters ("constants"), and an error term. The error term is treated as a random variable. It represents unexplained variation in the dependent variable. The parameters are estimated so as to give a minimum error; this is evaluated by using the squares method,  $(\delta\varepsilon)^2$  - where  $\delta\varepsilon$  represents the error.

The following results are obtained:

**Table 3.3** Different coefficients when a regression analysis is applied

Experimental data source	Results when applying equation 4: $P/P_0 = A\sigma^n$	Error $(\delta\varepsilon)^2$	Results when applying equation 6: $P/P_0 = AM^m\sigma^n$	Error $(\delta\varepsilon)^2$
<b>Scahufale 2000</b>	A=0.92699314 n=0.77566222	$(\delta\varepsilon)^2 = 0.00697698$	A=1.0359223 m=0.100607 n=0.851122	$(\delta\varepsilon)^2 = 0.03848871$
<b>Bruening 1992</b>	A=0.932825 n=0.739667	$(\delta\varepsilon)^2 = 0.01634715$	A=1.120684 m=0.167927 n=0.754832	$(\delta\varepsilon)^2 = 0.03437704$
<b>Russel 1996</b>	A=1.390323	$(\delta\varepsilon)^2 = 0.092389$	A=1.725115739	$(\delta\varepsilon)^2 = 0.000361$

	n=0.966059		m=0.26739557 n=0.96569268	
<b>Loftin 1980</b>	A=0.973603215 n=0.912149765	$(\delta\varepsilon)^2 = 0.03808482$	A=1.08883 m=0.09121 n=0.924106	$(\delta\varepsilon)^2 = 0.010849$

For a correct interpretation of the coefficients, we first analyze the values for the first equation. This equation takes into account the influence of the height only, and not the influence of the speed.

The  $A$  coefficient can be observed on the vertical axes, as it provides a fit of the curve for  $\sigma = 1$ . We have two cases: in some graphs  $A$  is greater than 1, and in others the  $A$  coefficient is smaller than 1. In the second case the results are justified by the hypothesis that the data we used represents a power rating in cruise smaller than the take-off power. The  $A$ , greater than one, value shows that the data we used represents the cruise speed greater than 0, which means that, in the respective case, the power is not rated relatively to power take-off. Usually the engine is flat rated at low altitudes for obtaining i.e. constant power at either climb or cruise over a wide range of speeds.

The term  $\sigma$  provides the corresponding variation of the power with height, so that height becomes a dimensionless number. Also it is the parameter which influences directly the combustion.

The  $n$  coefficient comes from the engine data from the manufacturer, and it may vary differently from one engine to another. This also depends very much on what control program is used for the respective engine. However, the value of  $n$  will be smaller when EAS is kept constant than when  $M$  is kept constant with altitude. This happens because an increase of height at constant EAS gives an increase of Mach number; the power at this altitude would be greater than if  $M$  would be kept constant.

If we take into account the variation with speed also, we have to analyze the second equation and correspondingly the terms and coefficients  $A$ ,  $m$  and  $n$ .

As we know, once the speed grows, the pressure increases also; correspondingly the shaft power of a turboprop increases. If we include both variations, with speed and height, we obtain for the power of a turboprop the second equation. All three coefficients are interrelated. Coefficient  $m$  also depends on engine data. Correlated with the other two, it gives proper results with experimental data.

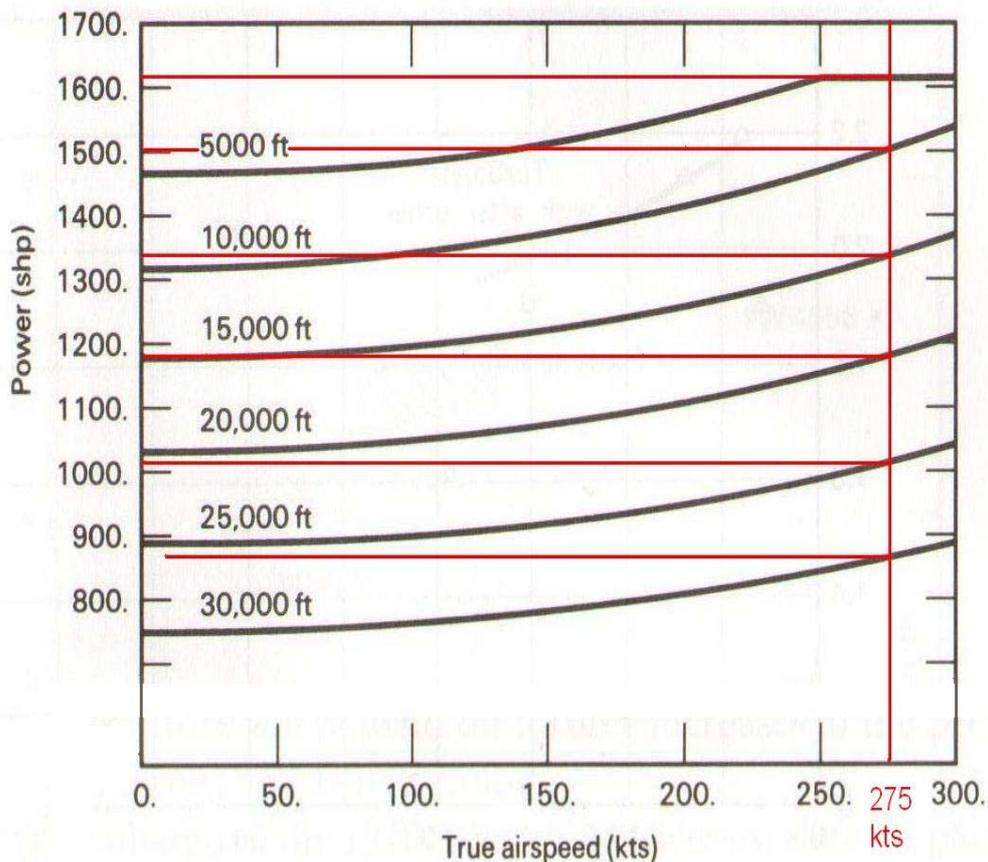
The ATR-72 is equipped with a Pratt&Whitney PW-127F engine. This engine is part of the PW-120 family, for which experimental graphs exist in the university libraries (**McCormick 1995**).

Having the variation of the cruise power with altitude, the experimental equation can be easily obtained using *Microsoft EXCEL*. The approximation is linear with height and comes from Fig. 3.6:

$$P_{CR} / P_{TO} = -5 \cdot 10^{-5} H + 0.877 \quad (3.5.10)$$

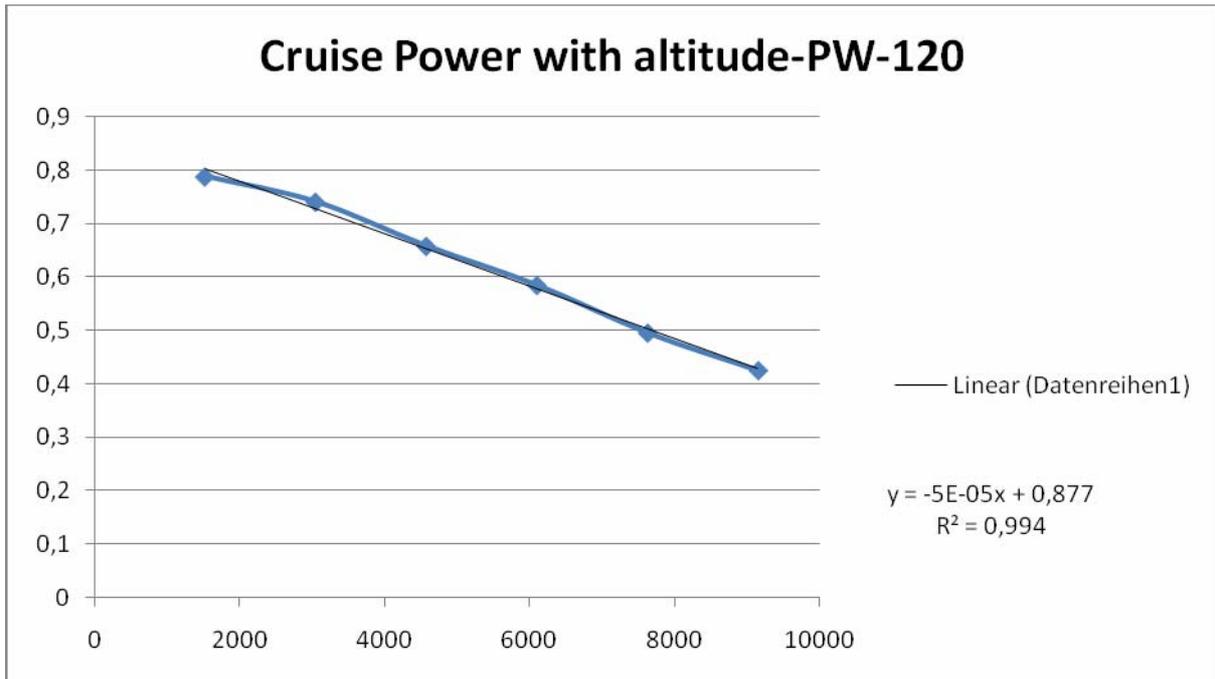
If we transform the linear variation from equation (3.5.10) into the form of the equation 4, the following coefficients give proper results:

$$\begin{aligned} P_{CR} / P_{TO} &= A \cdot \sigma^n \\ A &= 0.9; n = 0.728 \end{aligned} \quad (3.5.11)$$



**Fig.3.6** Maximum Cruise Power for the PW 120 turboprop, (McCormick 1995)

For obtaining the corresponding variation with altitude, we consider the cruise speed  $V_{CR} = 275 \text{ kts}$  (see Fig. 3.7).



**Fig. 3.7** The variation of Cruise Power with altitude for PW 120 turboprop

### 3.5.4 Wing Loading

As we said in the beginning of the chapter, for the cruise flight, when applying the equation lift equal to weight, we can obtain the analytic expression of the wing loading:

$$\frac{m_{MTO}}{S_W} = \frac{C_L \cdot q}{g} = \frac{C_L \cdot M^2}{g} \cdot \frac{q}{M^2} \quad (3.5.12)$$

For the cruise flight we do not take into account the fuel consumption; we use the mass from take-off in order to have a safety margin when sizing the aircraft. For the  $C_L$  coefficient, we can use the formula (3.5.8), with the proper considerations, which are the assumption that the aircraft is flying with maximum lift to drag ratio, or minimum drag.

The  $q/M$  report is obtained by replacing each term:

$$\frac{q}{M^2} = \frac{\frac{1}{2} \cdot \rho \cdot V^2}{\frac{V^2}{a^2}} = \frac{1}{2} \cdot \rho \cdot a^2 \quad (3.5.13)$$

The speed of sound varies with pressure and density, as we know from thermodynamics:

$$a^2 = \gamma \frac{P}{\rho} \quad (3.5.14)$$

We have now the final formula for the wing loading, for cruise flight, as follows:

$$q / M^2 = p(H) \cdot \gamma / 2 \quad (3.5.15)$$

$$\frac{m_{MTO}}{S_w} = \frac{C_L \cdot M_{CR}^2}{g} \cdot \frac{\gamma}{2} \cdot p(H) \quad (3.5.16)$$

or, if we consider instead of Mach number, the cruise speed the wing loading becomes:

$$\frac{m_{MTO}}{S_w} = \frac{C_L \cdot \rho_0 \cdot V_{CR}^2 \cdot \sigma(H)}{2 \cdot g} \quad (3.5.17)$$

The results we obtain depend on the height and can be represented into a chart, together with the results from paragraph 3.5.3.

### 3.5.5 Propeller Efficiency

For the estimation of the propeller efficiency we use the diagram from Fig. 3.8. According to this figure, in order to determine the parameter power-to-density and surface, we need to approximate the diameter of the propeller. From the manufacturer data, this diameter is  $d_p = 3.93m$ . We can now calculate:

$$S = \frac{d_p^2 \cdot \pi}{4} = 12.13m^2 \quad (3.5.18)$$

$$Parameter = \frac{P_{max,cr}(H_{CR})}{\rho(H_{CR}) \cdot S_{disc}} = \frac{1267.6107}{0.722 \cdot 12.13} = 144.739kW \cdot m / kg \quad (3.5.19)$$

We calculated  $P_{max,cr}$  using the analytic formula from Fig. 3.7. With the help of this parameter and with a cruise speed of 141.2m/s we can now read the value of the efficiency for cruise:

$$\eta_{P,CR} = 0.8593 \quad (3.5.20)$$

In the same way we can read the values for the propeller efficiency for all the flight phases; the results are included in Table 3.4.

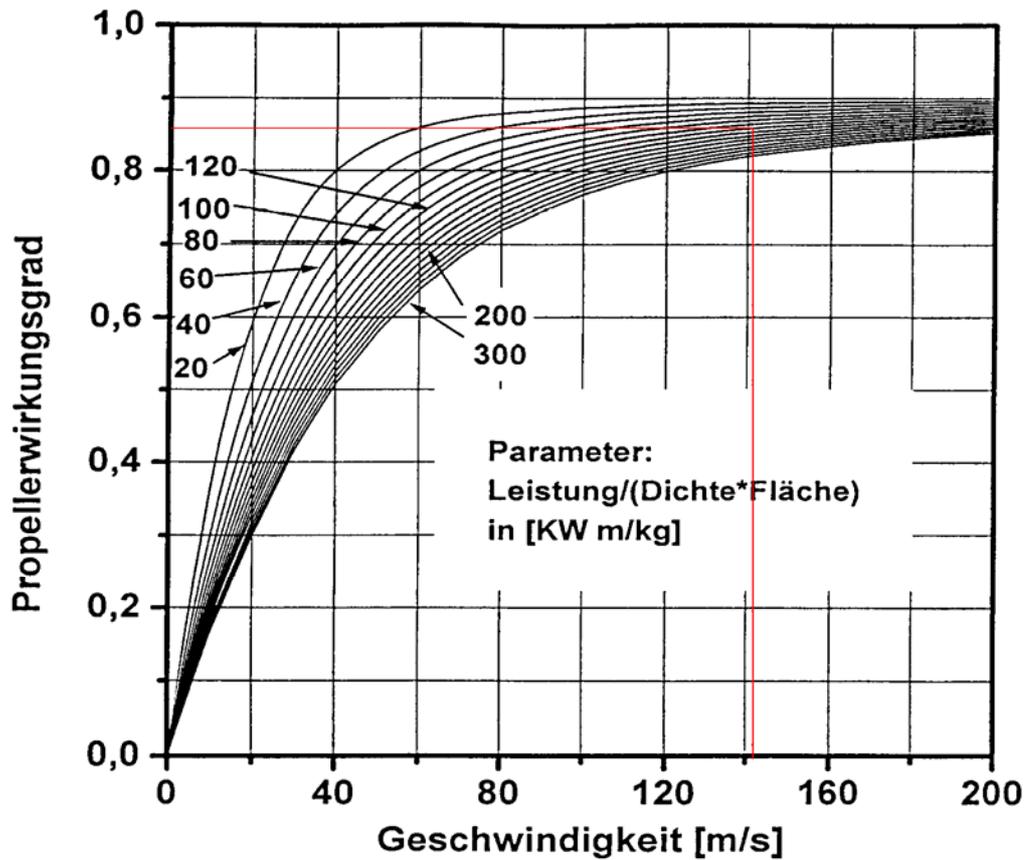


Fig.3.8 Propeller Efficiency Diagram (Scholz 2000)

Table 3.4 Propeller efficiency for different flight phases

Flight Phase	Velocity	Propeller Efficiency
Take-off	$\bar{V} = 44.1 \text{ m/s}$	$\eta_{P,TO} = 0.64545$
Second Segment	$V_2 = 62.37 \text{ m/s}$	$\eta_{P,2nd} = 0.7303$
Missed Approach	$V_2 = 62.37 \text{ m/s}$	$\eta_{P,MA} = 0.7303$
Cruise	$V_{CR} = 141.94 \text{ m/s}$	$\eta_{P,CR} = 0.8593$

### 3.5.6 Results

The results we obtain for cruise are shown in Table 3.5 and afterwards represented in the matching chart.

Table 3.5 Results for cruise

H[m]	T[k]	$\sigma$	$m_{MTO} / S_W$	$P_{CR} / P_{S,TO}$	$P_{S,TO} / m_{MTO}$
0	288.150	1.000000	1158	0.877	93.741
500	284.900	0.952853	1091	0.852	96.492

1000	281.650	0.907426	1027	0.827	99.409
1500	278.400	0.863675	966	0.802	102.508
2000	275.150	0.821557	908	0.777	105.806
2500	271.900	0.781028	853	0.752	109.323
3000	268.650	0.742047	801	0.727	113.083
3500	265.400	0.704573	752	0.702	117.110
4000	262.150	0.668563	704	0.677	121.435
4500	258.900	0.633980	660	0.652	126.091
5000	255.650	0.600782	617	0.627	131.118
5500	252.400	0.568930	577	0.602	136.563
6000	249.150	0.538388	569	0.577	142.480
6500	245.900	0.509115	526	0.552	148.933
7000	242.650	0.481077	486	0.527	155.999
7500	239.400	0.454235	449	0.502	163.767
8000	236.150	0.428555	415	0.477	172.351
8500	232.900	0.404000	383	0.452	181.883
9000	229.650	0.380537	354	0.427	192.532
9500	226.400	0.358131	327	0.402	204.506
10000	223.150	0.336749	303	0.377	218.067

### 3.6 Matching Chart

The two dimensions represented in the matching chart are power to weight ratio and wing loading for the cases we studied so far. Every flight phase we have studied delivers corresponding terms; putting them together gives us the possibility to optimize the sizing of the aircraft. We are allowed to do so, because for all the calculations we always used the maximum take-off mass, so that the comparison could be made possible. As we mentioned in the

When choosing the design point we have to consider achieving the smallest possible thrust to weight ratio at the highest possible wing loading. The ideal case is when all the lines come together.

The resulting design point has the following coordinates:

$$\begin{aligned} \frac{P_{S,TO}}{m_{MTO}} &= 192 \frac{W}{kg} \\ \frac{m_{MTO}}{S_w} &= 354 \frac{kg}{m^2} \end{aligned} \tag{3.6.1}$$

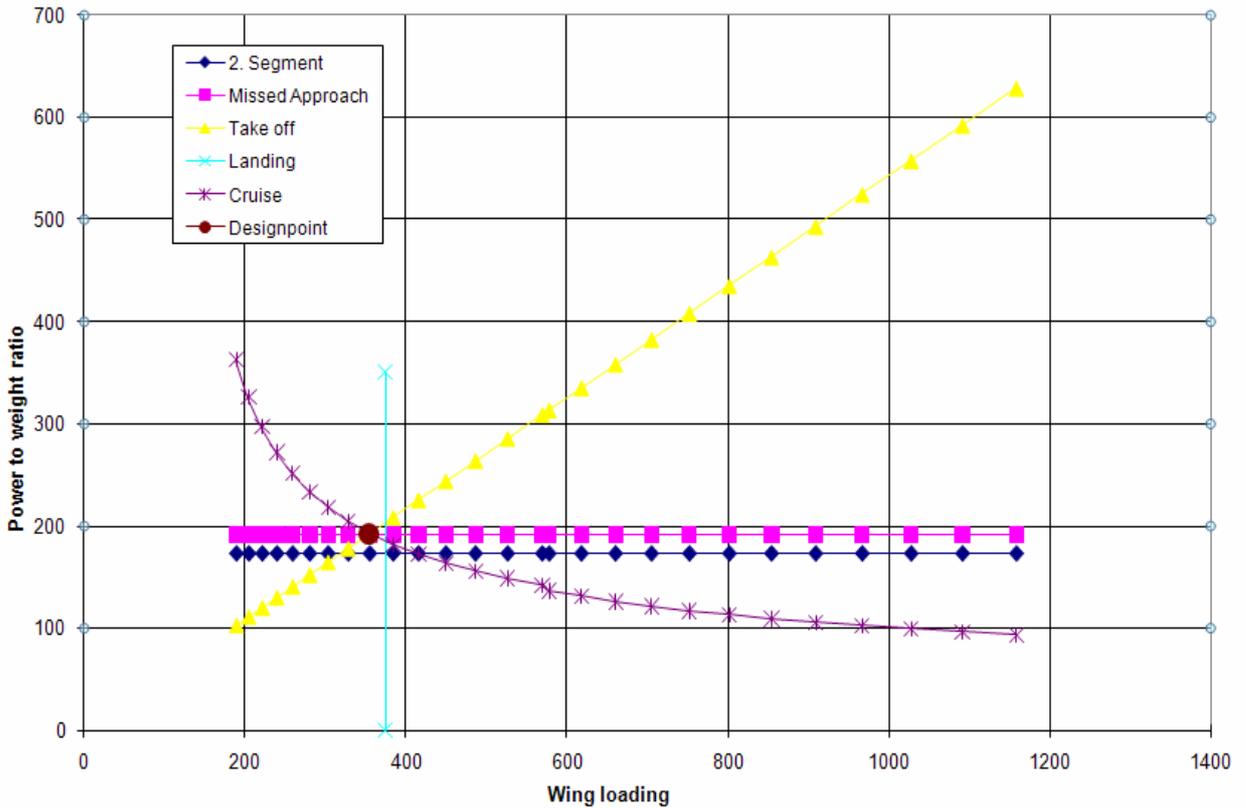


Fig. 3.9 Matching chart for the preliminary sizing of the ATR 72

### 3.7 Maximum Take-off Mass

An estimation for maximum take-off mass is also required. This mass represents the sum of operating empty mass, payload and fuel mass.

$$m_{MTO} - m_F - m_{OE} = m_{PL} \quad (3.7.1)$$

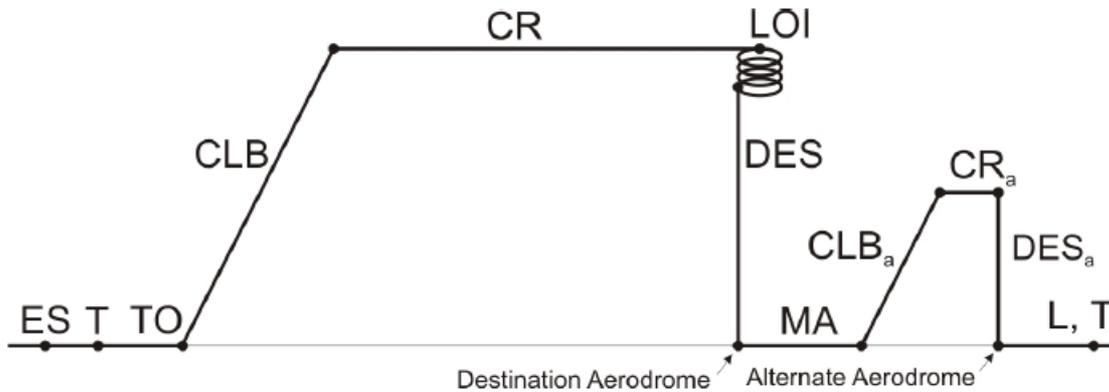
If we redistribute these terms and divide by maximum take-off mass we obtain the following:

$$m_{MTO} = \frac{m_{PL}}{1 - \frac{m_F}{m_{MTO}} - \frac{m_{OE}}{m_{MTO}}} \quad (3.7.2)$$

The relative fuel mass and operating empty mass become design parameter, which are to be calculated next.

#### 3.7.1 Relative Fuel Mass

This parameter needs an estimation in order to insert it in equation (3.7.2), for obtaining the maximum take-off mass. The fuel is used during all flight phases, including taxiing, as shown in Fig. 3.9.



**Fig. 3.10** Typical flight phases of a civil transport flight mission

The mission fuel fraction  $M_{ff}$  is a parameter that helps calculating the fuel consumption for the entire flight. It is obtained with the help of mission segment mass fractions, defined for every flight phase. A mission segment mass fraction for segment  $i$  is the report  $m_{i+1}/m_i$ , where  $i$  refers to the mass from the beginning of the flight phase and  $i+1$  refers to the mass at the beginning of the next flight phase. The parameter  $1 - \frac{m_{i+1}}{m_i}$  gives then the relative fuel consumption for phase  $i$ .

$$M_{ff} = \frac{m_{SO}}{m_T} \cdot \frac{m_T}{m_L} \cdot \frac{m_L}{m_{DES}} \cdot \frac{m_{DES}}{m_{CR,alt}} \cdot \frac{m_{CR,alt}}{m_{CLB}} \cdot \frac{m_{CLB}}{m_{MA}} \cdot \frac{m_{MA}}{m_{DES}} \cdot \frac{m_{DES}}{m_{LOI}} \cdot \frac{m_{LOI}}{m_{CR}} \cdot \frac{m_{CR}}{m_{CLB}} \cdot \frac{m_{CLB}}{m_{TO}} = \frac{m_{SO}}{m_{TO}} \quad (3.7.3)$$

The fuel mass consumed during the flight will become now:

$$m_F = m_{TO} - m_{SO} = m_{TO} \cdot \frac{m_{TO} - m_{SO}}{m_{TO}} = m_{TO} \cdot (1 - M_{ff}) \quad (3.7.4)$$

The  $SO$  index is the mass at the end of the flight ‘after switch off’.

It is necessary now to determine the mission fractions for all the flight phases.

**Table 3.5** Mission segment mass fractions (based on Roskam I 1997)

Phase	M <sub>ff</sub> per flight phases	
	index	value
take-off	TO	0.995
climb	CLB	0.985
descent	DES	0.985
landing	L	0.995

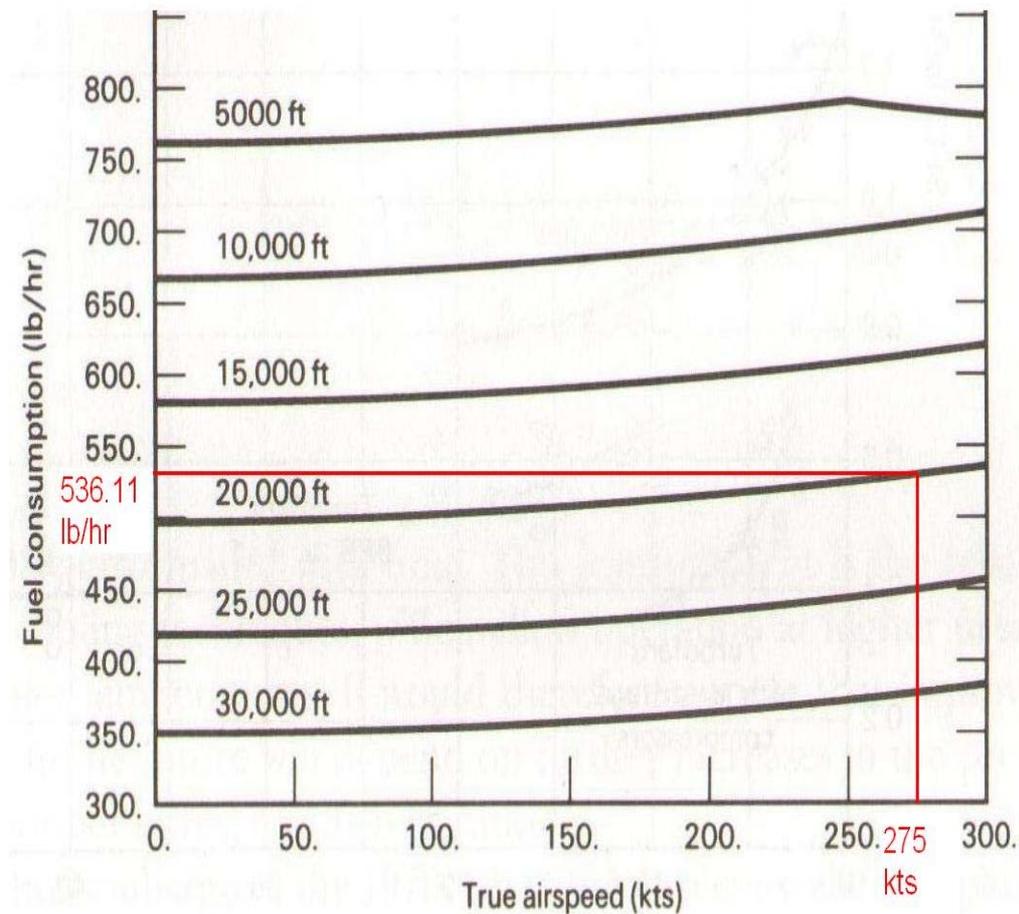
start-up	ES	0.990
taxi	T	0.995

For cruise and loiter we have to first determine the Breguet range factor. For a propeller driven aircraft the factor is:

$$B_s = \frac{L/D \cdot \eta_{p,CR}}{SFC_p \cdot g} = \frac{19.71 \cdot 0.859}{7.5 \cdot 10^{-8} \cdot 9.81} = 23021044m \quad (3.7.5)$$

In this formula  $SFC_p$  refers to the performance-specific fuel consumption and  $\eta_{p,CR}$  is the propeller efficiency. In order to appreciate the specific fuel consumption, we either use the statistics, or data from the manufacturer of the engine; this last source is of course more reliable.

For the ATR 72, which is equipped with a P&W 127 engine, we have a graphic variation for the specific fuel consumption as follows (McCormick 1995):



**Fig.3.10** Fuel Consumption with airspeed for the PW 120 (McCormick 1995)

We can extract from the graph the fuel consumption for cruise; by dividing to power we obtain the specific fuel consumption:

$$SFC_P = \frac{536.11 \text{ lb/hr}}{1200 \text{ hp}} = \frac{536.11 \cdot 0.453592 \text{ kg}}{1200 \cdot 746 \cdot 3600 \text{ W} \cdot \text{s}} = 0.75 \cdot 10^{-7} \text{ kg}/(\text{W} \cdot \text{s}) \quad (3.7.6)$$

The fuel efficiency for cruise, we can extract from Fig. 3.8 diagram, respectively from Table 3.4. With these dates we can obtain the Breguet factor and afterwards, using the next formula, we can estimate the mission segment mass fraction for the cruise phase:

$$\frac{m_{LOI}}{m_{CR}} = e^{-\frac{S_{CR}}{B_S}} \quad (3.7.7)$$

In this formula,  $S_{CR}$  represents the distance covered in the cruise phase.

For a number of 70 passengers, according to the payload range diagram, the range is 800NM. The value for the mission fraction, for the range of  $R = 800 \text{ NM}$ , becomes:

$$M_{ff,CR} = e^{-\frac{R}{B_{S,P}}} = 0.938 \quad (3.7.8)$$

We also have to take into account the fuel consumption for the reserve distance:

$$M_{ff,RES} = e^{-\frac{R_{RES}}{B_{S,P}}} = 0.993 \quad (3.7.9)$$

According to **FAR Part 121** the reserve distance for domestic flight is:  $R_{RES} = 200 \text{ NM}$ . Also, for the same category, the loiter time is:  $t_{loiter} = 2700 \text{ s}$ .

The mission fuel fraction for loiter comes from:

$$M_{ff,LOI} = e^{-\frac{t}{B_i}} \quad (3.7.10)$$

The Breguet factor,  $B_i = \frac{B_S}{V_{CR}} = 162184 \text{ s}$ , so that the mission fraction becomes

$$M_{ff,LOI} = 0.983.$$

In a standard mission, the general fuel fraction becomes:

$$M_{ff,std} = M_{ff,TO} \cdot M_{ff,CLB} \cdot M_{ff,CR} \cdot M_{ff,DES} \cdot M_{ff,L} = 0.901 \quad (3.7.11)$$

The fuel fraction which takes into account the reserves is:

$$M_{ff,res} = M_{ff,CLB} \cdot M_{ff,RES} \cdot M_{ff,LOI} \cdot M_{ff,DES} = 0.948 \quad (3.7.12)$$

The final mission fuel fraction is now the result from the next multiplication:

$$M_{ff} = M_{ff,res} \cdot M_{ff,std} = 0.894 \cdot 0.948 = 0.853 \quad (3.7.13)$$

We have at the moment all the data for obtaining the relative fuel mass:

$$m_F / m_{MTO} = 1 - M_{ff} = 0.147 \quad (3.7.14)$$

### 3.7.2 Relative Operating Empty Weight

This parameter can be either estimated from statistics (**Jenkinson 1999**) or we calculate the report from existing data, as follows:

$$\frac{m_{OE}}{m_{MTO}} = \frac{12950}{22800} = 0.5679 \quad (3.7.15)$$

### 3.7.3 Payload

The payload is the sum of the passenger's weight and the weight of their baggage (hand baggage and cargo baggage):

$$m_{PL} = n_{SEAT} \cdot m_{PAX,BAG} + m_{CARGO} \quad (3.7.16)$$

The number of seats in the configuration we have chosen, (see next chapter), is  $n_{seat} = 70$ . The weight of a passenger, according to statistics, for short and medium range aircrafts is shown in Table 3.6. We choose a value of 95 kg for our calculations, according to the payload-range diagram.

We consider the purpose of the aircraft to be only transportation of passengers; that means the cargo mass remains zero. The cargo volume was determined (see next chap-

ter):  $V_{CARGO} = 9.49m^3$ . This volume is only used for the passenger's baggage, which is included in the average value of 95 kilograms.

These assumptions give the following result for the payload mass:

$$m_{PL} = 70 \cdot 95 = 6650kg \quad (3.7.17)$$

**Table 3.7** Passenger and baggage average weight

Short and medium range aircrafts	
Medium weight of a passenger	79.4kg
Medium weight of hand baggage per pax	13.6 kg
Total baggage, $m_{PAX,BAG}$	93kg

### 3.8 Design Parameters

The final values of the design parameters are now easily obtained with the help of the previously calculated fractions, as it can be observed in the following table.

**Table 3.8** Design Parameters

Design Parameter	Formula
Maximum take-off mass	$m_{MTO} = \frac{m_{PL}}{1 - \frac{m_F}{m_{MTO}} - \frac{m_{OE}}{m_{MTO}}} = \frac{6650}{1 - 0.147 - 0.5679} = 23296.272kg$
Maximum landing mass	$m_{ML} = m_{MTO} \cdot \frac{m_{ML}}{m_{MTO}} = 23296.272 \cdot 0.98 = 22830.347kg$
Maximum operating empty weight	$m_{OE} = m_{MTO} \cdot \frac{m_{OE}}{m_{MTO}} = 23296.272 \cdot 0.568 = 13231.874kg$
Wing surface	$S_W = m_{MTO} / \frac{m_{MTO}}{S_W} = 23296.272 / 374.32 = 62.237m^2$
Take-off power (both engines)	$P_{S,TO} = m_{MTO} \cdot \frac{P_{S,TO}}{m_{MTO}} = 23296.272 \cdot 203.094 = 4731.321kW$
Take-off power, one engine	$P_{S,TO,E} = \frac{P_{S,TO}}{n_E} = 2365.66kW$
Necessary fuel mass	$m_{F,erf} = m_{MTO} \cdot \frac{m_F}{m_{MTO}} = 23296.272 \cdot 0.147 = 3414.398kg$
Wing span	$b = \sqrt{A \cdot S_W} = \sqrt{12 \cdot 62.237} = 27.32m$

It is necessary to include other two mission segments when calculating the necessary fuel mass; these are start-up and taxi, which by now had no meaning for the design. According to **Roskam I 1997** and Table 3.5, the total necessary fuel is:

$$m_{F,nec} = m_{MTO}(1 - M_{ff,ES} \cdot M_{ff,T} \cdot M_{ff}) = 23296.272(1 - 0.99 \cdot 0.95 \cdot 0.853) = 3711.632kg \quad (3.8.1)$$

The necessary volume for the fuel mass, when the density is  $\rho_F = 800kg / m^3$  becomes:

$$V_{F,nec} = \frac{m_{F,nec}}{\rho_F} = \frac{3937.854}{800} = 4.268m^3 \quad (3.8.2)$$

The following condition must also be respected: the maximum landing weight of the aircraft has to have a value of at least the sum between the operating weight empty, payload and reserve fuel:

$$m_{ML} \geq m_{OE} + m_{PL} + m_{F,res} \quad (3.8.3)$$

$$22830.347kg \geq 21103.73kg$$

The results show that the aircraft is correctly sized.

### 3.9 Comparing values

The ATR-500 which is now in use has the following characteristics; the next table shows the difference between the original design and the design presented in this thesis:

**Table 3.9** Comparing values

Parameter	Original value	Calculated value	Deviation [%]
Wingspan [m]	27.05	27.32	0.99%
Wing surface [m <sup>2</sup> ]	61	62.237	2%
Maximum take-off mass [kg]	22800	23296.272	2%
Maximum operating empty weight [kg]	12950	13231.874	2%
Wing loading [kg/m <sup>2</sup> ]	373.77	374.317	0.15%
Power to weight ratio [W/kg]	179.9	179.84	0.03%

## 4 Fuselage Design

A very important aspect concerning the fuselage design is related to market expectations, company strategies and comfort trends. All these should be put in balance with requirements regarding weight, drag or costs.

For passenger aircrafts one must consider, for example, having a good access to cabin and cargo compartments for ground operations, and also a lightweight structure when forming a pressure vessel for high altitude cruise, with acceptable air pressure in cabin.

Other requirements are related to ergonomics, environment, stowage (summing payload containment requirements- pax and cargo); minimum drag is another requirement concerning aerodynamics; minimum weight is also a requirement of leverage for tail control surfaces.

The majority of space in the fuselage is dedicated to the payload, while systems and equipment is to be positioned rather in niche zones.

For ATR 72-500 the standard versions includes 70 passengers seats at a pitch of 31 inches with a straight partition. Two cargo compartments are available: one in front and a rear one.

### 4.1 Fuselage cross section

The parameters for the fuselage design will follow the logic way from “requirements to solution”. The design begins by choosing the number of seats per row or the cabin width.

If we consult the literature in this domain, according to **Roskam III 1997**, we have to choose the slenderness parameter,  $\lambda_F = \frac{l_F}{d_F}$ . A minimum drag is obtained for a slenderness of 6, but

if we consider the influence of the empennage, according to **Roskam III 1997**, the value of 8 is optimal.

After choosing the number of the passengers,  $n_{PAX} = 70$ , we can calculate the number of seats per row:

$$n_{SA} = 0.45 \cdot \sqrt{n_{PAX}} \quad (4.1.1)$$

The result is 3.76, which means a number of 4 passengers per row, according to Fig. 4.1.

We also have to consider **CS** norms, which state that for a number of 4 passengers per row a single aisle is indicated (**CS 25.815**).

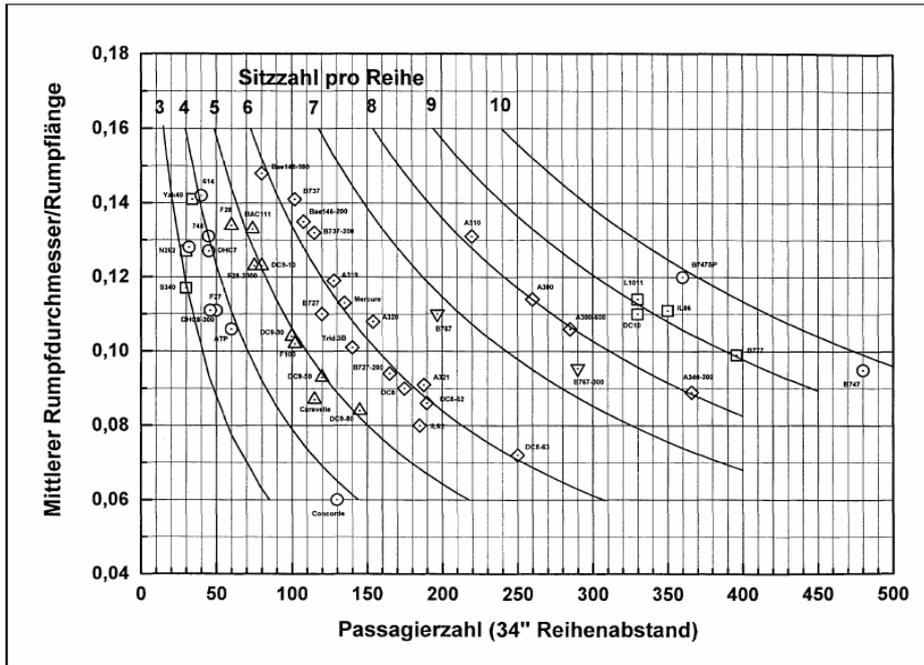


Fig. 4.1 Number of seats/row as a function of passenger number and slenderness (Markwardt 1998)

A next step implies the inner configuration of the fuselage. It has been proved that good results concerning seats and aisles sizing are obtained with **Raymer's 2006** reference data. In order for the proper passenger comfort to be achieved, and also all other requirements concerning volume and costs, we obtain the following results, which also consider **CS** norms:

$$\text{Aisle width}=18 \text{ in} \quad (4.1.2)$$

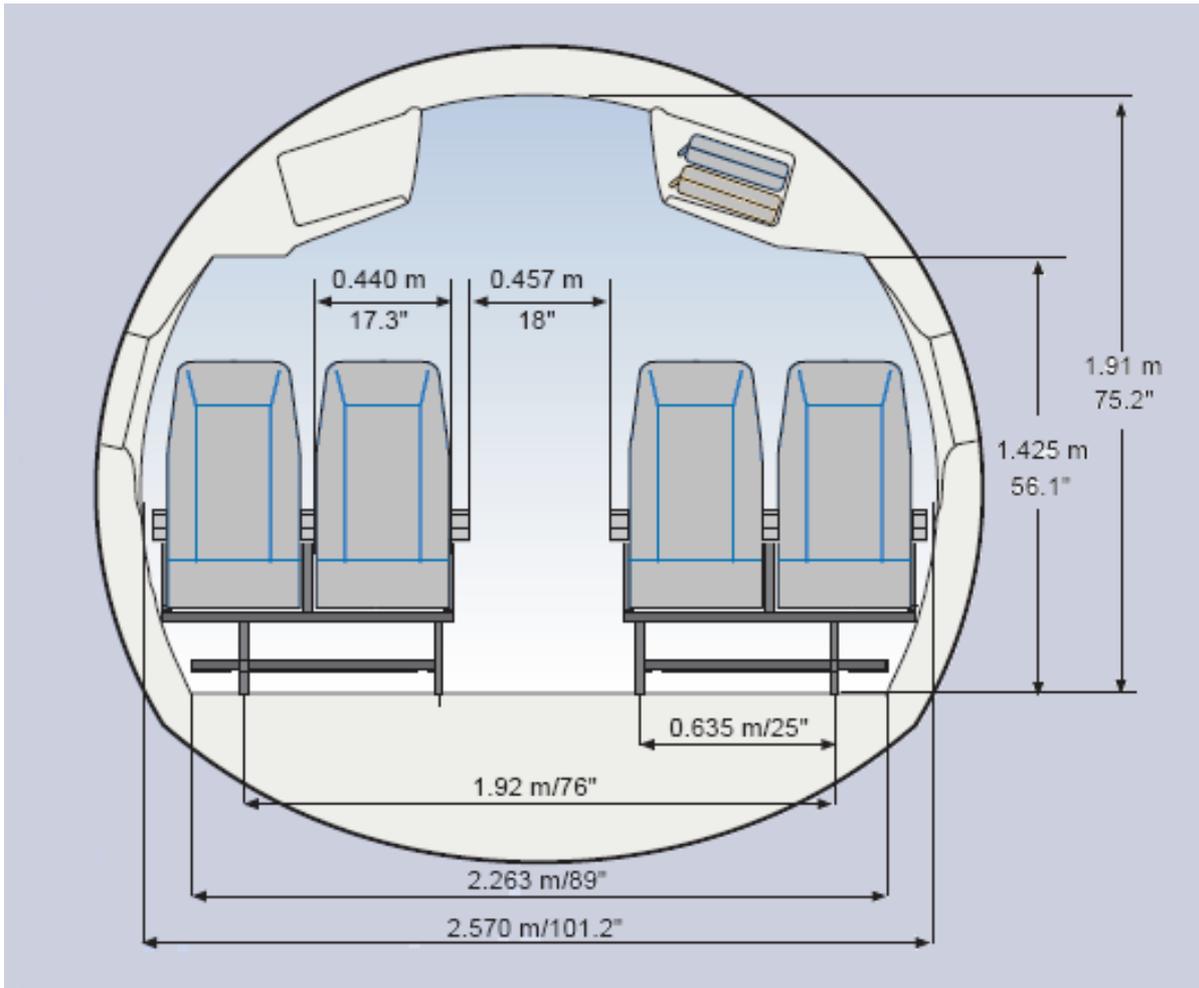
$$\text{Seat width}=17.3 \text{ in} \quad (4.1.3)$$

$$\text{Armrest width}=2 \text{ in} \quad (4.1.4)$$

These dates provide us a bench width of *40.6 inches*.

The number of chairs and aisles are the same as the ATR 72-500 which is now in use.

The diameter of the fuselage is another parameter which needs to be designed. If we take into account a distance of 0.025 meters between the seat and the wall, the inner diameter of the fuselage becomes:



**Fig. 4.2** Cross Section view and dimensions (ATR 2008)

$$d_{F,I} = (40.6 \times 2 + 18) \text{in} \cdot 0.0254 \frac{\text{m}}{\text{in}} + 2 \times 0.025 \text{m} = 2.57 \text{m} \quad (4.1.5)$$

The outer diameter of the fuselage can be calculated with an empirical formula, after **Markwardt 1998**:

$$d_{F,O} = d_F = 0.048 \text{m} + 1.045 \cdot d_{F,I} = 2.77 \text{m} \quad (4.1.6)$$

According to **Raymer 2006** we can choose a *74.5 inches* cabin height and a *0.55 meters* floor lowering.

**Schmitt 1998** gives us the formula for the floor deck thickness; according to the author, we can approximate it in this way:

$$f_{th} = 0.035 \cdot d_F = 0.096 \cong 0.1 \text{m} \quad (4.1.7)$$

## 4.2 Cabin Layout

In the first paragraph we have decided to take into account a number of 70 passengers accommodated in 4 rows and 1 aisle in between.

An important aspect concerning the comfort of the passengers is related to the distance between the chairs. This distance varies with the types of classes. The business or first class passengers should have more comfort; this fact is translated into more space: “pay more to get more” has become a common idea among market demands.

In our design study we consider a single class, the tourist class, and a larger number of passengers.

According to **Raymer 2006** a distance of 31” is appropriate for this class.

Where an emergency exit is present, the seats will be so arranged that an easy escape to be made possible.

When we calculate the length of the cabin, we must consider a factor of 1.1 meters (**Raymer 2006**), which comes from statistics:

$$k_{CABIN} = 1.1m \quad (4.2.1)$$

$$l_{CABIN} = k_{CABIN} \cdot \frac{n_{PAX}}{n_{SA}} \quad (4.2.2)$$

$n_{SA}$  represents the number of seats abreast; we obtain the following results:

$$l_{CABIN} = 1.1 \cdot \frac{70}{4} = 19.25m \quad (4.2.3)$$

The length of the fuselage can be calculated according to **Schmitt 1998** with the next formula:

$$l_F = l_{CABIN} + 1.4 \cdot d_F + 4m = 27.13m \quad (4.2.4)$$

Having the length and the fuselage diameter, we can obtain the true slenderness of the aircraft, by dividing these two values:

$$\lambda_F = 9.79 \cong 10 \quad (4.2.5)$$

**Schmitt 1998** indicates how the nose length and tail length should be calculated:

$$l_{BUG} = 1.4 \cdot d_F = 3.88m \quad (4.2.6)$$

$$l_{HECK} = 3 \cdot d_F = 8.31m \quad (4.2.7)$$

$$\phi = \arctan \frac{1}{3} = 18.43^\circ \quad (4.2.8)$$

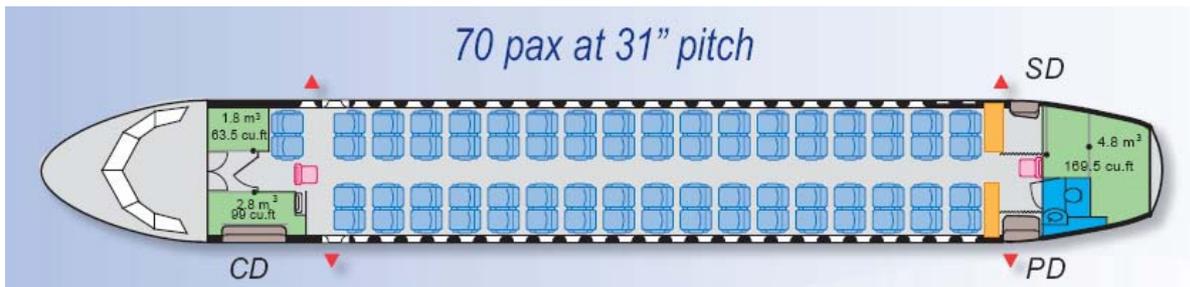
Afterwards we can define the surface of the cabin:

$$S_{CABIN} = l_{CABIN} \cdot d_{F,I} = 49.47m \quad (4.2.9)$$

This surface must meet the requirements and therefore we have to check if the space is sufficient for sustaining the maximum payload.

According to the cabin standards (**Schmitt 1998**) for a number of 70 passengers 2 Toilets are required, with a surface of:

$$S_{LAVATORY} = 1.1m \quad (4.2.10)$$



**Fig. 4.3** Seating configuration and cargo compartments (front and rear) (ATR 2008)

In order to define the galley surface, a factor must be defined (**Marckward 1998**):

$$k_{GALLEY} = 16m^2 \quad (4.2.11)$$

$$S_{GALLEY} = k_{GALLEY} \cdot \frac{n_{PAX}}{1000} + \frac{1}{2}m^2 = 1.62m^2 \quad (4.2.12)$$

The wardrobe area should also meet comfort requirements; according to **Marckward 1998**:

$$S_{WARDROBE} = 0.03m^2 \cdot \left(1 - 3 \cdot \frac{n_{AISLE}}{n_{PAX}}\right) \cdot n_{PAX} = 2.01m^2 \quad (4.2.13)$$

The aisle surface of the floor can also be calculated with the formula:

$$S_{AISLE} = 18in \cdot 0.0254 \frac{m}{in} \cdot l_{CABIN} = 8.80m^2 \quad (4.2.14)$$

where 18'' represent the aisle width.

The surface of the seats is:

$$S_{SEAT} = (40.6 \times 2)in \cdot 0.0254 \frac{m}{in} \cdot 31in \cdot 0.0254 \frac{m}{in} \cdot \frac{n_{PAX}}{n_{SA}} = 28.42m^2 \quad (4.2.15)$$

where 31'' represent the seat pitch.

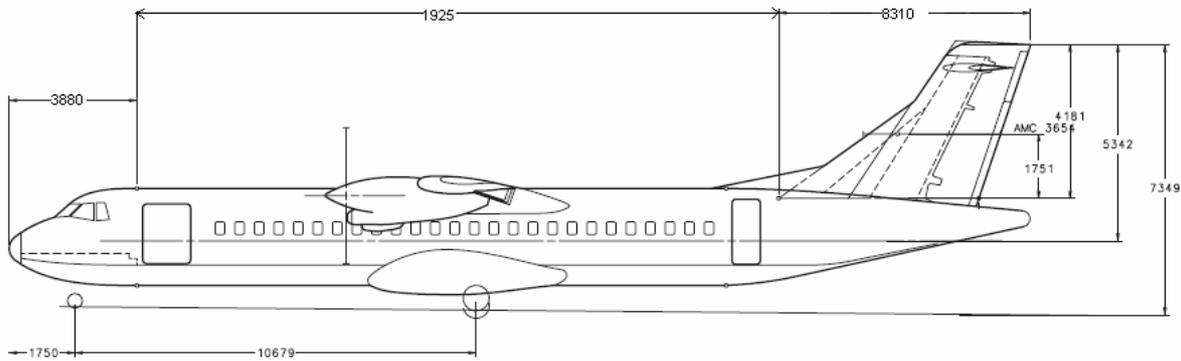
An additional surface needs to be calculated, in order that we take into account the seats near the emergency exits, for example:

$$S_{ADDITIONAL} = ((n_{AISLE} + 1) \cdot 40 + 30 + 9)in \cdot 0.0254 \frac{m}{in} \cdot (d_{F.I} - 20in \cdot 0.0254 \frac{m}{in}) = 6.39m^2 \quad (4.2.16)$$

After summing all these parameters we obtain the required surface, and we can check if the cabin surface is sufficient:

$$S_{CABIN,needed} = 2 \cdot S_{LAVATORY} + S_{GALLEY} + S_{WARDROBE} + S_{AISLE} + S_{SEAT} + S_{ADDITIONAL} = 49.44m^2 \quad (4.2.17)$$

The value we have obtained is very close to the one from (4.2.9).



**Fig. 4.4** Lateral view and dimensions

### 4.3 Cargo compartments

The maximum payload for a passenger we consider it to be 11.05 kg; the medium weight of a passenger is chosen to be 79.4 kg (**Loftin 1980**).

$$m_{BAGGAGE} / n_{PAX} = 11.05kg \quad (4.3.1)$$

$$m_{PAX} / n_{PAX} = 79.4kg \quad (4.3.2)$$

For a number of 70 passengers the total baggage weight is:

$$m_{BAGGAGE} = \frac{m_{BAGGAGE}}{n_{PAX}} \cdot \frac{m_{PAX}}{n_{PAX}} = 773.5kg \quad (4.3.3)$$

**Torenbeek 1988** gives us the approximation for the baggage density  $\rho_{BAGGAGE} = 170kg / m^3$  and cargo density  $\rho_{CARGO} = 160kg / m^3$ . In this way we can obtain the necessary volume for carrying payload:

$$V_{BAGGAGE} = m_{BAGGAGE} / \rho_{BAGGAGE} = 4.55m^3 \quad (4.3.4)$$

$$V_{CARGO} = m_{CARGO} / \rho_{CARGO} = 9.49m^3 \quad (4.3.5)$$

For each passenger a hand baggage volume of 0.05 cubic meters is required; this implies a total volume of 3.5 cubic meters for the total number of 70 passengers.

The total necessary volume has to be greater or at least equal to the minimum volume as we can see in the next formula:

$$V_{OVERHEAD\_STOWAGE} = 3.5m^3 \quad (4.3.6)$$

$$V_{CARGO\_COMPARTMENT} \geq (V_{BAGGAGE} + V_{CARGO}) - V_{OVERHEAD\_STOWAGE} = 10.54m^3 \quad (4.3.7)$$

The final value for the cargo compartment volume is calculated with the help of  $k_{CARGO\_COMPARTMENT} = 0.35$ , which is a constant that comes from statistics, the surface is calculated as  $S_{CARGO\_COMPARTMENT} = 1.26m^2$ .

$$V_{CARGO\_COMPARTMENT} = l_F \cdot S_{CARGO\_COMPARTMENT} \cdot k_{CARGO\_COMPARTMENT} = 12.21m^3 \quad (4.3.8)$$

In the Preliminary Sizing sheet we have considered the cargo compartment being empty, but in the same time a larger weight per passenger (95kg)

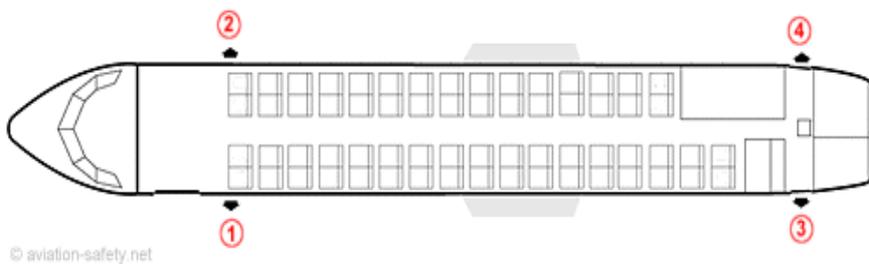
## 4.4 Emergency Exits

According to **CS 25-809** there are 2 types of emergency exits that are required for a number of 40 to 79 passengers: type I and III for each side of the fuselage. At least one of them has to

be above the waterline and at least with the dimensions of type III. Flight crew exits have to be located in the flight crew area. Such exits shall be of sufficient size and so located as to permit rapid evacuation by the crew. This is also valid for the passenger emergency exits.

**Type I** is a floor level exit with a rectangular opening of not less than 24 inches (61cm) wide by 48 inches (1.219 m) high, with corner radii not greater than one-third the width of the exit.

**Type III** is a rectangular opening of not less than 20 inches (51cm) wide by 36 inches (91cm) high, with corner radii not greater than one-third the width of the exit, and with a step-up inside the airplane of not more than 20 inches (51cm). If we locate the exit over the wing, the step-down outside the airplane may not exceed 27 inches (69cm).



- ① emergency exit 0.91m x 0.51m
- ② emergency exit 0.91m x 0.51m
- ③ pax door 1.75m x 0.82m
- ④ service door 1.22m x 0.61m

**Fig. 4.5** Emergency exit locations and dimensions (sample seating configuration) ([www.aviation-safety.net](http://www.aviation-safety.net) 2008)

## 4.5 Water line

As it is referred in paragraph 4.4, in the case of an emergency ditching, the doorsteps of the emergency exits must be above the waterline. This implies the necessity of calculating where the waterline lies.

We consider the situation when the ditching happens after take-off; that means the total weight of the aircraft is approximately equal to the maximum take-off weight:  $m_{MTO} = 22800kg$ . For a water density of:  $\rho_{WATER} = 1000kg / m^3$  we find the replaced volume of:  $V_{WATER} = 22.8m^3$ .

The doorsteps are situated at a height of  $h_{DOOR} = 1.07m$  above the lower part of the fuselage.

The opening angle, depending on  $h_{DOOR}$ , is calculated at  $\theta = 153.7^\circ$ . With this dates we can calculate a surface of a circle segment of the fuselage cross section:

$$S_{Segment} = \frac{h_{DOOR}}{6 \cdot d_F \cdot \sin \theta} \cdot (3 \cdot h_{DOOR}^2 + 4 \cdot d_F^2 \cdot \sin^2 \theta) = 2.35 m^2 \quad (4.4.1)$$

The cylindrical part of the fuselage has a length of 14.94 meters; having the surface implies we can calculate the volume of the fuselage:  $V_{CYL} = S_{segment} \cdot l_{CYL} = 35.17 m^3$ .

The nose can be approximated with a cone; the length of the cone is 3.4 meters, so the volume will be:

$$V_{CONE} = l_{nose} \cdot \frac{h_{DOOR}^2 \cdot \pi}{3} = 4.07 m^3 \quad (4.4.2)$$

The total volume is:

$$V_{TOT} = V_{CYL} + V_{NOSE} = 39.24 m^3 \geq V_{WATER} = 22.8 m^3 \quad (4.4.3)$$

The results show that the **CS 25.807** requirements are fulfilled.

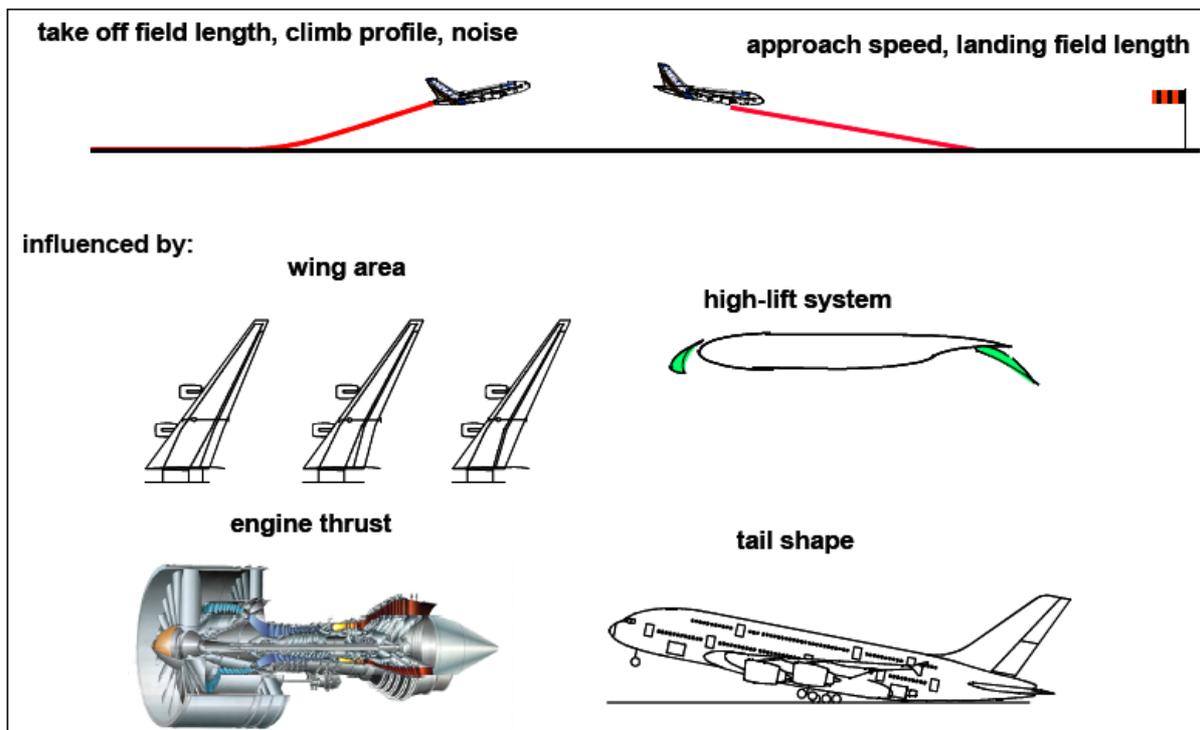
## 5 Wing Layout

The redesign of the wing has to be driven by all requirements categories: cruise requirements, airport requirements-take-off and landing, taxi and terminal, operation and costs requirements. Also, when redesigning a typically aircraft, the approach has to include considerations related to the design of the aircraft family it is part of.

Even if the main purpose of the wing is to produce lift in order to carry the payload, the wing design plays an important role in satisfying the requirements earlier mentioned:

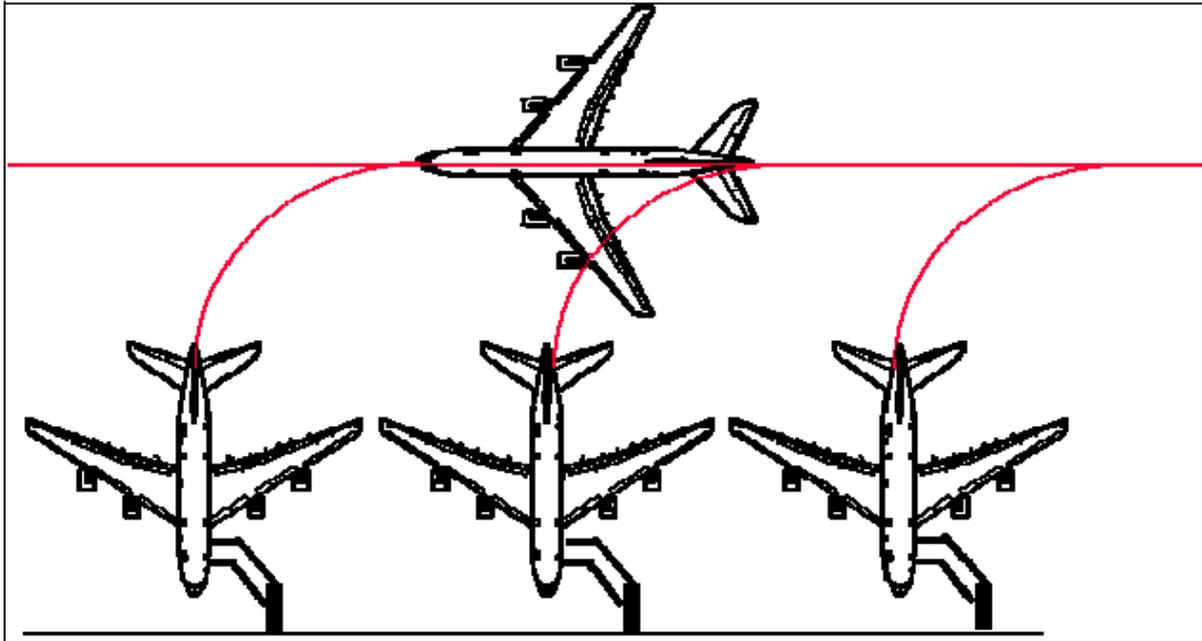
- reach cruise level within geographical or political boundaries
- reach cruise level within given time
- sufficient altitude at beginning of cruise on top of weather and obstacles
- sufficient climb capability to reach next flight level (for adaptation of cruise altitude for best efficiency).

The take-off and landing field length are influenced also by the wing design and high lift systems; other parameters like engine thrust and tail shape are also important, as we can see below. These parameters have to meet the airports infrastructures.



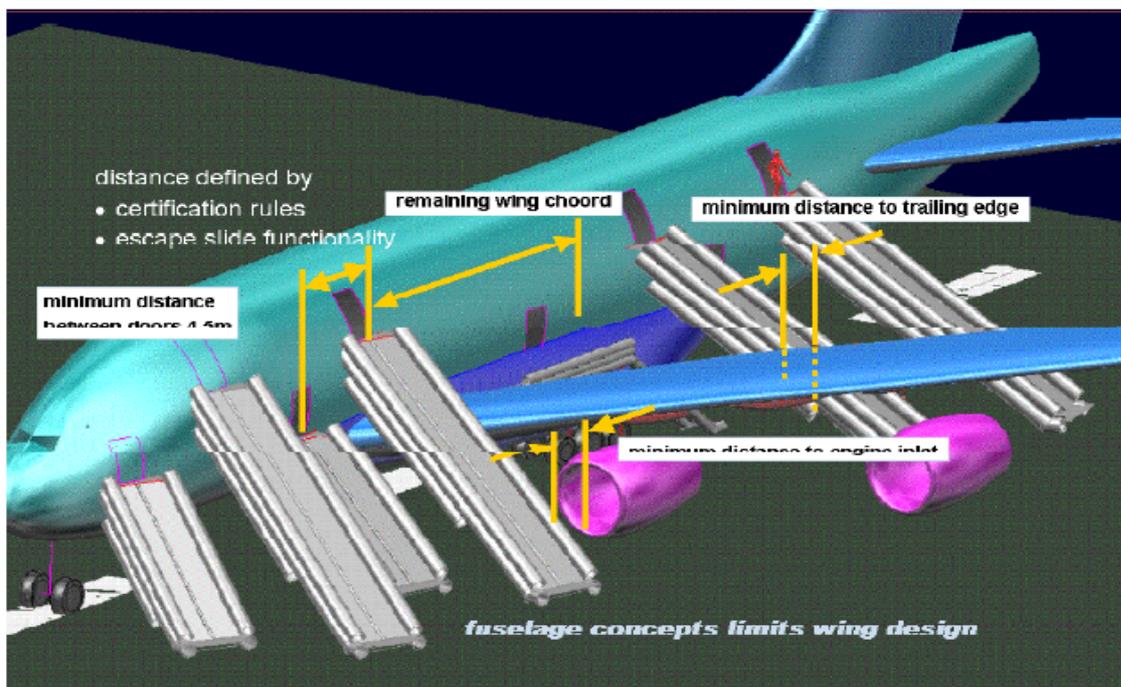
**Fig. 5.1** Aircraft parameters influencing take-off and landing (Böttger 2008)

When the aircraft is being operated on an airport, there are limitations regarding the wing span: the short range aircrafts have to fit in an available space of 36 m at the passenger bridge terminal.



**Fig. 5.2** Aircraft span limited by airport requirements (Scholz 2007)

Regarding certification requirements, we have to consider the necessary space for a secure inflation of the slides at the passengers' doors. The available space for wing and engine becomes limited. (see Fig. 5.3)



**Fig. 5.3** The fuselage concepts limits the wing design (Böttger 2008)

Another aspect which has to be taken into account is the integration of the engines and servicing capability and accessibility of the aircraft.

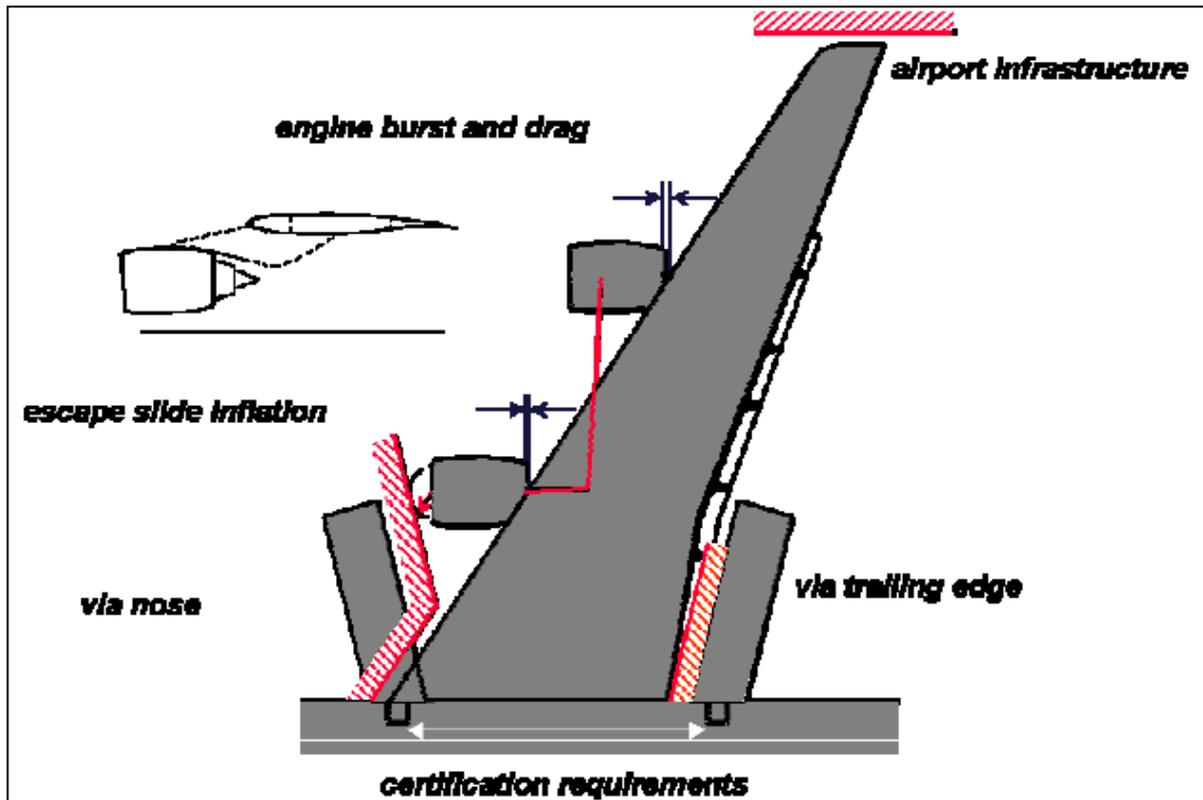


Fig. 5.4 Boundary conditions inflict strong limitations on wing optimization (Böttger 2008)

According to these requirements we can settle the main parameters of the wing already.

## 5.1 Wing Parameters

In chapter 3 the wing was described with parameters like wing area  $S_w$  and wing aspect ratio  $A_w$ . Further we will need to define other sets of parameters like: wing section or planform.

The information obtained in the preliminary sizing can be summed up in the following table:

Wing area:	$S_w = 62.2m^2$
Wing span:	$b = \sqrt{A_w \cdot S_w} = 27.32m$
Lift Coefficient, cruise speed	$C_L = 0.813$
Cruise speed	$V_{CR} = 141.94m/s$ or $M_{CR} = 0.443$

These parameters have a direct influence on the wing efficiency; the wing area affects lift capability, high lift system or fuel tank volume. The preliminary sizing also provides values for wing loadings. In a general design of a wing, these values are first approximated via statistics, and afterwards improved.

The relative position of the wing towards the fuselage has to take into account the diameter of the propeller; accordingly we can only choose a high wing position (see more details in the next paragraph).

For continuing the design of the wing, other parameters have to be determined (see Definitions paragraph):

Taper ratio	$\lambda_w$
Sweep angle	$\varphi_{25,w}$
Thickness ratio	$(t/c)_w$
Airfoil	
Dihedral angle	$\nu_w$
Incidence angle	$i_w$
Wing twist	$\varepsilon_t$

In order to determine these parameters, some other values are also necessary:

Take-off/Landing/Cruise	Lift coefficients and Lift-to-Drag ratio
Take-off/Landing	High lift systems, Lift curve slope $dC_L/d\alpha$
Overall flight	Fuel tank volume
	Flight characteristics, stalling behavior, flight in turbulence
	Landing gear actuation and stowage
	Wing mass
	Production costs

## 5.2 Position of the wing

For aircrafts which fly at lower speeds, the possibility to choose a braced wing exists. However the advantage of a 30% reduction of the weight must be balanced with a considerable increase of drag, due to the form of the wing and interference. For this reason we choose the standard configuration of a cantilevered wing.

The type of the aircraft we are redesigning must be build for regional transportation of passengers. When choosing one of the three variants: high wing, mid wing or low wing, we have to consider some important aspects. A high wing position gives the passengers a good visibility, but it becomes more difficult to integrate the landing gear into the wing; the landing gear becomes longer and accordingly heavier. A mid wing gives the best values for interference drag, but it cannot be used, as it is not able to provide a continuous free cross section for the cabin or the hold. Also the high positioned wing has a stabilizing effect around the rolling axis, but a destabilizing effect on Dutch roll.

When balancing these effects we decide to choose a high positioned wing. If we do not choose the position of the landing gear to be on the wing, due to its high weight, then, in order to insert the landing gear into the fuselage, fairings are required. This gives a relatively high drag, but this will be compensated by the wing geometry and characteristics, and also, because the aircraft flies at smaller speeds, the effect of the fairings is acceptable.

A summary of the effects of the wing position is underlined in Table 5.1.

**Table 5.1** Summary evaluation of wing position in relation to the fuselage (**Scholz 2008**)

	High wing	Mid wing	Low wing
Interference drag	average	low	high
Stability around the longitudinal axis	stable	neutral	unstable (requires dihedral for stability)
Visibility from cabin and cocpit	good	average	poor
Landing gear: on the wing on the fuselage	long and heavy high drag	- -	short and light -
Loading	easy	average	requires steps and loading aids

### 5.3 Design Mach Number

For typical transportation aircrafts (jets mostly, which fly at higher speeds) the term  $M_{DD}$  (drag divergence Mach number) is used as a design parameter for the wing. This parameter is defined by means of critical Mach number  $M_{crit}$ ; when the speed increases locally on the airfoil until  $M=1$ , the  $M_{crit}$  is reached; the consequence is an increase of the drag with the so called wave drag. The  $M_{DD}$  factor is the Mach number where the wave drag reaches the value of 0.002 (according to the definition applied by Airbus and Boeing). At  $M_{crit}$  the wave drag is zero so the  $M_{DD}$  is greater than  $M_{crit}$ , depending on the airfoil section. According to **Raymer 2006**:  $M_{DD}=0.08M_{crit}$ .

The supercritical airfoils were developed to reduce the wave drag, minimize the shock waves and increase the critical Mach number, all these without affecting in an unacceptable way the flight at low speeds.

In the case of the ATR 72, which flies at lower speeds, we may assume that the design Mach number is equal to the cruise Mach number, which gives  $M_{DD} = M_{CR} = 0.44$ .

## 5.4 Wing Sweep

The drag divergence Mach number increases with increasing sweep angle. The primary reason of using swept wings is exactly that to reduce the effects of transonic or supersonic flow over the wing. The reason is this one: for a swept wing, the distance from the leading edge to trailing edge is shorter when measured perpendicular to the leading edge; the shock wave is formed also by the air speed flowing perpendicular to the LE. The distance is shorter, so the velocity is smaller, so the shock waves do not form any more. Thus we obtain an increase of the critical Mach number.

For the regional turboprop aircrafts, a larger wing sweep would be unnecessary, and it would only increase the weight of the wing. A lower wing sweep or even a zero value for the sweep angle is more suitable.

The lift coefficient also drops for wings with airfoils which have a less sharply defined leading edge (as it is the case for the AT 72; sharp LE airfoils are used for high speed aircrafts). The next relation is valid:

$$C_{L,max,swept} = C_{L,max,unswept} \cdot \cos \varphi_{25} \quad (5.4.1)$$

The smaller the lift coefficient, the greater the wing surface has to be; this gives the weight increase, as we said before. Also, forward swept wings become heavier than the aft swept wings, due to the required stiffening of the forward swept wing which is more exposed to divergence. However, the use of new composite materials minimizes the danger of divergence. The wing sweep influences also the stall behavior of the wing. A safe precaution for a wing is to insure that first the inner wing stalls, and afterwards the outer wing, so that the ailerons remain functional. This happens for forward swept wings, due to a better lift distribution for this case. In the same time, for an aft swept wing, the so called *tip stall* occurs (the boundary layer separates itself at the wing tip); *stall fences* are used for aft swept wings to prevent this. Another consequence of the tip stall, which makes the wing stall even faster, is a nose up moment induced by the tips, which are situated behind the center of gravity. The pitching moment changes and the pilot has to correct it promptly.

Swept wings also have to allow an appropriate installation of the landing gear, near the center of gravity of the aircraft; this sometimes requires additional areas for the wing.

Another aspect concerning sweep angle is related to the lift curve slope, which becomes smaller if the sweep angle increases. This implies a smaller visibility for approach (small curve slope implies a greater pitch attitude) and a larger angle for take-off.

A positive argument for aft swept wings is a better stability around the longitudinal axis. However aft swept wings have a destabilizing effect on the Dutch roll, effect which might be overcome by an electronic yaw damper.

For forward swept angles a positive dihedral is required in order to compensate the instability around the longitudinal axis, produced by the sweep angle.

In order to achieve an optimum for the wing sweep, an aircraft can be build up with variable wing sweep. This improves the take-off and landing characteristics, minimal drag for cruise and an optimum E during the flight. However, the mechanism is heavy and expensive and modifies the aerodynamics of the wing, fact which requires larger empennages.

**Raymer 2006** suggests another approach: the use of ‘oblique wings’, which means one wing is aft swept and the other forward. The main disadvantage would be related to the difficulty to control the aircraft, but this can be overcome by a computerized flight control system. A significant advantage would be a smaller wave drag due to a better volume distribution (argument valid for higher speed aircrafts).

A better view over all these effects is shown in the Table 5.2 and Table 5.3.

**Table 5.2** Evaluation of wing sweep  $\varphi_{25}$  (**Scholz 2008**)

	Large sweep	Small sweep
Critical Mach number	large	small
Maximum lift coefficient	small	large
Lift curve slope	small	large
Pitch attitude angle during approach	large (poor visibility from cockpit)	small (good visibility from cockpit)
Flight in turbulent air	smooth	bumpy
Required angle for rotation	large	small
Integration of landing gear	difficult	minor problems
Wing mass	large	small

**Table 5.3** Evaluation of type of wing sweep (Scholz 2008)

	Forward swept	No sweep	Aft sweep
Risk: tip stall	none	none	large
Risk: pitch up	minor	none	large
Maximum lift coefficient	small	large	very small
Risk: one-sided stall	minor	very minor	large
Risk: divergent wing deflection	yes	no	no
Wing mass	very large	small	large
Stability around longitudinal axis	unstable (requires positive dihedral)	neutral	stable

For the ATR 72, as shown in the precedent paragraphs, a small forward sweep angle is most suitable; the aircraft design can be optimized so that disadvantages shown in Table 5.3 are overcome by appropriate means, so that the requirements are fulfilled. Let us remember that, among other arguments, the efficiency of the airplane on short range is the main aspect to take into account. A 25% wing sweep of 3 degrees is suitable in the moment for the designing process.

## 5.5 Wing Aspect Ratio

The aspect ratio was determined in the Preliminary Sizing chapter and has the value:

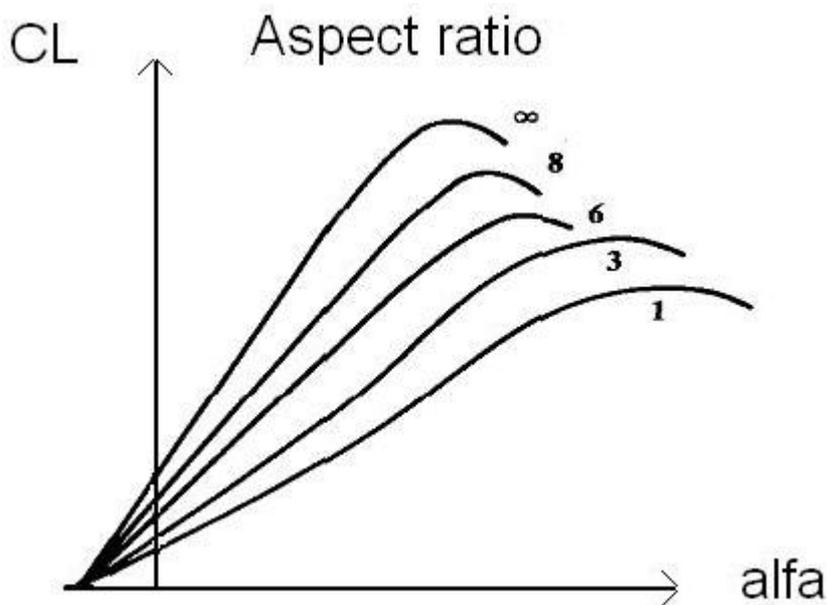
$$A = \frac{b^2}{S_w} = \frac{27.32^2}{62.237} = 11.99 \approx 12. \text{ This is an important parameter for the wing, as we earlier discussed, because it influences other design parameters.}$$

When looking at the induced drag formula:  $C_L^2 / (\pi \cdot A \cdot e)$  we can conclude that the higher aspect ratio the airplane has, the smaller the induced drag becomes. In the same time the wing mass increases. Also, for the same wing surface, a higher aspect ratio requires a larger wing span. The wing span becomes a requirement when operating the aircraft in an airport (see first paragraph).

An inverse proportionality is also valid for the drag due to lift. At the wing tip, the pressure difference between lower and upper surface of the wing is lower, so that the lift is lower; a circular movement of the air is induced (a vortex), which increases the drag. If the aspect ratio is higher, for a constant wing surface, the tip is situated further than for a wing with a lower aspect ratio, which means a smaller surface gets affected by the vortex. On the other hand a lower  $A$  wing will stall at a higher angle of attack than a wing with a higher aspect ratio. One reason why the empennages usually have a smaller aspect ratio is that the airplane can still be controlled, even if the wing already stalls.

The tank volume, as we will later show, decreases if A increases:  $V_{\text{tank}} \approx \frac{1}{\sqrt{A}}$ .

The aspect ratio also influences the lift curve slope,  $C_{L,\alpha} = \frac{dC_L}{d\alpha}$ , proportionally: if A is small, the lift slope decreases. This has consequences, as discussed in paragraph 5.5, on the visibility from cockpit, at take-off and landing: if  $C_{L,\alpha}$  is small, then the angle of attack is large and the visibility is poor. In the same time, if  $C_{L,\alpha}$  is greater due to A greater, then the aircraft becomes more sensitive to vertical gusts; also a larger angle for rotating the aircraft at take-off is required.



**Fig. 5.5** Effect of aspect ratio on lift curve slope (Raymer 2006)

All the conclusions can be summarized in the next table.

**Table 5.4** Evaluation of wing aspect ratio (Scholz 2008)

	Large aspect ratio	Small aspect ratio
Induced drag	small	large
Lift to drag ratio	large	small
Lift curve slope	large	small
Pitch attitude angle during approach	small (good visibility from cockpit)	large (poor visibility from cockpit)
Flight in turbulent air	bumpy	smooth
Required angle for rotation	small	large
Wing mass	large	small
Tank volume	small	large
Span (for constant wing surface)	large	small

## 5.6 Taper Ratio

The taper ratio  $\lambda = c_t / c_r$  is primarily set to achieve a semielliptical lift distribution in the spanwise direction. We know that the smallest induced drag is obtained when the lift distribution has an elliptical form. The usual values lie between 0.1 and 0.6, the higher values being valid for unswept aircrafts with higher aspect ratio, as it is the case of the ATR 72. A low value for the taper ratio is structurally beneficial. In the same time it gives a high local lift loading at the tip of the wing, which produces a tip stall tendency. The ailerons also input a limit to the tip chord. According to **Howe 2000** the taper ratio should respect the next inequality:

$$\lambda \geq 0.2 A^{1/4} \cos^2 \varphi_{1/4} = 0.2 \cdot 12^{1/4} \cos^2(3^\circ \cdot \pi / 180^\circ) = 0.371 \quad (5.6.1)$$

**Torenbeek 1986** gives a different first approximation for the optimum taper ratio for the smallest induced drag. This is valid for positive swept wings:

$$\lambda_{opt} = 0.45 \cdot e^{-0.036 \cdot \varphi_{2s}} = 0.45 \cdot e^{-0.036 \cdot 3^\circ} = 0.404 \quad (5.6.2)$$

For a taper ratio of  $\lambda = 0.45$  the lift distribution, represented by the function  $c \cdot C_L = \text{func}\left(\frac{y}{b/2}\right)$ , is very close to elliptical (the drag is only 1% higher) if the wing is unswept.

If  $\lambda$  is small, the pressure center moves towards the inner wing, the bending moment becomes correspondingly smaller and the mass of the wing can become also smaller.

The wing tip chord has to be big enough to ensure the integration of the ailerons mechanism. For the ATR 72, a double tapered wing, with an inner wing of  $\lambda = 1$  and an outer wing with a small sweep angle would achieve an optimum design. On the inner rectangular wing we set the turboprop engines; the production costs are minimized, and the aerodynamic efficiency is achieved. The taper ratio for the outer wing has to be larger than 0.45, due to the  $3^\circ$  sweep angle we chose before. According to Howe the upper limit is 0.6. In order to prevent the tip stall and for a good efficiency of the ailerons we choose a value as large as possible; the value of 0.59 is suitable. A summary of the taper ratio effects can be seen in the next table.

**Table 5.5** Taper ratio evaluation (Scholz 2008)

	Small taper ratio	Large taper ratio
Tip stall	bad	good
Tank volume at $t/c=\text{const}$	large (optimum for $\lambda = 0$ )	small
Production costs	-	advantage only for $\lambda = 1$
Installation space for aileron	small	large
Wing mass	small	large
Induced drag in the case of	large sweep: small	small sweep: small

## 5.7 Geometry of the Double Tapered Wing

In the last paragraph we concluded over the next two values:  $\lambda_i = 1$  and  $\lambda_o = 0.59$ . For a double tapered wing the kink relative span position must be defined; (the kink is the imaginary line between the inner and the outer wing).

In the preliminary sizing chapter we calculated the values for wing span, wing surface, and consequently the aspect ratio. The next relation is valid:

$$A = \frac{b^2}{S} = \frac{2b}{c_r[(1-\lambda) \cdot \eta_k + \lambda_i + \lambda]} \text{ with } \eta_k = \frac{y_k}{b/2} \quad (5.6.3)$$

We assumed that the engines will be positioned on the inner wing; in order to ensure a minimum necessary space to integrate the engines, the  $y_k$  coordinate must be approximated with the value of:

$$y_k = \frac{d_f}{2} + d_e + MF \quad (5.6.4)$$

where MF is a margin factor at least equal to the span of the propeller and  $d_e$  is the engine diameter.

The formula gives a result equal to 4.73 meters. This gives a value of 0.346 for the relative span position of the kink:  $\eta_k = 0.346$ .

We can use now formula (5.6.3) to extract the root chord and afterwards the tip chord:

$$c_r = \frac{2b}{A[(1-\lambda)\eta_k + \lambda_i + \lambda]} = \frac{2 \cdot 27.32}{12[(1-0.59) \cdot 0.346 + 1 + 0.59]} = 2.626m \quad (5.6.5)$$

$$c_i = \lambda c_r = 0.59 \cdot 2.626 = 1.556m \quad (5.6.6)$$

The mean aerodynamic chord can be evaluated by means of the aerodynamic chord of each inner and outer wing, and the corresponding surfaces:

$$c_{MAC} = \frac{c_{MAC,i} \cdot S_i + c_{MAC,o} \cdot S_o}{S} \quad (5.6.7)$$

$$S = S_i + S_o = \frac{b^2}{A} = \frac{b}{2} c_r [(1 - \lambda)\eta_k + \lambda_i + \lambda] \quad (5.6.8)$$

The surface we can either calculate with (5.6.8) formula, or use the value from the preliminary sizing. The formula (5.6.8) gives the following result:

$$S = \frac{27.32}{2} \cdot 2.626 [(1 - 0.59) \cdot 0.346 + 1 + 0.59] = 62.187m^2 \quad (5.6.9)$$

The inner surface is a rectangular surface, so the next formula is valid:

$$S_i = y_k \cdot c_r = 4.73 \cdot 2.626 = 12.42m^2 \quad (5.6.10)$$

The outer surface results as a difference between the two:

$$S_o = S - S_i = 62.187 - 12.42 = 49.767m^2 \quad (5.6.11)$$

The general expression for the mean aerodynamic chord is showed in formula (5.6.12):

$$c_{MAC} = \frac{2}{S} \int_0^{b/2} c^2 dy \quad (5.6.12)$$

In order to apply formula (5.6.7) we need to evaluate the mean aerodynamic chord for both inner and outer wing. Because the inner wing is rectangular, with  $\lambda = 1$ , we can consider the wing to be similar to a simple tapered wing, but having the extra  $S_i$  surface. The formula for the  $c_{MAC}$  for a simple tapered wing is as follows:

$$c_{MAC} = \frac{2}{3} \cdot \frac{1 + \lambda + \lambda^2}{1 + \lambda} \quad (5.6.13)$$

The results for both inner and outer wings are showed below:

$$c_{MAC,i} = c_r = 2.626m \quad (5.6.14)$$

$$c_{MAC,o} = \frac{2}{3} \cdot 2.626 \frac{1+0.59+0.59^2}{1+0.59} = 2.1368m \quad (5.6.15)$$

The mean aerodynamic chord for the wing of our design becomes:

$$c_{MAC} = \frac{c_{MAC,i} \cdot S_i + c_{MAC,o} \cdot S_o}{S} = \frac{2.626 \cdot 12.42 + 2.1368 \cdot 49.767}{62.187} = 2.2345m \quad (5.6.16)$$

The position of the aerodynamic chord can also be determined; it is also necessary first to evaluate the position of each mean chord for each inner and outer wing:

$$y_{MAC} = \frac{y_{MAC,i} \cdot S_i + (y_k + y_{MAC,o}) \cdot S_o}{S_i + S_o} \quad (5.6.17)$$

The general formula for  $y_{MAC}$  is given by:

$$y_{MAC} = \frac{2}{S} \int_0^{b/2} c \cdot y dy \quad (5.6.18)$$

It is again possible to use the formulas from the simple tapered wing:

$$\frac{y_{MAC}}{b/2} = \frac{1 - \frac{c_{MAC}}{c_r}}{1 - \lambda} = \frac{1}{3} \left( \frac{1 + 2\lambda}{1 + \lambda} \right) \quad (5.6.19)$$

The position of the inner mean aerodynamic chord is easy to estimate:

$$y_{MAC,i} = \frac{y_k}{2} = \frac{4.73}{2} = 2.365 \quad (5.6.20)$$

The position of the outer aerodynamic chord we estimated with formula (5.6.19):

$$y_{MAC,o} = \frac{1}{3} \left( \frac{1 + 2 \cdot 0.59}{1 + 0.59} \right) = 0.4573 \quad (5.6.21)$$

The position of the mean aerodynamic chord of the wing when using (5.6.17) becomes:

$$y_{MAC} = \frac{2.365 \cdot 12.42 + (0.4573 + 4.73) \cdot 49.767}{62.187} = 4.6234 \quad (5.6.22)$$

## 5.8 Dihedral Angle

A positive dihedral angle leads to positive stability around the longitudinal axis. The moment produced by the dihedral in a turn produces a stable spiral mode and an unstable Dutch roll.

The stability around the longitudinal axis is in the same time sustained by an aft swept wing, as we discussed, and also by a high wing configuration. It appears it is no longer necessary to use the benefit of a positive dihedral if we decide over a high positioned wing in our design study. Another positive argument for our decision comes from statistics (**Scholz 2008**): moving the wing position by “one step” (e.g. from high to middle, from middle to low) achieves roughly as much as a  $3.5^\circ$  dihedral. As an observation, the combination of a high positioned wing with an aft sweep angle has to be counteracted by a negative dihedral. For a high unswept wing a small dihedral, between 0 and 2 degrees is possible, especially if clearance of engines and wing tips from the ground is necessary. But it is not the case of our design.

## 5.9 Wing Twist

In order to define the wing twist we need to explain first the notion of angle of incidence. The angle of incidence is not the same with angle of attack, according to English literature. It is defined as the angle between the chord line and a reference line of the fuselage, which is for example the cabin floor. For passenger aircrafts the cabin floor usually has to be horizontal in cruise flight so that the service to the passenger is easier to realize.

The geometrical wing twist is defined as the difference between the angle of incidence from tip and the angle of incidence from root. There is also an aerodynamic twist which takes into account the changes in the zero lift line along the span.

The twist angle is negative when the incidence angle of the chord decreases in the direction of the wing tip; that gives a lower lift distribution at the tip, thus closer to an elliptical one, which prevents the tip stall. A negative twist angle of  $-3^\circ$  is a proper data for the preliminary design, according to statistics. (**Scholz 2008**)

## 5.10 Wing Relative Thickness. Choosing the Airfoil

The relative thickness has an important effect on the maximum lift coefficient, also on drag, structural weight of the wing and stall characteristics. The drag increases with increased thickness. The structural weight varies approximately inversely with the square root of  $(t/c)$ . (The wing represents approximately 15% of the total empty mass; the thickness ratio would

increase the empty mass by about 6%). For thickness ratios up to 0.12- 0.14 the maximum lift coefficient increases with the thickness. Another obvious parameter influenced by the thickness is the fuel tank volume, which also increases with thickness.

According to **Howe 2000** for incompressible flow a thickness of 0.2 for the root chord gives acceptable drag; for the tip chord thickness the value is typically two-thirds of the value from the root thickness.

The estimation of the relative thickness meets the problem of choosing the optimum equation for best results. The thesis “Mach number, relative thickness, sweep and lift coefficient of the wing -An empirical investigation of parameters and equations” written by **Ciornei 2005** brings up different methods to estimate the thickness ratio and also ranks these methods. The equations we choose to estimate, which are on the first positions on the ranking, are given by **Torenbeek 1998**, **Howe 2000** and also using a nonlinear regression and *EXCEL Solver*.

The maximum relative thickness for an average spanwise position on the wing is given by **Torenbeek 1986** as follows:

$$(t/c) = 0.3 \cdot \cos \varphi_{25} \cdot \left[ \left( 1 - \left( \frac{5 + M_{DD,eff}^2}{5 + (k_M - 0.25 \cdot C_L)^2} \right)^{3.5} \right) \cdot \frac{\sqrt{1 - M_{DD,eff}^2}}{M_{DD,eff}^2} \right]^{2/3} \quad (5.10.1)$$

The position of the thickness determined with (5.10.1) is, according to **Jenkinson 1999**, given by the next formula:

$$t/c = \frac{3(t/c)_t + (t/c)_r}{4} \quad (5.10.2)$$

The values for  $k_M$  factor are as follows:

**Table 5.6** Typical values for  $k_M$  factor

$k_M=1.00$	For conventional airfoils; maximum $t/c$ at about 0.3c
$k_M=1.05$	High-speed (peaky) airfoils, 1960-1970 technology
$k_M=1.12-1.15$	Supercritical airfoils

The value  $M_{DD,eff}$  is an effective value:

$$M_{DD,eff} = M_{DD} \cdot \sqrt{\cos \varphi_{25}} \quad (5.10.3)$$

In the precedent paragraphs we settled the value  $M_{DD}$  as being equal to  $M_{CR}=0.443$  and the sweep angle  $\varphi_{25}=3^\circ$ . This gives a value very close to  $M_{CR}$ , due to the small sweep angle. Eventually the  $M_{DD,eff}=0.443$ .

The design lift coefficient which comes up in the equation was determined in the Preliminary Sizing chapter and has the value of  $C_L=0.8$ . This value is the value of the lift coefficient when the  $L/D$  ratio is maximum. This is obtained when the tangent to the polar is drawn from the origin.

The parameter  $k_M$  is being chosen to fit the airfoil data. According to Table 5.6 the value 1.00 should be good enough, but for airfoils with a maximum  $t/c$  situated at 30% of chord. The purpose of this paragraph is that to understand how the airfoil of our wing should look like. The position of the maximum thickness should be nevertheless positioned more close to the leading edge.

The results are summed up in the (5.10.4) relation; for the  $k_M$  we choose the value of 1.00, according to Table 5.6:

$$(t/c) = 0.3 \cdot \cos\left(3^\circ \cdot \frac{\pi}{180^\circ}\right) \cdot \left[ \left( 1 - \left( \frac{5 + 0.443^2}{5 + (k_M - 0.25 \cdot C_L)^2} \right)^{3.5} \right) \frac{\sqrt{1 - 0.443^2}}{0.443^2} \right]^{2/3} = 0.327 \quad (5.10.4)$$

The value we obtain with **Torenbeek 1986** is very high and it cannot fit with our design. The explanation is this one: our design case is that of a regional turboprop that flies at low speeds. If our aircraft would of flown at high speeds, having the characteristics we designed by now, then the necessary thickness would have to be 32% to prevent the shock waves from forming. This is of course not our case.

**Howe 2000** proposes an equation that sets a relation between the airfoil relative thickness, lift coefficient and design Mach number in a different way:

$$M_{DD,eff} = A_F - 0.1C_L - t/c \quad (5.10.5)$$

From the form of the formula we can understand that  $A_F$  is a coefficient which gives the value of the drag divergence Mach number for an airfoil with zero angle of attack (so zero lift coefficient) and zero relative thickness. We already settled the drag divergence Mach number<sup>1</sup> as being equal to the cruise Mach number. For usual airfoils Howe gives a value of 0.8-0.95 for  $A_F$  coefficient. The thickness results as follows:

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<sup>1</sup> This term does not make sense for an aircraft that flies at low speed, but we keep the author's term, having in mind in the same time that we refer at the design Mach number which is equal to cruise Mach number

$$(t/c) = A_F - 0.1C_L - M_{CR} = 0.8 - 0.1 \cdot 0.8 - 0.443 = 0.277 \quad (5.10.6)$$

The value we obtain is rather high, which means the results are also more suitable for aircrafts that fly at higher speeds.

A third option is the case of a nonlinear regression (**Ciornei 2005**). In this case the model of the equation is build as a nonlinear combination of parameters. For the curve fitting, the Power Family Equation was chosen to be used (as it is referred in **Ciornei 2005**), while taking into account the variation of relative thickness with  $M_{DD}$ ,  $\cos\varphi$ ,  $C_L$ ,  $k_M$  and a proportionality coefficient, called  $k_t$ . Other forms like Polynomial Family or Taylor Series Equations can also be used.

The form is as follows:

$$(t/c) = k_t \cdot M_{DD}^t \cdot \cos\varphi_{25}^u \cdot C_L^v \cdot k_M^w \quad (5.10.7)$$

Based on a large number of aircrafts, including aircrafts that fly at lowers speed, the nonlinear regression gives the following values for the coefficients (**Ciornei 2005**), values which give back good results and fit with the studied aircrafts:

$$\begin{aligned} k_t &= 0.127 \\ t &= -0.204 \\ u &= 0.573 \\ v &= 0.065 \\ w &= 0.556 \end{aligned} \quad (5.10.8)$$

For the  $k_M$  parameter, new values are fit in the case of the nonlinear regression. See Table 5.7:

**Table 5.7** Parameter  $k_M$  used for equation (5.10.7) from nonlinear regression (**Ciornei 2005**)

Parameter	Value
$k_M$ for conventional	0.921
$k_M$ for peaky	0.928
$k_M$ for older supercritical	1.017
$k_M$ for modern supercritical	0.932

The results obtained when applying the nonlinear regression equation are as follows:

$$(t/c) = k_t \cdot M_{DD}^t \cdot \cos\varphi_{25}^u \cdot C_L^v \cdot k_M^w = 0.127 \cdot 0.443^{-0.204} \cdot \cos(30^\circ \cdot \pi / 180^\circ)^{0.573} \cdot 0.8^{0.065} \cdot 0.921^{0.556} = 0.141 \quad (5.10.9)$$

Suitable for our design case is the value of 0.921 for  $k_M$ , which applies for conventional airfoils, according to Table 5.7.

The value of 14% for the average thickness is acceptable for our design. Next we need to find the thickness for the root and tip sections. The spanwise position of the 14% thickness we discovered applies, according to **Jenkinson 1999**, in a section given by (5.10.2).

If we add Howe's supposition which says:  $(t/c)_t = \frac{2}{3}(t/c)_r$ , then together with (5.10.2) we can form a system of 2 equations and 2 unknown parameters which give final results for root and tip relative thickness.

$$0.141 = \frac{3 \cdot \left[ \frac{2}{3}(t/c)_r \right] + (t/c)_r}{4} \Rightarrow (t/c)_r = \frac{4}{3} \cdot 0.141 = 0.188 \quad (5.10.10)$$

$$(t/c)_t = \frac{2}{3} \cdot 0.188 = 0.125 \quad (5.10.11)$$

We can choose now final values for the relative thickness:

$$(t/c)_r = 18\% \quad (5.10.12)$$

$$(t/c)_t = 13\% \quad (5.10.13)$$

## 5.11 Lift curve slope

There are various methods for calculating the increase of the lift coefficient with the angle of attack. According to **DATCOM 1978** the next formula is valid:

$$C_{L,\alpha} = \frac{2\pi A}{2 + \sqrt{\frac{A^2 \cdot \beta^2}{\kappa^2} \left( 1 + \frac{\tan^2 \varphi_{50}}{\beta^2} \right) + 4}} \quad (5.11.1)$$

where  $\beta = \sqrt{1 - M^2}$  and  $\kappa = \frac{c_{L,\alpha}}{2\pi/\beta}$  is a factor that needs further approximation for  $c_{L,\alpha}$  and for simplicity can be approximated with 1. In this case, the formula becomes:

$$C_{L,\alpha} = \frac{2\pi A}{2 + \sqrt{A^2(1 + \tan^2 \varphi_{50} - M^2) + 4}} \quad (5.11.2)$$

For estimating  $\varphi_{50}$  we use the next formula:

$$\tan \varphi_{50} = \tan \varphi_{25} - \frac{4}{A} \left[ \frac{50-25}{100} \cdot \frac{1-\lambda}{1+\lambda} \right] = \tan(3^\circ \cdot \frac{\pi}{180^\circ}) - \frac{4}{12} \left[ \frac{25}{100} \cdot \frac{1-0.59}{1+0.59} \right] = 0.031 \text{ rad} = 0.178^\circ \approx 0^\circ \quad (5.11.3)$$

This gives the following result for the lift curve slope:

$$C_{L,\alpha} = \frac{2\pi \cdot 12}{2 + \sqrt{12^2(1+0-0.443^2) + 4}} = 5.826 \quad (5.11.3)$$

**Howe 2000** gives a different approximation:

$$\frac{dC_L}{d\alpha} = \frac{A}{[(0.32 + 0.16A / \cos \varphi_{25}) \{1 - (M \cos \varphi_{25})^2\}^{1/2}]} \quad (5.11.4)$$

When replacing the corresponding values we obtain the following:

$$C_{L,\alpha} = \frac{12}{[(0.32 + 0.16 \cdot 12 / \cos(3^\circ \cdot \pi / 180^\circ)) \{1 - (0.443 \cos(3^\circ \cdot \pi / 180^\circ))^2\}^{1/2}]} = 5.966 \quad (5.11.5)$$

The results are very similar.

## 5.12 Incidence angle

The incidence angle is the angle between the chord line of the wing root and a reference line of the fuselage, in our case the cabin floor. It should be so chosen, that the drag in cruise flight is minimized. This implies a parallel direction of the flow to the longitudinal axis of the fuselage. Another aspect is the maneuvering of the trolleys; if the cabin floor is not horizontal in cruise flight, this becomes a difficult task for the flight attendants.

According to **Roskam II 1997** these requirements give the following equation:

$$i_w = \frac{C_{L,CR}}{C_{L\alpha}} + \alpha_0 - 0.4 \cdot \varepsilon_t \quad (5.12.1)$$

For the lift curve slope and necessary lift in cruise flight we have already approximated the values:

$$C_{L,\alpha} = 5.89 \quad (5.12.2)$$

$$C_{L,CR} = 0.813 \quad (5.12.3)$$

The twist we have chosen to be  $\varepsilon_t = -3^\circ$ .

The  $\alpha_0$  is the angle of attack at zero wing lift; the factor 0.4 bears the contribution of the tapered wing.

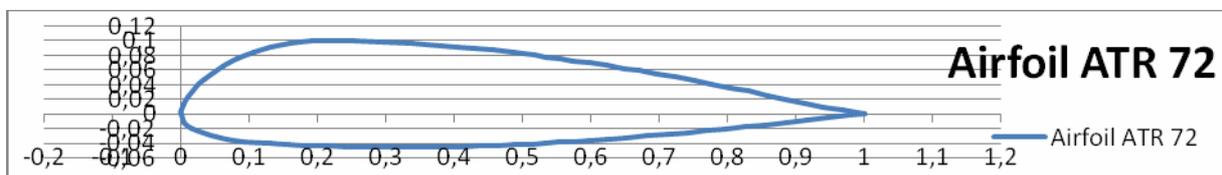
**Abbot and von Doenhoff 1959** in *Theory of Wing Sections* quote two approximate methods for the angle of zero lift. The first method belongs to **Pankhurst** and has the form indicated in (5.12.4):

$$\begin{aligned} \alpha_{L=0} &= \sum A(U + L) \\ c_{m_c/4} &= \sum B(U + L) \end{aligned} \quad (5.12.4)$$

In the previous formula, U and L represent the upper and lower ordinates of the wing section in fractions of chord. A and B are constants given in Table 5.8.  $c_{m_0}$  is the moment coefficient.

The solution proposed by **Pankhurst** gives, according to **Abbot and Doenhoff**, good results for airfoils for which the mean line ordinates are represented satisfactorily by  $(U+L)/2$ . The method is not suitable for airfoils with large curvatures of the mean line near the trailing edge, or for thick, highly cambered sections.

The results from paragraph 5.10 pointed to an airfoil similar to NACA 43018 for the root and NACA 43013 for the tip (see Fig.5.2). The maximum camber for our airfoil is situated at approximately 25% of the chord and has a value of 0.02806. This implies that the method is suitable for our design case.



**Fig. 5.2** Chosen airfoil for the ATR 72 design case, NACA 43018mod

The airfoil data give the following values for the upper and lower ordinates of the wing section:

**Table 5.8** Values for the constants in equation (5.12.4) (**Abbot 1959**)

x	A	B
0	1.45	-0.119
0.025	2.11	-0.156
0.05	1.56	-0.104
0.1	2.41	-0.124
0.2	2.94	-0.074
0.3	2.88	-0.009
0.4	3.13	0.045
0.5	3.67	0.101
0.6	4.69	0.170
0.7	6.72	0.273
0.8	11.75	0.477
0.9	21.72	0.786
0.95	99.85	3.026
1	-164.90	-4.289

**Table 5.9** Upper and lower ordinates of wing section in fractions of chord

x	U	L
0	0	0
0.025	0.041	-0.027
0.05	0.0656	-0.0315
0.1	0.0849	-0.0398
0.2	0.0998	-0.0434
0.3	0.0993	-0.0451
0.4	0.092	-0.0442
0.5	0.080	-0.0405
0.6	0.0698	-0.0359
0.7	0.0551	-0.028
0.8	0.0363	-0.02044
0.9	0.0148	-0.00795
0.95	0.0098	-0.00461
1	0	0

When applying (5.12.4) the following results are obtained:

$$\alpha_{L=0} = \sum A(U + L) = 2.00238^\circ \cong 2^\circ \quad (5.12.5)$$

The incidence angle now becomes:

$$i_w = \frac{0.813}{5.89} + 2^\circ - 0.4 \cdot (-3^\circ) = 3.338^\circ \square 4^\circ \quad (5.12.6)$$

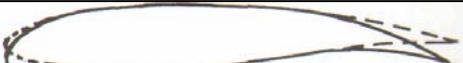
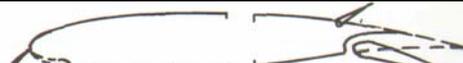
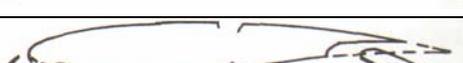
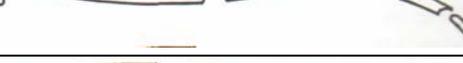
## 6 High Lift Systems and Maximum Lift Coefficients

The high lift systems are required in landing and take-off configurations, to ensure higher lift coefficients. This is achieved by increasing the airfoil camber and the wing area. The high lift devices also have an influence over the boundary layer: the pressure distribution improves and the energy of the flow increases.

The high lift devices can be used either for the trailing edge or for the leading edge.

There are different types of flaps, which were used in the history of aircraft development, from which we can choose (See Table 6.1).

**Table 6.1** Airfoils and high lift devices (Howe 2000)

Leading Edge	Example	Trailing Edge
		High speed airfoil
		Advanced subsonic airfoil
		Variable camber airfoil
Plain flap		Split flap
Droop nose		Plain flap
Kruger flap		Single slotted flap with spoiler air brake
Vented Kruger flap		Double slotted flap
Slat		Triple slotted flap
		Fowler flap

As we can notice from the table some devices change only the camber (the plain flaps) and others, more complex, like multi slotted flaps, change also the length of the chord, by increasing it. All these devices occupy, according Howe's statistics, between 20% and 40% of the chord and have a maximum angle of 35° to 45°.

However, using slots can create problems by introducing additional drag.

When choosing our high lift device, we need to know how much the lift coefficient is improved. From the preliminary sizing chapter we already know the necessary lift coefficient for both landing and take-off:

$$C_{L,max,L} = 2.44 \quad (6.1)$$

$$C_{L,max,TO} = 1.952 \quad (6.2)$$

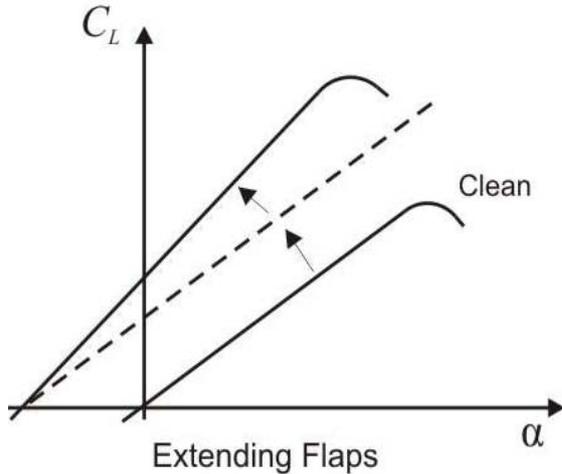
The statistics proposed by **Howe 2000** and **Raymer 2006** give data about each device type effectiveness (See Table 6.2):

**Table 6.2** Comparative statistical data of **Howe 2000** and **Raymer 2006**

<b>Howe 2000</b>			<b>Raymer 2006</b>	
High lift device	Lift coeffi cient- 2D	Lift coeffi cient- 3D	High lift device	$\Delta C_{l,max}$
Basic airfoil-subsonic	1.6	1.5		
Basic airfoil-sharp nose	1.0	0.95		
Plain trailing edge flap: 20%c 40%c	0.8 1.1	0.55 0.75	Plain and split	0.9
Split flap (no gap), t/c=0.15; 20%c 40%c	0.9 1.4	0.6 0.95		
Single slotted flap: 20%c 40%c	1.2 1.8	0.8 1.2	Slotted	1.3
Double-slotted flap: 40%(+26%)c	2.5	1.65	Double slotted	$1.6c'/c$
Triple slotted flap: 40%c overall	2.9	1.9	Triple slotted	$1.9c'/c$
Fowler flaps: 20%c 40%c	1.2 1.8	0.8 1.2	Fowler	$1.3c'/c$
Fowler plus split flap: 40%c	2.2	1.45		
Plain leading edge flap: 15%c	0.5	0.4		
Vented slat: 18%c	1.0	0.85		
Kruger flap: 20%c	0.8	0.65		
Vented Kruger flap: 20%c	1.0	0.85	Note: $c'$ -airfoil chord with extended flap	

As we can see from (6.1), the required lift coefficient is suitable, according to Table 6.2, for a double slotted flap, which gives a 2.5 lift coefficient. The slats give a high energy flow which keeps the boundary layer attached, so the lift coefficient increases. A triple slotted flap would give even more lift, but the flap kinematic becomes too complicated for a not necessary lift improvement. Also, leading edge devices can be used for further increasing the lift coeffi-

cient. Another reason is this one: the flaps increase the lift coefficient, but, in the same time, they tend to give a smaller stall angle; the leading edge flaps increase lift, but also increase the stall angle. Also, the slats (slotted leading edge flaps) increase both camber and wing area. If we make up our mind for a double slotted flap, then the slats are no longer necessary; if we choose a single slatted flap, then a further increase of the lift coefficient would be necessary. However the implementation process of both slats and flaps would be less efficient than in the case of using just the double slotted flap (among others, the production and maintenance costs would grow). The original ATR 72-500 uses also a double slotted flap, without slats.



**Fig. 6.1** Comparison of lift curves in the case of extending flaps with the clean wing (Raymer 2006)

## 6.1 Maximum Lift Coefficient of the Wing using DATCOM

In the Preliminary Sizing Chapter the lift coefficient in landing configuration was determined (see relation (6.1)). We can assume that, when taking into account the high lift systems, the new lift coefficient is 10% larger:

$$C_{L,\max} = 1.1 \cdot C_{L,\max,INITIAL\ SIZING} = 1.1 \cdot 2.44 = 2.684 \quad (6.1.1)$$

$$C_{L,\max,INITIAL\ SIZING} = C_{L,\max,L}$$

The design of the high lift devices has to take into account the necessary trimming of the horizontal tail, due to the pitching moments induced by the flap camber. The factor 1.1 considers exactly these effects.

The maximum lift coefficient of the clean wing can also be determined with help of **DATCOM 1978**:

$$C_{L,max,clean} = \left( \frac{C_{L,max}}{c_{L,max}} \right) \cdot c_{L,max,clean} + \Delta C_{L,max} \quad (6.1.2)$$

The first term of the equation comes from the **DATCOM 1978** diagram showed in Fig. 6.2. The  $\Delta C_{L,max}$  is equal to zero for Mach numbers smaller than 0.2, as it is the case in our design:

$$M_L = \frac{V_{APP}}{a_{SL}} = \frac{60.4}{340.294} = 0.177 < 0.2 \quad (6.1.3)$$

We have to determine now the lift coefficient of the clean airfoil. We can do this also according to **DATCOM 1978**<sup>2</sup> diagrams.

$$c_{L,max,clean} = (c_{L,max})_{base} + \Delta_1 c_{L,max} + \Delta_2 c_{L,max} + \Delta_3 c_{L,max} \quad (6.1.4)$$

In this formula the term  $\Delta_1 c_{L,max}$  is a correction term which takes into account the position of the maximum camber, when the maximum camber is situated at 30% of the chord, which is also very close to our case. We can read the value of  $\Delta_1 c_{L,max} = 0.075$  from the diagram in Fig. 6.2. For the term  $\Delta_2 c_{L,max}$  we can read the value from the diagram in Fig. 6.3; this is also a correction factor, but for thicknesses with different values than 30%. As we know from chapter 5, the position of our maximum thickness for the modified NACA airfoil is very close to 30% (it is in fact 27%). The value we can read of the graph is  $\Delta_2 c_{L,max} = -0.08$ .

The term  $\Delta_3 c_{L,max}$  takes into account the influence of Reynolds number, when the Reynolds number is different from  $9 \cdot 10^6$ .

We can calculate the Reynolds number for the wing using the mean aerodynamic chord as reference chord (calculated in the previous chapter) and the maximum take-off speed as reference speed, also obtained in the Preliminary Sizing chapter.

$$R_{n,MAC} = \frac{\rho V c_{MAC}}{\mu} = \frac{0.00237 \cdot 180.4 \cdot 2.2345 \cdot 3.28}{3.737 \cdot 10^{-7}} = 8.3 \cdot 10^6 \quad (6.1.5)$$

In the formula (6.8) the units are in the American system of units (feet).

The Reynolds number we calculated for the wing has the value of  $8.3 \cdot 10^6$ , which is close enough of the reference value, so that we can consider this correction term equal to zero.

<sup>2</sup> DATCOM comes from Data Compendium and contains 'handbook methods' which are used as tools for initial design steps

In order to read this diagrams we used the estimation of another parameter, called *leading-edge sharpness parameter*,  $\Delta y$ . For a five digit NACA airfoil, as it is our case,  $\Delta y$  has the value:

$$\Delta y = 26.0 \cdot (t/c) = 26.0 \cdot 0.14 = 3.64 \quad (6.1.6)$$

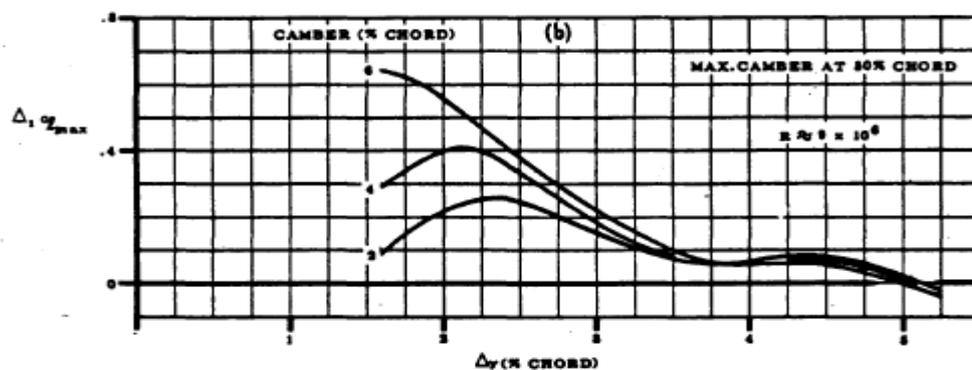


Fig. 6.2 Diagram for correction term  $\Delta_1 c_{L,max}$  when camber airfoil and position of maximum camber are considered (DATCOM 1978)

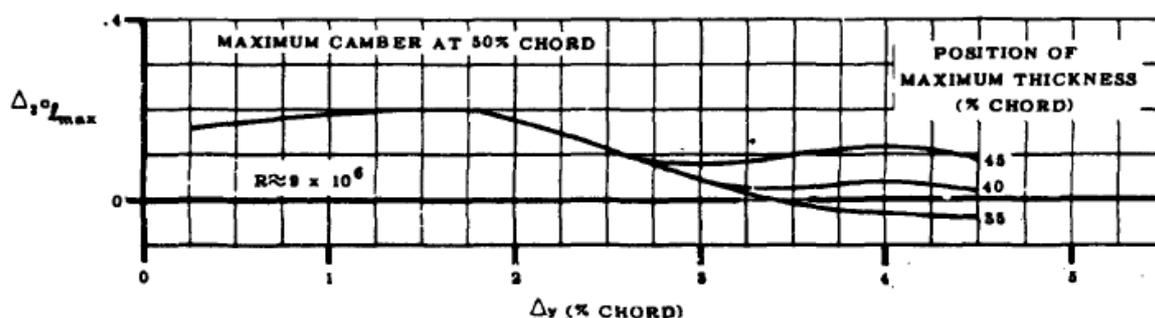


Fig. 6.3 Diagram for correction term for the position of maximum thickness (DATCOM 1978)

For our airfoil the  $(c_{L,max})_{base}$  has the value of 1.575, which can be read from the diagram in Fig. 6.4.

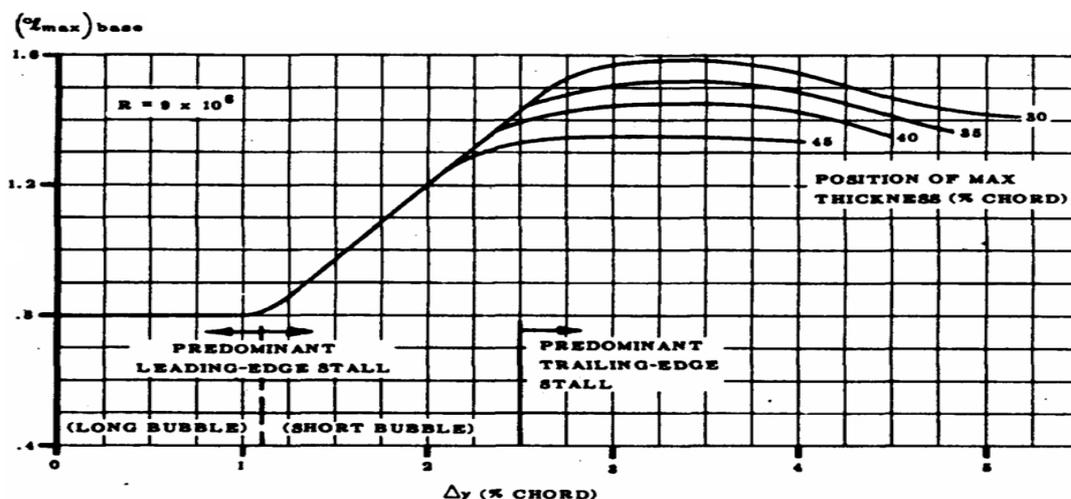


Fig. 6.4 Maximum lift coefficient of an airfoil, at  $9 \cdot 10^6$  Reynolds number (DATCOM 1978)

With these parameters we can set a final value for  $c_{L,max,clean}$  of our airfoil which is given by the sum:

$$c_{L,max,clean} = 1.575 + 0.075 - 0.08 + 0 = 1.57 \quad (6.1.7)$$

The term  $\left(\frac{C_{L,max}}{c_{L,max}}\right)$  can be read from Fig. 6.5. The value is: 0.875.

### SUBSONIC SPEEDS

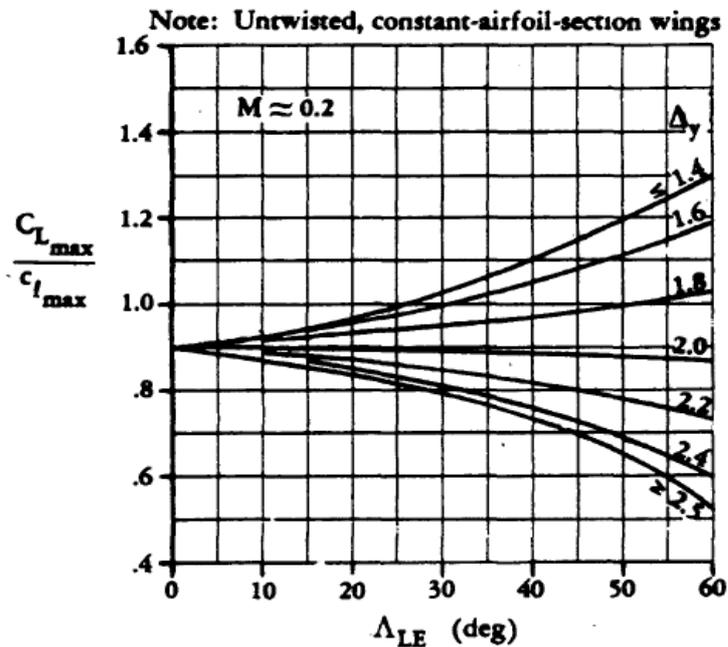


Fig. 6.5 Maximum lift of tapered wings with a high aspect ratio in subsonic speeds (DATCOM 1978)

We are now able to give a result from the relation (6.4):

$$C_{L,max,clean} = 0.875 \cdot 1.57 + 0 = 1.374 \quad (6.1.8)$$

If to the value of the parameter from (6.9) we add the flap effect, then we obtain a new value for the lift coefficient:

$$C_{L,max} = C_{L,max,clean} + \Delta C_{L,max,f} \quad (6.1.9)$$

The last parameter from the above formula shows the influence of the flaps on the lift coefficient and can be determined also according to **DATCOM 1978**:

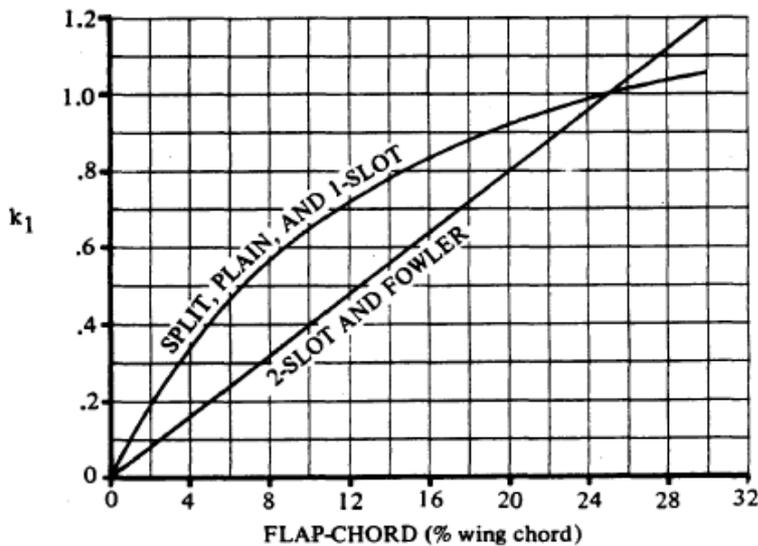
$$\Delta C_{L,\max,f} = \Delta c_{L,\max,f} \cdot \frac{S_{W,f}}{S_W} \cdot K_\varphi \quad (6.1.10)$$

Again we need first to estimate the increase of the lift coefficient due to the flaps first for the airfoil, in order to use it for the entire wing. In this case we need the following:

$$\Delta c_{L,\max,f} = k_1 k_2 k_3 (\Delta c_{L,\max})_{base} \quad (6.1.11)$$

The  $(\Delta c_{L,\max})_{base}$  term represents the maximum increase in the lift coefficient due to the flap, when the flap occupies 25% of the chord. The factors  $k_1$ ,  $k_2$  and  $k_3$  represent corrections brought to the  $(\Delta c_{L,\max})_{base}$  term, when taking into account different parameters from flap angle until kinematics of the flap mechanism. We can already settle the details of our flap design, due to the fact that it makes sense to choose those flap configurations that bring the correction factors as close as possible to 1.

The  $k_1$  factor is different than one if the flaps occupy another value than the value of 25% of the chord and can be read from Fig. 6.6. According to **Howe's 2000** statistics, a value between 20 and 40% gives proper effects. **Howe's 2000** suppositions are based on statistics. But we take into account that the design process is iterative; we settle the value of  $k_1$  to 1, so the flap occupies 25% of the chord; if the flap surface turns out not to be sufficient, we begin a new iteration.



**Fig. 6.6** Flap chord correction factor,  $k_1$  (**DATCOM 1978**)

The  $k_2$  factor is different than 1 if the flap deflection differs from the reference value. For the 2 slotted flap the reference angle is  $50^\circ$ . According to **Howe 2000**, we can choose a maximum actual angle of  $35^\circ$ . Again, we make these assumptions having in mind that, if the value is not

sufficient, a new iteration will be required. From Fig. 6.7 we choose the value of: 0.825, accordingly with the value of  $35^\circ$  angle.

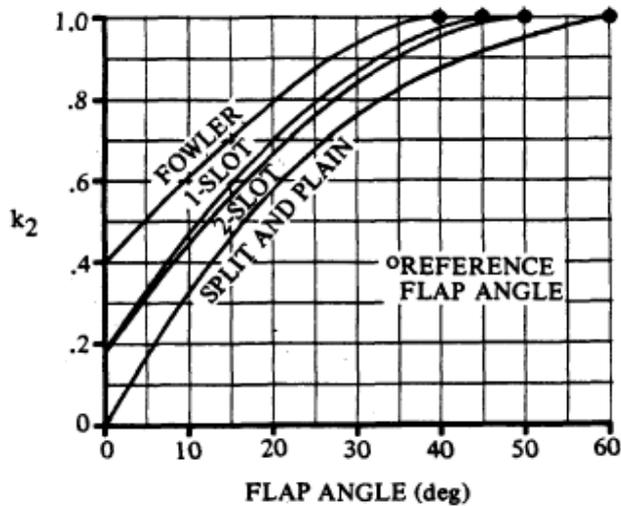


Fig. 6.7 Flap angle correction factor; the reference values are marked by dots (DATCOM 1978)

The  $k_3$  factor takes care of the flap kinematics. The parameter  $\frac{\text{actual flap angle}}{\text{reference flap angle}}$  can have the value  $\frac{35^\circ}{50^\circ} = 0.7$  (the  $50^\circ$  value comes from Fig. 6.7 and the  $35^\circ$  value we already settled according to Howe). The value we read from Fig. 6.8 is 0.8.

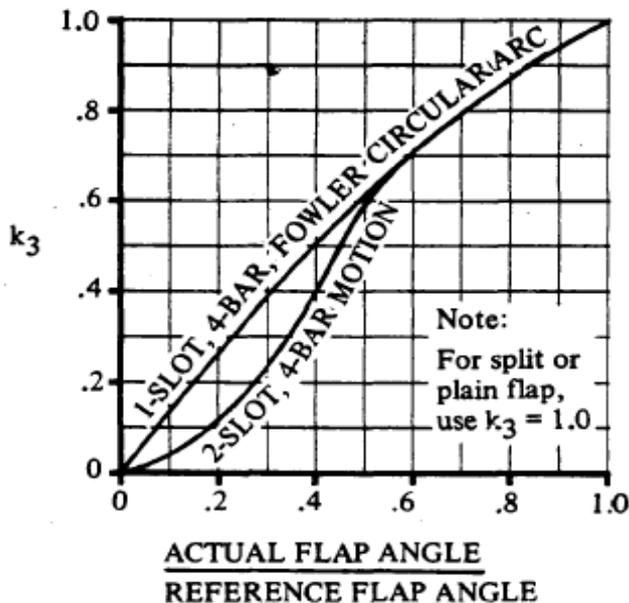


Fig. 6.8 Flap motion correction factor (DATCOM 1978)

The value of  $(\Delta C_{L,\max})_{base}$  can also be read from a diagram (see Fig. 6.9). In our design the case A is suitable, as we chose a double slotted flap, with a NACA 5 digit profile.

The value is:  $(\Delta c_{L,max})_{base} = 1.65$  for a thickness of 14%, which is the average thickness.

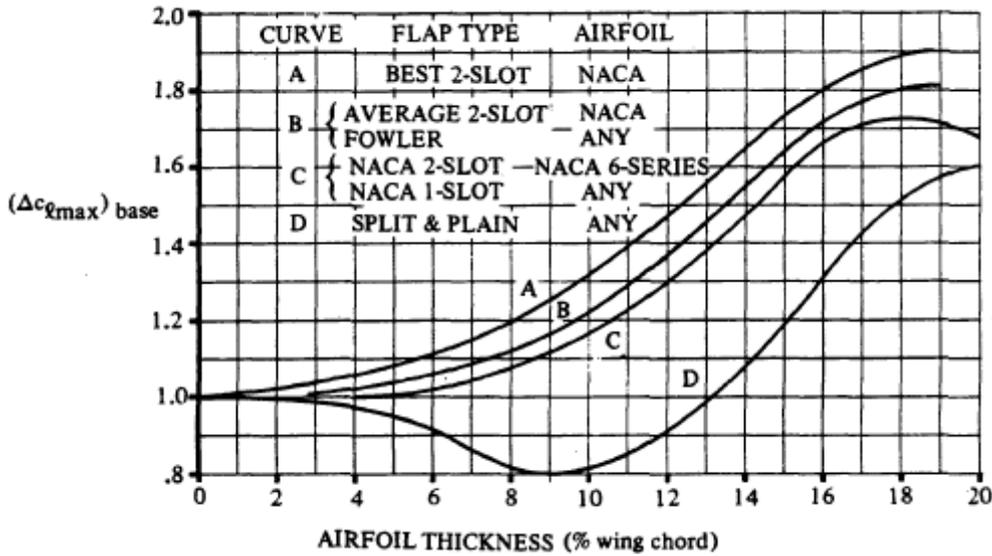


Fig. 6.9 Maximum lift increments for 25% flaps at a reference flap angle (DATCOM 1978)

We can now apply the formula (6.1.11) for calculating the final value for the lift increment due to flaps:

$$\Delta c_{L,max,f} = k_1 k_2 k_3 (\Delta c_{L,max})_{base} = 1 \cdot 0.825 \cdot 0.8 \cdot 1.65 = 1.089 \quad (6.1.12)$$

After obtaining the term  $\Delta c_{L,max,f}$  for the airfoil, we have to do the same for the wing. In order to achieve this, we need an approximation for  $K_\phi$  and for  $\frac{S_{W,f}}{S_W}$ , according to (6.1.10). For the  $K_\phi$  parameter, which takes care of the influence of the sweep angle we can either use Fig. 6.10 or the formula (6.1.13):

$$K_\phi = (1 - 0.08 \cos^2 \phi_{25}) \cdot \cos^{3/4} \phi_{25} \quad (6.1.13)$$

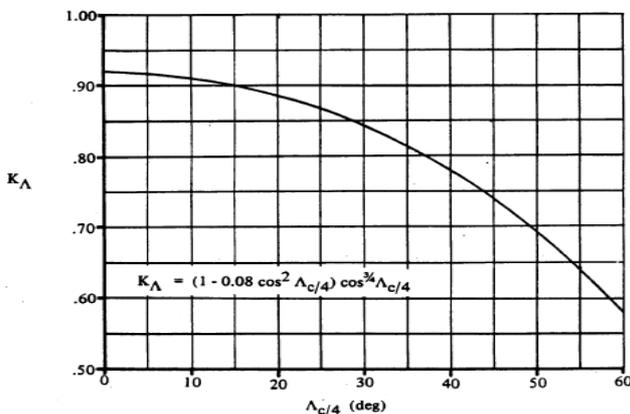
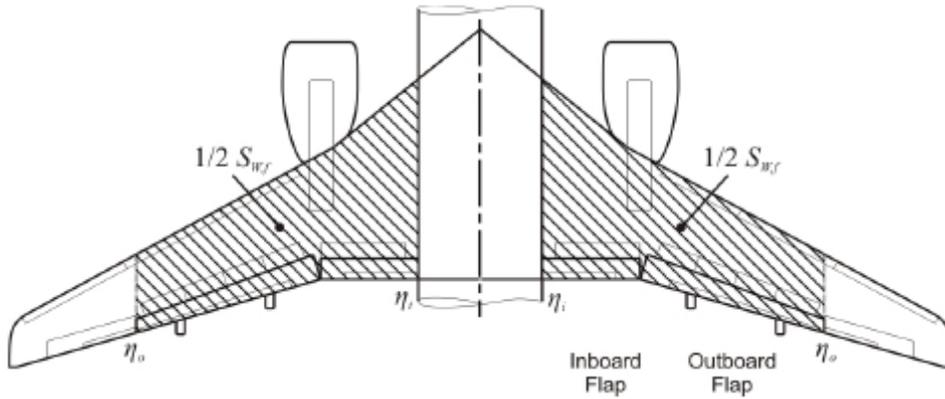


Fig. 6.10 Correction factor that takes the sweep into account (DATCOM 1978)

In our design we considered a very small sweep angle, due to the fact that our regional aircraft flies at lower speeds. The sweep angle is  $3^\circ$ ; for this reason we can make the assumption that  $\cos 3^\circ \approx 1$ . In this case, when using the formula, the value of  $K_\varphi$  becomes:

$$K_\varphi = 1 - 0.08 = 0.92.$$

For estimating  $\frac{S_{W,f}}{S_W}$  we need first to define  $S_{W,f}$ . For that see Fig. 6.8:



**Fig. 6.8** Definition of  $S_{W,f}$  (Scholz 2008)

The approximate formula for  $1/2 S_{W,f}$  should have this form:

$$S_{W,f} = S_W - S_{W,a} - d_f \cdot c_r \quad (6.1.14)$$

where  $S_{W,a}$  is the surface of the wing which corresponds to the ailerons length, the  $d_f$  is the diameter of the fuselage and the  $c_r$  is the root chord.

We assume that the ailerons occupy a length of 25% of the semispan; an approximate area would be given by:

$$S_{W,a} = 2 \cdot \left( 0.25 \cdot \frac{b}{2} \right) \cdot (1.2 \cdot c_t) = 2 \cdot \left[ 0.25 \cdot \frac{27.32}{2} \cdot 1.2 \cdot 1.556 \right] = 2 \cdot 6.376 = 12.752 m^2 \quad (6.1.15)$$

We have now a final value for  $S_{W,f}$ :

$$S_{W,f} = S_W - S_{W,a} - d_f \cdot c_r = 62.187 - 2 \cdot 6.376 - 2.77 \cdot 2.626 = 42.178 m^2 \quad (6.1.16)$$

We can now apply formula (6.1.10) for estimating the maximum increase of the lift coefficient of the wing, which takes into account the influence of the flaps:

$$\Delta C_{L,\max,f} = \Delta c_{L,\max,f} \cdot \frac{S_{W,f}}{S_W} \cdot K_\varphi = 1.089 \cdot \frac{42.178}{62.187} \cdot 0.92 = 0.679 \quad (6.1.17)$$

The maximum lift coefficient of the wing is obtained when we add to the lift coefficient of the *clean* wing the increase of the lift due to the flaps, according to (6.1.9):

$$C_{L,\max} = C_{L,\max, \text{clean}} + \Delta C_{L,\max,f} \quad (6.1.18)$$

The value we obtain is:

$$C_{L,\max} = 1.374 + 0.679 = 2.053 \quad (6.1.19)$$

At the beginning of the chapter we assumed that the required lift coefficient, which has to be insured by the wing and flap geometry, should be 10% greater than the value from the preliminary sizing of the aircraft (see relation (6.1.1)). The increase of 10% is due to the fact that in landing and take-off conditions, the pitching moment induced by the flaps influences the empennage; the reaction of the tail has to be counteracted with a sufficient amount of lift.

From the total increase of lift due to the flap, only 95% is used, because the use of the flaps brings the necessity of trimming the aircraft, and therefore negative lift is created.

The parameters of the high lift system have to satisfy the next relation:

$$0.95 \cdot \Delta C_{L,\max,f} \geq C_{L,\max} - C_{L,\max, \text{clean}} \quad (6.1.20)$$

$$0.645 = 0.95 \cdot 0.679 \geq 2.684 - 1.374 = 1.31 \quad (6.1.21)$$

## 6.2 Conclusions and Corrections

As we can see, the relation (6.1.21) is not valid; the lift provided by the lift system is not sufficient to fulfill the lift requirements, necessary to insure the coherency of the parameters from the preliminary sizing. However the process of designing is an iterative one.

To provide sufficient lift, an alternative is to introduce leading edge flaps or leading edge slotted flaps (or slats).

The slots are fixed slats placed in front of the wing's leading edge. By increasing the curvature of the top of the airfoil, the increase of the lift coefficient should be sufficient for the lift requirements in our design case. It is also possible to increase the wing area, by using slats or

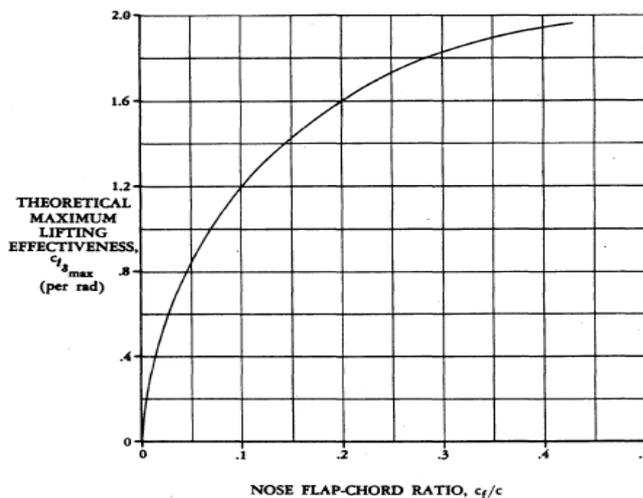
slotted leading edge flaps. This increases lift even more and also a higher stall angle is possible.

The increase in the maximum lift coefficient due to the slats or flaps on the leading edge is given by:

$$\Delta c_{L,\max,s} = c_{l,\delta,\max} \eta_{\max} \eta_{\delta} \delta_f \frac{c'}{c} \quad (6.2.1)$$

The first term from the formula is the theoretically maximum flap efficiency, given in **Fig. 6.9**. The  $\eta_{\max}$  is an empirical factor that accounts for the *parameter leading edge radius over relative thickness*,  $\frac{LER}{t/c}$ . The  $\eta_{\delta}$  is also an empirical factor that takes into account the actual angle of deflection compared to the reference angle of deflection.  $\delta_f$  is the angle of deflection of the slat or leading edge flap and  $\frac{c'}{c}$  is the ratio of the chord with deflection and the chord without deflection.

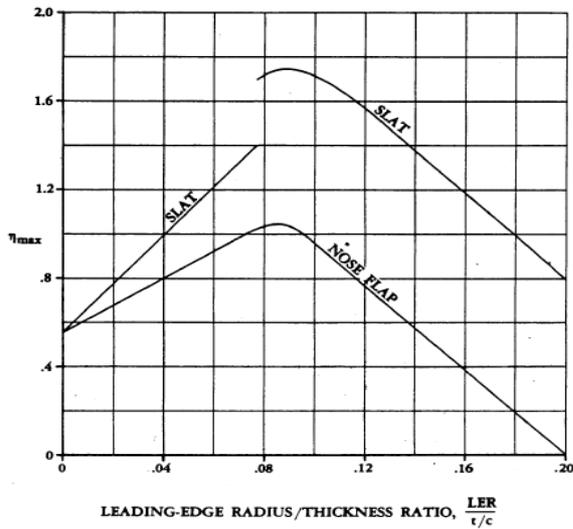
According to **Howe 2000** a plain leading edge slat gives an increment of 0.5 in lift coefficient; this value is not sufficient. **Raymer 2006** shares the opinion that slotted leading edge flaps (slats) are more suitable for both low and high speed aircrafts; the  $\Delta C_{l,\max}$  for slats has a value of  $0.4 c' / c$ . They prevent premature airflow separation due to the flaps and delay the stall. We choose accordingly a leading edge slotted flap.



**Fig. 6.9** Theoretical maximum flap effectiveness (**DATCOM 1978**)

For this type of leading edge high lift devices Howe appreciates a statistical value of 16% to 20% for the nose flap chord ratio  $c_f/c$ . For a value of 20% we can read the value of 1.6 for the  $c_{l,\delta,\max}$ .

The next term,  $\eta_{\max}$ , can be read from Fig. 6.10:



**Fig. 6.10** Empirical factor accounting for  $\frac{LER}{t/c}$  (Scholz 2008)

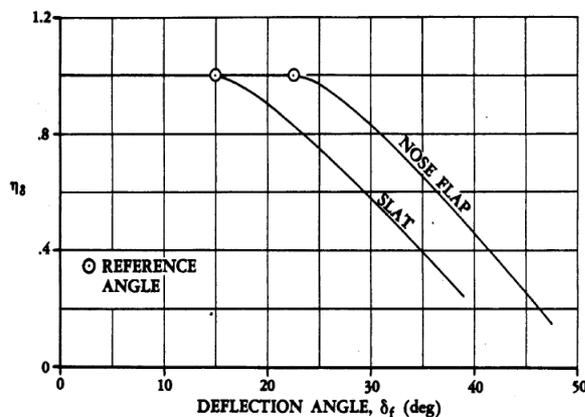
The leading edge radius has an approximate value of:  $LER = 0.07$  (from Fig.5.2); this gives:

$$\frac{LER}{t/c} = \frac{0.07}{0.14} = 0.5 \text{ that allows us to read the value of: } \eta_{\max} = 1.2 \text{ for the empiric factor.}$$

An usual deflection angle for the slats is  $30^\circ$  (Howe 2000); a smaller value of  $25^\circ$  is suitable for our design.. We can now read the value of the factor  $\eta_\delta$  from Fig. 6.11:  $\eta_\delta = 0.78$ .

For estimating  $c'/c$  a simple geometric problem must be solved by applying *Pitagora* theorem in a triangle with one angle of  $90^\circ$  and one angle of  $30^\circ$ . This gives:

$$\frac{c'}{c} = 1 + \sqrt{c_f^2 - (c_f/2)^2} = 1 + \sqrt{0.2^2 - (0.2/2)^2} = 1.173 \tag{6.2.2}$$



**Fig. 6.11** Empirical factor accounting for the real deflection factor in contrast to the reference angle (DATCOM 1978)

We can now obtain the increase of the lift coefficient of the airfoil due to the slats:

$$\Delta C_{L,\max,s} = 1.6 \cdot 1.2 \cdot 0.78 \cdot 25^\circ \cdot \pi / 180^\circ \cdot 1.173 = 0.766 \quad (6.2.3)$$

For the entire wing we need to make other assumptions. According to **Raymer 2006** the slats give the following lift increment for the wing:

$$\Delta C_{L,\max,s} = \Delta C_{L,\max,s} \cdot \frac{S_{W,s}}{S_W} \cdot \cos \varphi_{H,L} \quad (6.2.4)$$

In this formula, the  $\frac{S_{W,s}}{S_W}$  represents the area ratio for the slats; if we assume the slats occupying the same length from the wing span as the flaps, then this ratio is equal to the one obtained in (6.1.15) and (6.1.16). If this proves not to be necessary, we make a new iteration and increase the surface, so that the lift increases.

$$\frac{S_{W,s}}{S_W} = \frac{42.178}{62.187} = 0.678 \quad (6.2.5)$$

The subscript  $H,L$  comes from hinge line; we can assume the sweep angle  $\varphi_{H,L}$  is small so that we can approximate  $\cos \varphi_{H,L} \approx 1$ .

Finally we obtain:

$$\Delta C_{L,\max,s} = 0.766 \cdot 0.678 \cdot 1 = 0.519 \quad (6.2.6)$$

This lift increment is again not sufficient to satisfy the relation (6.1.20), which means we have to increase the available area for the slats, also in front of the ailerons. We can assume that the slats should occupy 80% of the semispan; this gives an increase in the lift coefficient due to the slats:

$$\Delta C_{L,\max,s} = 0.766 \cdot 0.8 \cdot 1 = 0.613 \quad (6.2.7)$$

We are now ready to test our design, by using (6.1.20). the maximum lift coefficient of the wing has now the value:

$$C_{L,\max} = C_{L,\max, \text{clean}} + \Delta C_{L,\max, f} + \Delta C_{L,\max, s} = 1.374 + 0.679 + 0.613 = 2.666 \quad (6.2.8)$$

$$\begin{aligned} 0.95 \cdot \Delta C_{L,\max,f} &\geq C_{L,\max} - C_{L,\max,clear} \\ 0.95 \cdot 0.679 + 0.613 &= 1.258 \geq 1.31 = 2.684 - 1.374 \end{aligned} \quad (6.2.9)$$

The relation is still not valid, but it is very close to being fulfilled. This means that our design of the high lift system can face the requirements; an explanation for which the relation has a small error is related to the method applied, which uses rough safety margins.

## 7 Empennage general design

This chapter provides a general view over the empennage configuration; a more detailed approach will be provided in chapter 9.

The main function of the empennage is that both vertical and horizontal tailplane create forces which create moments around the vertical axis (yaw) and, respectively, around the lateral axis (pitch). As a consequence the empennages ensure the stability and control of the aircraft and also the trimming.

The certification regulations (**CS 25.171 to CS 25.181**) refer at the stability as the ability of the aircraft to return to its original flying position when a disturbance interferes.

The controllability of the aircraft also has to satisfy the requirements (according to **CS 25.143 to CS 25.149**) in all critical flight states.

For the shape of the aircraft we choose a conventional tail as it gives several advantages like the downwash of the wing is relatively large in the area of the horizontal tailplane and also it allows a larger height of the vertical tailplane.

There are design rules which should be respected in order for the design to be accordingly to the requirements (see next table).

**Table 7.1** Design rules (**Scholz 2008**)

	Item	Design rule	Effect if the rule is not applied
a)	Position of the horizontal tailplane	Not to lie in slipstream	structure fatigue due to tail buffeting the tail buffeting gives noise in the cabin when changing engine performance the trimming changes considerable
b)	Lever arm	As large as possible	the area increases, so the weight and drag increases
c)	Aspect ratio of the horizontal tail plane	About half the aspect ratio of the wing	
d)	Critical Mach number	0.05 higher than $M_{cr}$ of the wing	the efficiency of the tail assembly will be not guaranteed at high speed
e)	Sweep of the horizontal tailplane	$5^\circ$ larger than the sweep of the wing, as a consequence of d)	the tail would stall early (in the same time with the wing)
f)	Sweep of the vertical	$35^\circ$ to $55^\circ$ for high	a larger sweep increases stall angle,

	tailplane	speeds and less than 20° for low speeds	but decreases the max lift coefficient
g)	Airfoils	Symmetrical for vertical tailplanes Symmetrical with 9% to 12% relative thickness for horizontal tailplane	If an asymmetrical is chosen it would not be able to give negative lift, as required for control and stability
h)	Left and right elevators	Have to be connected	The tendency to flutter increases The advantage of joint actuation is lost
i)	Dihedral angle	So that the empennage is positioned outside the engine slipstream	
j)	Incidence angle	2°-3° downwards to create negative lift, when the horizontal tail is fixed	

According to this table we can now mention some design parameters, as follows:

The aspect ratio of the horizontal tailplane:  $A_H = 0.5A_w = 0.5 \cdot 12 = 6$

The aspect ratio of the vertical tailplane:  $A_V = 1.3 - 2$ ; we choose the value of  $A_V = 1.6$

The taper ratio of the horizontal and vertical tailplane:  $\lambda = 0.3 - 0.6$ ; we choose the following values:

$$\begin{aligned}\lambda_H &= 0.6 \\ \lambda_V &= 0.6\end{aligned}\tag{7.1}$$

The sweep angle of the horizontal tailplane:  $\varphi_{25} = 5^\circ + 3^\circ = 8^\circ$

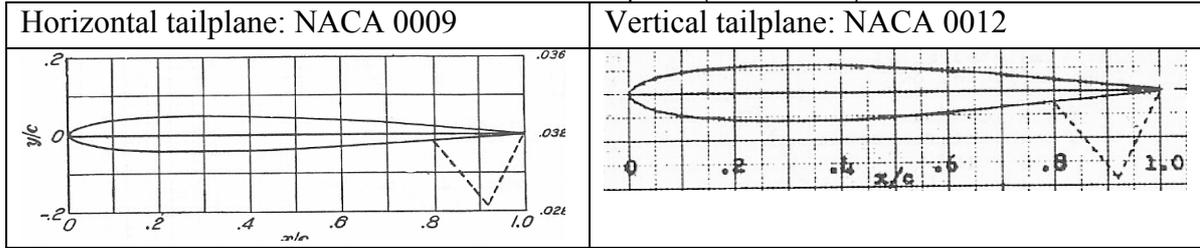
The sweep angle of the vertical tailplane:  $\varphi_{25} = 25^\circ$

The dihedral angle for the horizontal tailplane is, according to **Roskam II 1997**, between  $-15^\circ$  and  $0^\circ$  and for the vertical tailplane between  $75^\circ$  and  $90^\circ$ . we choose the following values:

$$\begin{aligned}V_H &= 80^\circ \\ V_V &= 0^\circ\end{aligned}\tag{7.2}$$

The incidence angle for both horizontal and vertical tailplane can have a sufficient value of  $0^\circ$  (**Roskam II 1997**).

The airfoil for the horizontal tailplane should be symmetrical, with a lower thickness ratio as the wing; the same hypothesis is valid for the vertical tailplane; we choose accordingly, from **Abbot 1959** the next airfoils:

**Table 7.2** Airfoils of the horizontal and vertical tailplane (**Abbot 1959**)

Another important parameter for our design is the tail volume. This parameter is defined by means of the horizontal or vertical tailplane area multiplied by the lever arm of the horizontal, respectively vertical tailplane. The tail volume coefficient is defined as follows:

$$C_H = \frac{S_H \cdot l_H}{S_W \cdot c_{MAC}} \quad (7.3)$$

$$C_V = \frac{S_V \cdot l_V}{S_W \cdot b} \quad (7.4)$$

The lever arms are defined as the distance between the aerodynamic centers of the wing and horizontal, respectively vertical tailplane. Instead of the coordinates of the aerodynamic centers, the 25% point on the mean aerodynamic chord can be used in aircraft design. However, the lever arms can be only approximated when the position of the wing is fixed; this will be established in the next chapter.

At the moment, we can estimate the lever arms positions, according to **Raymer 2006**, so that we can obtain the tailplanes areas and, correspondingly, the wing spans.

**Raymer 2006** estimates from his statistics, that an average position for  $l_H$  and  $l_V$  is situated, for an aircraft configuration with the engines on the wing, at about 50% to 55% of fuselage length. This gives the following values:

$$l_V = l_H = 50\% \cdot 27.13 = 13.565m \quad (7.5)$$

The values for the tail volume coefficients are given by **Raymer 2006** from statistics only for jets. For the regional turboprops we have to make our own statistic. When including the original ATR 72 and other turboprops like Fairchild Dornier 328, we find a little higher values:

$$C_H = 1.05 \quad (7.6)$$

$$C_V = 0.119 \quad (7.7)$$

We can now estimate the areas of both horizontal and vertical tailplanes:

$$S_H = \frac{C_H \cdot S_W \cdot c_{MAC}}{l_H} = \frac{1.05 \cdot 62.187 \cdot 2.2345}{13.565} = 10.756m^2 \quad (7.8)$$

$$S_V = \frac{C_V \cdot S_W \cdot b}{l_V} = \frac{0.119 \cdot 62.187 \cdot 27.32}{13.565} = 14.904m^2 \quad (7.9)$$

The wing spans are accordingly:

$$b_H = \sqrt{A_H \cdot S_H} = \sqrt{6 \cdot 10.756} = 8.033m \quad (7.10)$$

$$b_V = \sqrt{A_V \cdot S_V} = \sqrt{1.6 \cdot 14.904} = 4.883m \quad (7.11)$$

The elevator and rudder can be considered as plain flaps. Elevators are usually deflected downwards between  $15^\circ$  and  $25^\circ$  and upwards between  $25^\circ$  and  $35^\circ$  (**Scholz 2008**). For the rudder a maximum deflection of  $25^\circ$  to  $35^\circ$  is a usually value.

The chord occupies approximately 25% to 40% of the chord of the tail (either horizontal or vertical).

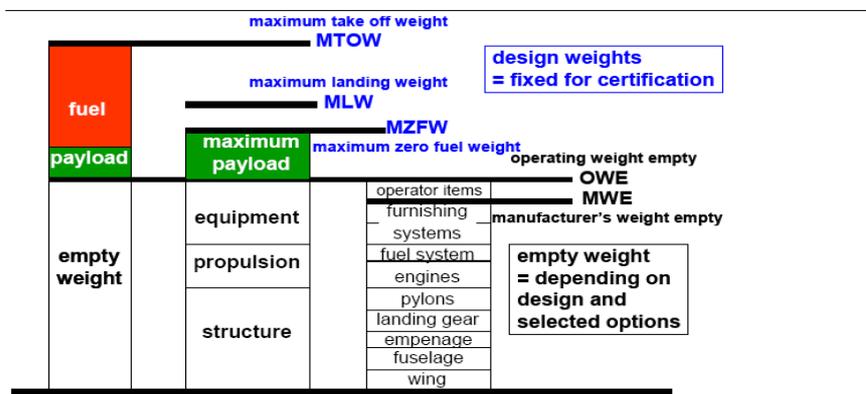
## 8 Prediction of Mass and CG-Location

The purpose of the aircraft is to carry payload. Therefore the other design weights have to be correspondingly sized so that the mission of the aircraft can be fulfilled. In a preliminary phase of a design it is recommended to use empirical or semi empirical methods to calculate the weight breakdown; a next step would be to perform a check by confronting with statistics and afterwards get into details and use a design based method, which considers loads, geometry and sizing in an optimal manner.

The methods less precise are the *Class I Methods*, and the ones which account for more details are the *Class II Methods*.

### 8.1 Class I Method for Weight Estimation and Weight Break-down (Raymer 2006)

A typical weight breakdown is shown in the next figure, which was presented at the Aircraft Design Short Course Mai 2008, by **Trahmer 2008**:



**Fig. 8.1** Weight Break Down (Trahmer 2008)

**Raymer 2006** divides in a similar way the main components of the weight break down. His method accounts for wetted and exposed areas multiplied by a factor in order to predict the mass for each component.

We first need to estimate the exposed area of the wing, empennage and wetted area of the fuselage. According to **Raymer 2006** the next formulas are valid:

$$S_{exposed,W} = 2 \cdot (S_W - S_{W,F}) = 2 \cdot \left( S_W - \left[ c_r - \frac{c_r - c_k}{y_k} \cdot \frac{d_F}{4} \right] \cdot d_F \right) \quad (8.1.1)$$

$$S_{exposed,W} = 2 \cdot \left( 62.187 - \left[ 2.626 - \frac{2.626 - 2.626}{4.73} \cdot \frac{2.77}{4} \right] \cdot 2.77 \right) = 109.83m^2$$

$$S_{exposed,tail} = 2 \cdot (S_H + S_V) = 2 \cdot (8.195 + 8.767) = 33.924m^2 \quad (8.1.2)$$

$$S_{wet,F} = \pi \cdot d_F \cdot l_F \cdot \left( 1 - \frac{2}{\lambda_F} \right)^{2/3} \cdot \left( 1 + \frac{1}{\lambda_F^2} \right)$$

$$\lambda_F = l_F / d_F = 9.794 \quad (8.1.3)$$

$$S_{wet,F} = \pi \cdot 2.77 \cdot 27.13 \cdot \left( 1 - \frac{2}{9.794} \right) \cdot \left( 1 + \frac{1}{9.794^2} \right) = 204.858m^2$$

For the engine weight estimation **Raymer 2006** proves to be not accurate enough:

$$m_{E,ges} = \frac{0.1448}{g} \cdot \left( \frac{T_{RO}}{2} \right)^{1.1} \cdot e^{-0.045\mu} \quad (8.1.4)$$

$$T_{RO} = \frac{P \cdot \eta}{V}$$

$$m_{E,ges} = \frac{0.1448}{9.8} \cdot \left( \frac{2051 \cdot 0.6454}{2 \cdot 54.12} \right) \cdot e^{-0.045 \cdot 5} = 395.293kg \quad (8.1.5)$$

**Roskam V 1997** makes a different estimation:

$$m_{E,ges} = 0.453shp^{0.907} = 0.453 \cdot 2051^{0.907} = 457.142kg \quad (8.1.6)$$

According to the data in Jane's for the PW 127F the weight is 481 kg; we can use this information to perform a check of our estimation. Accordingly, the **Roskam V 1997** method is more suitable.

We can now insert our data into **Raymer 2006**'s mass prognoses, as it is showed in Table 8.1.

**Table 8.1** Weight estimations on **Raymer 2006**'s Class I method

	Factor	Reference parameter[m <sup>2</sup> ]	Mass [kg]
Wing	49	$S_{exposed,W} = 109.83$	5381.472
Fuselage	24	$S_{wet,F} = 204.858$	4916.581
Empennage	27	$S_{exposed,tail} = 33.924$	915.948
Nose gear	0.006	$m_{MTO} = 23296.272$	139.778
Main gear	0.037	$m_{MTO} = 23296.272$	861.962

Structure	-		
Power plant	1.3	$m_E = 457.142$	594.282
Systems&items	0.17	$m_{MTO} = 23296.272$	3960.366
$m_{OE}$	-	-	$\Sigma = 16770.389$

With the result from Table 8.1 a new maximum take-off mass can be estimated by using the values for maximum payload and maximum fuel from Preliminary Sizing chapter:

$$m_{PL} = 6650kg \quad (8.1.7)$$

$$m_F = 3414.398kg$$

$$m_{MTO} = m_{OE} + m_F + m_{PL} = 16770 + 3414.398 + 6650 = 26844.941kg \quad (8.1.8)$$

Because the maximum operating empty weight came out larger than in the Preliminary Sizing chapter, correspondingly, the maximum take-off weight is larger.

## 8.2 Class II Method for Weight Estimation and Weight Break-down (Torenbeek 1986)

The *Class II* method proposed by **Torenbeek 1986** has a higher degree of accuracy and uses equations for estimating each component mass; the data are extracted from a large number of publications, especially from the *International Society of Allied Weight Engineers*. The process is an iterative one, so the calculations will be produced with the help of EXCEL.

### Wing mass, $m_W$

The equation for estimating the wing mass has the following form, for the category of aircrafts with a  $m_{MTO} > 5700kg$  :

$$\frac{m_W}{m_{MZF}} = 6.67 \cdot 10^{-3} \cdot b_s^{0.75} \cdot \left( 1 + \sqrt{\frac{b_{ref}}{b_s}} \right) \cdot n_{ult}^{0.55} \cdot \left( \frac{b_s / t_r}{m_{MZF} / S_W} \right)^{0.30} \quad (8.2.1)$$

The parameters used in the equation have the following meaning:

**Table 8.2** Necessary parameters for estimating wing mass

Parameter	Value	Observations
$b_{ref}$	1.905m	It is an input parameter necessary for calculations
$\varphi_{50}$	$\tan \varphi_{50} = \tan \varphi_{25} - \frac{4}{A} \left[ \frac{50-25}{100} \cdot \frac{1-\lambda}{1+\lambda} \right]$ $\tan \varphi_{50} = \tan(3^\circ \cdot \frac{\pi}{180^\circ}) - \frac{4}{12} \left[ \frac{25}{100} \cdot \frac{1-0.5927}{1+0.5927} \right] = 0.033$ $\varphi_{50} \cong 2^\circ$	Necessary for estimating the structural span
$b_s$ , structural span	$b_s = \frac{b}{\cos \varphi_{50}} = \frac{27.32}{\cos(2^\circ \cdot \pi / 180^\circ)} \cong \frac{27.32}{1} = 27.32m$	
$n_{lim}$	$n_{lim} = 2.5$	For aircrafts with $m_{MTO} \geq 22680$
$n_{ult}$ , ultimate load factor	$n_{ult} \approx 1.5 \cdot n_{lim} = 1.5 \cdot 2.5 = 3.75$	According to JAR 25.337
$t_r$ , airfoil thickness, root	$t_r = 0.18$	From wing design
$m_{MZF}$ , maximum zero fuel mass	$m_{MZF} = 19881.874kg$	From Preliminary Sizing chapter
$S_W$ , wing area	$S_W = 62.187m^2$	
$m_{MTO}$ , maximum take-off mass	$m_{MTO} = 23296.272kg$	From Preliminary Sizing chapter

When applying the data from Table 8.2, the following results are obtained:

$$\frac{m_W}{m_{MZF}} = 0.166 \quad (8.2.2)$$

This gives a wing mass of:

$$m_W = 0.16536 \cdot m_{MZF} = 0.166 \cdot 19881.9 = 3309.432kg \quad (8.2.3)$$

To this value some corrections are made:

+2% taking into account the spoilers;

- 5% if 2 engines are fixed on the wing and
- 5% if the landing gear is not fixed on the wing.

This gives a final value of:

$$m_W = m_W + 2\%m_W - 5\%m_W - 5\%m_W = m_W(1 - 0.08) = 3045kg \quad (8.2.4)$$

### **Fuselage mass, $m_F$**

In order to proceed with the calculations we have to evaluate the dive speed of the aircraft. According to **JAR 25.335**  $V_D$  can be estimated from  $M_D$  which has a value with 0.05 to 0.09 greater than  $M_C$ . All the design speeds are equivalent airspeeds (**JAR 23.335**). This gives the following:

$$\begin{aligned} V_{EAS} &= V_{TAS} \cdot \sqrt{\sigma} \\ \sigma &= \rho / \rho_0 \\ V_D &= M_D \cdot a \end{aligned} \quad (8.2.5)$$

The  $M_D$  has a value of:  $M_D = M_C + 0.09 = 0.53$ . The speed of sound at the cruise altitude of 17000 ft is:  $a = 319.789m/s$  and the sigma for the same altitude is:  $\sigma = 0.589$ . This gives a value for the dive speed of  $V_D = 130.076m/s$ .

The formula for estimating the fuselage weight, according to **Torenbeek 1986**, for aircrafts with a dive speed greater than 250kts or 128.6m/s is:

$$m_F = 0.23 \cdot \sqrt{V_D \cdot \frac{l_H}{w_F + h_F}} \cdot S_{F,wet}^{1.2} \quad (8.2.6)$$

The  $l_H$  is the lever arm of the horizontal tailplane which was estimated in chapter 7:  $l_H = 13.565m$ . The  $w_F$  and  $h_F$  are the fuselage width and height; they can be approximated with the fuselage diameter, estimated in chapter 4:  $d_F = 2.77m$ .

We now need to estimate the wetted area of the fuselage; the data required for this operation are calculated in the fuselage design chapter:

$$S_{wet,F} = \sqrt{l_{bug}^2 + (d_F/2)^2} \cdot \frac{d_F \cdot \pi}{2} + d_F \cdot l_{cyl} \cdot \pi + \sqrt{l_{heck}^2 + (d_F/2)^2} \cdot \frac{d_F \cdot \pi}{2} = 184.593m^2 \quad (8.2.7)$$

The final value for the fuselage mass is:  $m_F = 2323.432kg$ .

**Horizontal tailplane,  $m_H$  and Vertical tailplane,  $m_V$** 

For an aircraft with a diving speed greater than 250kts the next formulas are valid:

$$m_H = k_H \cdot S_H \cdot \left( 62 \cdot \frac{S_H^{0.2} \cdot V_D}{1000 \sqrt{\cos \varphi_{H,50}}} - 2.5 \right)$$

$$m_V = k_V \cdot S_V \cdot \left( 62 \cdot \frac{S_V^{0.2} \cdot V_D}{1000 \sqrt{\cos \varphi_{V,50}}} - 2.5 \right)$$
(8.2.8)

The necessary values for applying the formula have been estimated in the previous chapter. More details are included in the next table:

**Table 8.3** Parameters necessary to evaluate the empennage mass

Parameter	Value	Observations
$k_H$ , factor accounting for the elevator	$k_H = 1.1$	This value is applicable for trimmed elevator
$k_V$ , factor accounting for the rudder	$k_V = 1 + 0.15 \cdot \frac{S_H \cdot z_H}{S_V \cdot b_V} \approx 1$	
$b_V$ , span of the vertical tail	$b_V = 3.745$	
$\varphi_{H,50}$ and $\varphi_{V,50}$ , sweep angle at 50% of the chord	$\varphi_{H,25} = 8^\circ$ ; $\varphi_{V,25} = 25^\circ$ $\varphi_{H,50} = 6.917^\circ \approx 7^\circ$ ; $\varphi_{V,50} = 24.088^\circ \approx 24^\circ$	Values are obtained when applying the same formula from previous table
$S_H$ ; $S_V$ , areas of the tailplane	$S_H = 10.756m^2$ $S_V = 14.904m^2$	As estimated in the general design of the empennage

The results are:

$$m_H = 124.445kg$$

$$m_V = 178.607kg$$
(8.2.9)

**Landing gear mass,  $m_{LG}$** 

The equation for this approximation is indicated in (8.2.10).

$$m_{LG,N} \text{ or } m_{LG,M} = k_{LG} \cdot (A_{LG} + B_{LG} \cdot m_{MTO}^{3/4} + C_{LG} \cdot m_{MTO} + D_{LG} \cdot m_{MTO}^{3/2}) \quad (8.2.10)$$

**Torenbeek 1986** gives the values for the  $A$ ,  $B$ ,  $C$ ,  $D$  coefficients for the nose and main gear, as it is shown in the next table.

The landing gear mass is composed out of the nose gear mass and main gear mass:

$$m_{LG} = m_{LG,N} + m_{LG,M} \quad (8.2.11)$$

**Table 8.4** Coefficients used for landing gear mass calculation

Gear type	Gear component	$A_{LG}$	$B_{LG}$	$C_{LG}$	$D_{LG}$
Retractable gear	Main gear	18.1	0.131	0.019	$2.23 \cdot 10^{-5}$
	Nose gear	9.1	0.082	-	$2.97 \cdot 10^6$

The value of the  $k_{LG}$  constant is 1. The results are:

$$\begin{aligned} m_{LG,N} &= 174.285 \text{ kg} \\ m_{LG,M} &= 787.044 \text{ kg} \end{aligned} \quad (8.2.12)$$

The total mass of the landing gear is given by (8.2.11):

$$m_{LG} = 174.285 + 787.044 = 961.329 \text{ kg} \quad (8.2.13)$$

### Nacelle mass, $m_N$

**Torenbeek 1986** gives an estimative formula only for the turbofans. The nacelle of a turbofan is yet heavier than for a turboprop, due to the required geometry for sustaining the double flow; the cowl of a turboprop only covers the engine and does not require additional surfaces. Therefore we minimize the factor from the formula accordingly:

$$m_N = 0.0485 \cdot T_{TO} / g = 0.0485 \cdot \frac{P \cdot \eta}{V \cdot g} \quad (8.2.14)$$

For the take-off configuration, using the propeller efficiency and the power correspondingly, the result is:

$$m_N = 0.0485 \cdot \frac{4102000 \cdot 0.64545}{54.12 \cdot 9.81} = 241.865 \text{ kg} \quad (8.2.15)$$

### Installed Engine mass, $m_{E,inst}$

The formula is:

$$m_{E,inst} = k_E \cdot k_{thr} \cdot n_E \cdot m_E \quad (8.2.16)$$

The  $m_E$  mass is the mass of the engine, without auxiliary components, necessary for the integration of the engine. For our case, the weight of the engine is  $481kg$ .

The values of the constants are:

$k_E = 1.35$  for propeller driven aircrafts, with more than one engine and

$k_{thr} = 1.18$  which accounts for the reverse thrust of the turboprop

$n_E$  is the number of engines, which is 2

The results are:

$$m_{E,inst} = 1.35 \cdot 1.18 \cdot 2 \cdot 481 = 1532.46kg \quad (8.2.17)$$

### **Mass of the systems, $m_{SYS}$**

The estimative equation, according to **Torenbeek 1986**, is:

$$m_{SYS} = k_{EQUIP} \cdot m_{MTO} + 0.768 \cdot k_{F/C} \cdot m_{MTO}^{2/3} \quad (8.2.18)$$

The values of the constants are:

$k_{EQUIP} = 0.11$  for a twin engine propeller driven aircraft

$k_{F/C} = 0.88$  for the transport aircraft form the ATR class, taking into account the actuation way of the surface controls.

This gives a value of:

$$m_{SYS} = 3113.842kg \quad (8.2.19)$$

Another mass should be added to our chart which takes into account the weight of the seats and also other extra weights which were not included in our break down. An average value for each seat weight is 15 kg. This gives:

$$m_{sup} = n_{pax} \cdot m_{seat,extra} = 70 \cdot 15 = 1050kg \quad (8.2.20)$$

In order to check the assumptions a new total operating mass is calculated:

$$m_{OE} = m_W + m_F + m_H + m_V + m_{LG} + m_N + m_{E,inst} + m_{SYS} + m_{SUP} = 12833.62kg \quad (8.2.21)$$

The maximum take-off weight becomes:

$$m_{MTO} = \frac{m_{MPL} + m_{OE}}{M_{ff}} \quad (8.2.21)$$

The maximum payload comes from the number of passengers and we have the value of  $m_{MPL} = 6650kg$  from the Preliminary Sizing chapter. The fuel fraction was also calculated in this chapter and has the value of:  $M_{ff} = 0.853$ . The resulting maximum take-off weight is:  $m_{MTO} = 22841.29kg$ .

These values are very close to the value from Chapter 5, so no other iteration is required.

### 8.3 Center of Gravity Calculation

For the landing gear, nose gear and main gear the center of gravity is given by their position on the aircraft. Until now the position was not fixed in our design, but a first estimation can be made while having in mind other aircrafts as an example.

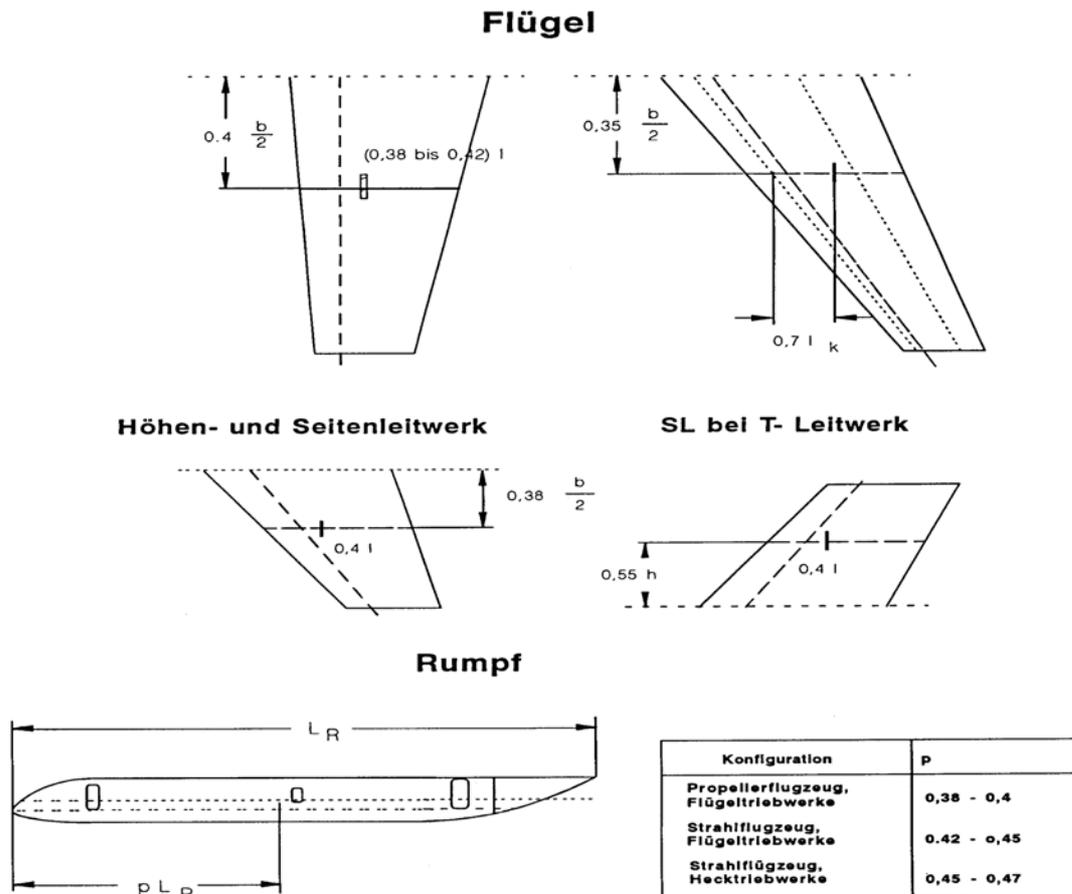
For the systems and equipments we can use the approximation of 40% to 50% of the length of the fuselage for the position of the CG. For propeller driven aircrafts with the engines on the wing, the percentage is a little lower, according to Fig. 8.2: between 38% and 40%.

The position of the CG, while taking into account all the components and their weight, can be approximated with the next formula:

$$x_{CG} = \frac{\sum m_i \cdot x_i}{\sum m_i} \quad (8.3.1)$$

In order to check how the weight increase of one of the components  $m_i$  influences the position of the center of gravity we have to apply the next formula:

$$\Delta x_{CG} = x_i \cdot \frac{\Delta m_i}{\sum m_i} \quad (8.3.2)$$



**Fig. 8.2** Position of CG for main mass groups after **Marckwardt 1998** with data from **Torenbeek 1988 (Scholz 2008)**

A similar situation, when a component moves its position, has to be verified, in order to understand how the center of gravity moves:

$$\Delta x_{CG} = \Delta x_i \cdot \frac{m_i}{\Delta m_i} \quad (8.3.2)$$

Another aspect which has to be taken into account is the position of the wing towards the fuselage, so that the center of gravity has the right position. By now we designed the wing, but did not make any assumptions about its exact location. In order to achieve this, **Scholz 2008** proposes the next method: first we divide the weights into two major components: the fuselage group (containing the fuselage, the empennage and the systems) and the wing group (which considers the wing, the landing gear and the engines). Next a calculation of the moment towards the leading edge of the mean aerodynamic chord is required:

$$(m_{WG} + m_{FG}) \cdot x_{CG,LEMAC} = m_{WG} \cdot x_{WG,LEMAC} + m_{FG} \cdot (x_{FG} - x_{LEMAC}) \quad (8.3.3)$$

For the distance between the LE from the mean aerodynamic chord position until the CG of the airplane can be assumed to be:  $x_{CG,LEMAC} = 0.25 \cdot c_{MAC}$ . This gives the following:

$$x_{LEMAC} = x_{FG} - x_{CG,LEMAC} + \frac{m_{WG}}{m_{FG}} (x_{WG,LEMAC} - x_{CG,LEMAC}) \quad (8.3.4)$$

For the calculations we use a scaled drawing of the aircraft for which we use Fig. 8.2. The results are included in the next table, separately for each group:

**Table 8.5** Estimation of the CG for the mass of the wing group

Massgroup-Wing	Mass [kg]	CG [m]	Mass·CG [kg·m]
Wing	3309.432	$0.4 \cdot 2.2345 + d_{C-FN}^3 = 12.24$	40507.4477
Main Gear	787.044	12.429	9782.1698
Engines	1532.466	10.12	15508.5559
Nacelle	241.865	10.12	1665.4282
$m_{WG}$	$\Sigma = 5870.807$	-	$\Sigma = 68245.85$
	Estimation of CG	$x_{WG} = \frac{\sum m_i \cdot x_i}{\sum m_i}$	11.625

**Table 8.6** Estimation of the CG for the mass of the fuselage group

Massgroup-Fuselage	Mass [kg]	CG [m]	Mass·CG [kg·m]
Fuselage	3373.612 <sup>4</sup>	$0.39 \cdot 27.13 = 10.58$	35692.815
Vertical Tail	178.607	$0.4 \cdot 2.839 + d_{V-FN}^5 = 23.99$	4393.7322
Horizontal Tail	124.445	$0.4 \cdot 1.389 + d_{H-FN}^6 = 25.95$	3229.3477
Nose Gear	172.285	1.75	301.4987
Systems	3113.865	11.5	35809.447
$m_{FG}$	$\Sigma = 6962.814$	-	$\Sigma = 79317.89$
	Estimation of CG	$x_{FG} = \frac{\sum m_i \cdot x_i}{\sum m_i}$	11.392

For estimating the position of the CG for the horizontal and vertical tail (according to Fig. 8.2) we needed first to estimate the mean aerodynamic chord. If we approximate the tails as being simple tapered wings, then we can use the same formulas as we used in chapter 5. The results are:

<sup>3</sup> This distance is the distance between the leading edge of the mean aerodynamic chord and the fuselage nose, where we settled the origin of the reference system and has the value of  $d_{C-FN} = 10.9m$

<sup>4</sup> The fuselage mass contains the supplementary mass from eq. (8.2.20)

<sup>5</sup> This distance is the same distance as specified in the first footnote, but for the vertical tail and has the value  $d_{V-FN} = 22.86m$

<sup>6</sup> This distance is the same distance as specified in the first footnote, but for the horizontal tail and has the value  $d_{H-FN} = 25.394m$

$$c_{MAC,V} = \frac{2}{3} \cdot 3.476 \cdot \frac{1+0.6+0.6^2}{1+0.6} = 2.839m \quad (8.3.5)$$

$$c_{MAC,H} = \frac{2}{3} \cdot 1.701 \cdot \frac{1+0.6+0.6^2}{1+0.6} = 1.389m \quad (8.3.6)$$

As we underlined earlier, we need to determinate the position of the wing by finding the value of  $x_{LEMAC}$ :

$$x_{WG} = x_{LEMAC} + x_{WG,LEMAC} \quad (8.3.7)$$

Until now we have estimated the position of the CG of the fuselage and wing group relative to the origin of the reference system, positioned at the nose of the aircraft.

The process is iterative. The first estimation is an assumption of  $x_{LEMAC} = 10.9m$  of the distance between the leading edge of the mean aerodynamic chord and nose of the fuselage. This assumption comes from statistics (see Fig. 8.2) and was used to already estimate the position of the CG of the wing, with the approximation of  $0.4c_{MAC}$ . Having this value we can determine the  $x_{WG,LEMAC}$ :

$$x_{WG,LEMAC} = x_{WG} - x_{LEMAC} = 12.24 - 10.9 = 1.34m \quad (8.3.8)$$

We can now apply the formula (8.3.4), considering the value of  $25\%c_{MAC}$  for the  $x_{CG,LEMAC}$ , which becomes:  $x_{CG,LEMAC} = 0.25 \cdot 2.2345 = 0.5586m$ . This gives the new value for the distance between the fuselage nose and the leading edge of the wing:

$$x_{LEMAC} = 10.9 - 0.5586 + \frac{5793.511}{6962.814} (1.34 - 0.5586) = 10.99 \approx 11m \quad (8.3.9)$$

The center of gravity of the whole aircraft is given by:

$$x_{CG} = x_{CG,LEMAC} + x_{LEMAC} = 0.5586 + 11 = 11.5586m \quad (8.3.10)$$

According to **Roskam II 1997**, the range of the CG location for a regional turboprop is between  $0.14$  and  $0.27 c_{MAC}$ . We can calculate this range according to **Roskam II 1997**:

$$x_{CG,most,fwd} = x_{CG} - 0.5 \cdot 0.27 \cdot c_{MAC} = 11.25m \quad (8.3.11)$$

$$x_{CG,most,aft} = x_{CG} + 0.5 \cdot 0.27 \cdot c_{MAC} = 11.86m \quad (8.3.12)$$

## 9 Empennage Sizing

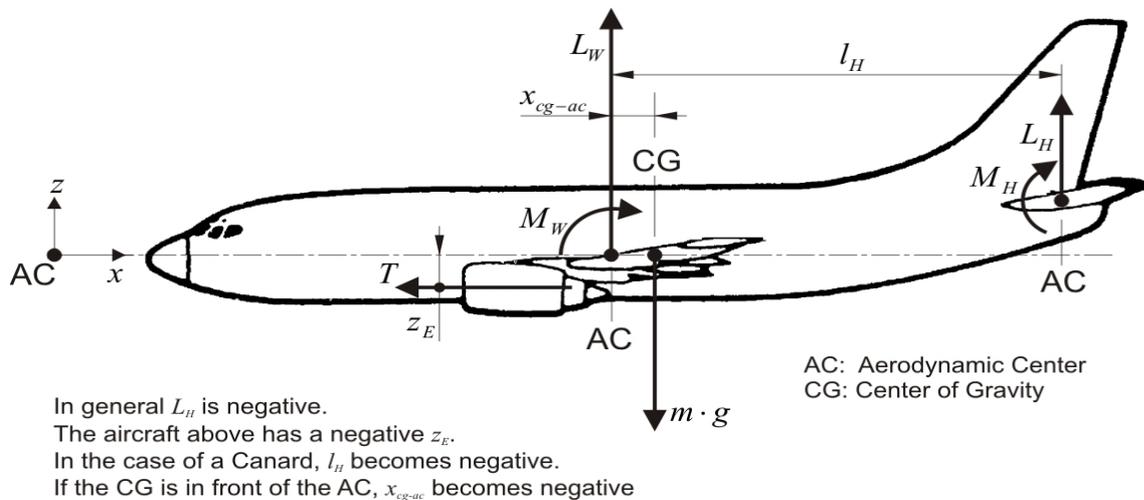
In the first approach (chapter 7) a general view over the main parameters of the empennage was required in order to have rough estimations for the surfaces so to estimate the center of gravity. A more detailed dimensioning, taking into account control and stability requirements, is now needed.

### 9.1 Horizontal Tailplane Sizing

#### 9.1.1 Sizing According to Control Requirement

By writing the moment equation around the lateral axis, in the center of gravity, we obtain the following equation:

$$M_{CG} = M_W + L_W \cdot x_{CG-AC} + M_F + M_E + M_N - L_H \cdot (l_H - x_{CG-AC}) + M_H \quad (9.1.1)$$



**Fig. 9.1** Forces, moments and lever arms to calculate pitching moment (Scholz 2008)

The significance of the parameters is shown in the above Figure, where AC is the aerodynamic center of the wing.

The term  $M_F$  is the pitching moment of the fuselage, neglectable for the control requirement. The term  $M_E$  is the pitching moment through the engines which is caused due to the fact that the thrust does not go directly through the center of gravity or the flow makes an angle with the propeller shaft; other reasons are changes caused in the lift of the wing or in the dynamic pressure or the change in the angle of attack of the horizontal tailplane.  $M_N$ , the moment due to

the nacelles, will not be taken into account. Also the  $M_H$  is not considered, because the horizontal tailplane gives a relatively small moment around the lateral axis.

When simplifying, the equation becomes:

$$M_{CG} = M_W + L_W \cdot x_{CG-AC} + T \cdot z_E - L_H \cdot (l_H - x_{CG-AC}) \quad (9.1.2)$$

For the lift of the horizontal tailplane and wing, the known formulas are valid:

$$\begin{aligned} L_W &= C_{L,W} \cdot q \cdot S_W \\ L_H &= C_{L,H} \cdot q \cdot S_H \end{aligned} \quad (9.1.3)$$

From the stability and control requirements, we know that the empennage has to generate negative lift. It makes sense to define the dynamic pressure ratio, which is smaller than one:

$$\eta_H = \frac{q_H}{q} \quad (9.1.4)$$

The expression of the pitching moment of the wing and around the CG is:

$$\begin{aligned} M_W &= C_{M,W} \cdot q \cdot S_W \cdot c_{MAC} \\ M_{CG} &= C_{M,CG} \cdot q \cdot S_{CG} \cdot c_{MAC} \end{aligned} \quad (9.1.5)$$

For the engine, when writing the moment, we have to take into account that the wing is high, the engines are integrated into the wing, and therefore the  $z_E$  distance is positive:

$$M_E = C_{M,E} \cdot q \cdot S_W \cdot c_{MAC} = -T \cdot z_E = -\frac{\eta_P \cdot P_S}{V} \cdot z_E \quad (9.1.6)$$

We have to have in mind that the turboprop engines have power as output, instead as thrust. The flight phase we have to consider when we proceed with the calculations has to be a critical one for our configuration. The case we consider critical, accounting for the fact that the engines are situated above the center of gravity ( $z_E > 0$ ), is: missed approach, maximum flap position, foremost center of gravity position, for which the propeller efficiency, speed and power settings are known.

If writing the expression of the total lift of the aircraft, as a sum between the lift generated by the wing and the lift generated by the empennage, and extracting correspondingly the moment coefficients, when inserting in equation (9.1.1) we obtain the following:

$$C_{M,CG} \cdot c_{MAC} = C_{M,W} \cdot c_{MAC} + C_{L,W} \cdot x_{CG-AC} + C_{M,E} \cdot c_{MAC} - \eta_H \cdot \frac{S_H}{S_W} \cdot C_{L,H} \cdot (l_H - x_{CG-AC}) \quad (9.1.7)$$

If dividing by  $c_{MAC}$  and changing the notation with:  $\overline{x_{CG-AC}} = \frac{x_{CG-AC}}{c_{MAC}}$  we obtain a simpler equation, which can be further transformed by inserting the expression of the  $C_{L,W}$  from the total lift and by writing the equilibrium equation:  $M_{CG}=0$ . This gives:

$$0 = C_{M,W} + C_L \cdot \overline{x_{CG-AC}} + C_{M,E} - C_{L,H} \cdot \eta_H \cdot \frac{S_H}{S_W} \cdot \frac{l_H}{c_{MAC}} \quad (9.1.8)$$

We can now notice that the area of the horizontal tailplane depends on the CG position of the wing linearly:

$$S_H / S_W = a \cdot \overline{x_{CG-AC}} + b \quad (9.1.9)$$

where the  $a$  and  $b$  constants are:  $a = \frac{C_L}{C_{L,H} \cdot \eta_H \cdot \frac{l_H}{c_{MAC}}}$  and  $b = \frac{C_{M,W} + C_{M,E}}{C_{L,H} \cdot \eta_H \cdot \frac{l_H}{c_{MAC}}}$ .

### Calculating parameters for control requirement

All the parameters from equation (9.1.9) have to be determined in order to plot the control line, which represents a border for the empennage sizing.

$C_{M,E}$  will have a negative value:

$$C_{M,E} = \frac{-1.97}{1/2 \cdot 1.225 \cdot 64.95^2 \cdot 62.187 \cdot 2.2345} \cdot \frac{0.73 \cdot 4102 \cdot 10^3}{64.95} = -0.2529 \quad (9.1.10)$$

In this calculation we considered the efficiency and power in cruise flight.

$\eta_H$  has a value between 0.85 and 0.9; the typical value is 0.9.

$C_{L,H}$  for calculating this parameter unknown data is required, such as the incidence angle of the horizontal tailplane. We establish the value of -0.5, accounting also for stall, due to simplicity reasons.

AC the aerodynamic center for wings with  $A \geq 5$  and  $\varphi_{25} \leq 35^\circ$  is equal with the aerodynamic center of the airfoil, which is roughly  $0.25\% c_{MAC}$ .

$C_{L,H}$  the lift coefficients can be calculated from relation (9.1.11):

$$\begin{aligned} C_{L,W} &= C_{L,\alpha,W} \cdot (\alpha + i_W - \alpha_{0,W}) \\ C_{L,H} &= C_{L,\alpha,H} \cdot (\alpha + i_H - \varepsilon - \alpha_{0,H}) \end{aligned} \quad (9.1.11)$$

In these formulas the  $\varepsilon$  represents the downwash angle caused by the wing and the  $\alpha_0$  is the zero lift angle of attack.

$$\alpha_{0,W} = \alpha_0 + \frac{\Delta\alpha_0}{\varepsilon_t} \cdot \varepsilon_t \quad (9.1.12)$$

The lift coefficient for the wing, however, was already estimated in the Preliminary Sizing Chapter, under the conditions of missed approach, and has the value of:  $C_{L,W} = 1.44$ .

$\overline{x_{CG-AC}}$  we have to consider the foremost position of the center of gravity, which is situated at 14% of the mean aerodynamic chord. In the same time the aerodynamic center is situated at 25% of the chord, which gives a value of 11% for  $\overline{x_{CG-AC}}$ , representing 0.2457 meters.

$C_{M,W}$  is the last value we need to estimate and for achieving this we make use of DATCOM methods; the value will be negative and increases if the flaps are extended.

The pitching moment of the wing according to **DATCOM 1978** has the following expression:

$$C_{M,W} = \left[ c_{M,0,flaped} \cdot \frac{A \cdot \cos^2 \varphi_{25}}{A + \cos \varphi_{25}} + \left( \frac{\Delta c_{m,0}}{\varepsilon_t} \right) \cdot \varepsilon_t \right] \cdot \frac{(c_{m,0})_M}{(c_{m,0})_{M=0}} \quad (9.1.13)$$

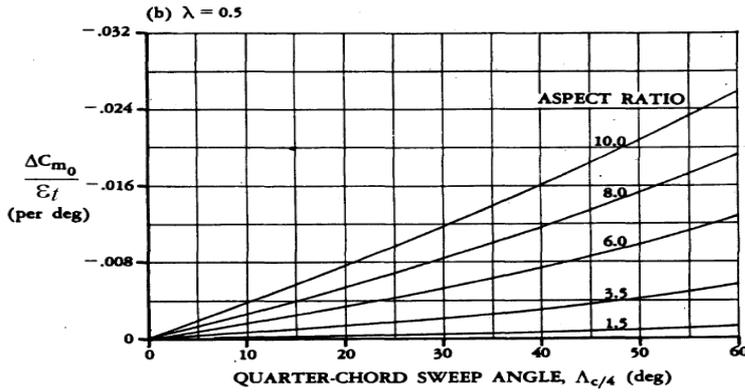
The terms are explained below:

$\left( \frac{\Delta c_{m,0}}{\varepsilon_t} \right)$  is the change to the pitching moment coefficient with respect to the aerodynamic center per degree of linear wing twist; we can read the value of -0.001 from Fig. 9.2 for the aspect ratio of 12 and the taper ratio of 0.5; the value is very small, due to the very small sweep angle of the wing.

$\frac{(c_{m,0})_M}{(c_{m,0})_{M=0}}$  is the Mach number influence; due to the fact that, in the case of missed approach, the Mach number is below 0.2, this term is equal to 1.

$c_{M,0,flaped}$  is the pitching moment of the airfoil at the aerodynamic center when the flaps are extended; for calculating this coefficient, we need the **DATCOM 1978** assumptions:

$$c_{M,0,flaped} = c_{M,0} + \Delta c_M \quad (9.1.14)$$



**Fig. 9.2** Influence of linear wing twist on pitching moment coefficient with respect to the wing aerodynamic center (**DATCOM 1978**)

The  $c_{M,0}$  can be taken from airfoil catalogues, like **Abbott 1959**; if the data is unavailable, such it is our case, a value of -0.1 is a good assumption.

The  $\Delta c_M$  value for slotted flaps is given by:

$$\Delta c_M = \Delta c_{L,flapped} \cdot \left( \frac{x_{AC}}{c_{MAC}} - \frac{x_{CP}}{c_{MAC}} \cdot \left( \frac{c'}{c} \right) \right) \quad (9.1.15)$$

The  $\frac{x_{CP}}{c_{MAC}}$  parameter has the value of 0.44 and represents the position of the center of pressure CP in relation to the length of the aerodynamic center.

For the  $\Delta c_{L,flapped}$ , which represents the increase in the maximum lift coefficient of the airfoil, due to flap extension, we have already estimated a value in the High Lift Devices chapter:

$\Delta c_{L,flapped} = 1.089$ ; in the same chapter we also have an estimation for  $\frac{c'}{c}$ , which has the value of 1.173. This information allows us to obtain the  $\Delta c_M$  value and the value of the pitching moment coefficient:

$$\Delta c_M = 1.089(0.25 - 0.44 \cdot 1.173) = -0.2898 \quad (9.1.16)$$

$$c_{M,0,flaped} = -0.1 - 0.2898 = -0.3898 \quad (9.1.17)$$

The pitching moment of the entire wing, as it is described by the formula (9.1.13) becomes:

$$C_{M,W} = \left[ -0.3898 \cdot \frac{12 \cdot \cos^2(3^\circ \cdot \pi / 180^\circ)}{12 + \cos(3^\circ \cdot \pi / 180^\circ)} - 0.001 \cdot (-3^\circ \cdot \pi / 180^\circ) \right] = -0.359 \quad (9.1.18)$$

We have to have in mind that after positioning the center of gravity of the aircraft and accordingly adjusting the wing position, the lever arm of the horizontal and vertical tailplane changes. By making a simple calculation having the distance from the nose until the leading edge of the mean aerodynamic chord, the length of the fuselage and the position of 25% for the aerodynamic centers for the wing and empennage, we obtain the following values:

$$\begin{aligned} l_H &= l_F - x_{LEMAC} - 1.5m = 27.13 - 11 - 1.5 = 14.63m \\ l_V &= l_F - x_{LEMAC} - 1.3m = 27.13 - 11 - 1.7 = 14.43m \end{aligned} \quad (9.1.19)$$

This allows us to calculate the  $a$  and  $b$  coefficients, and illustrate the linear variation:

$$a = \frac{C_L}{C_{L,H} \cdot \eta_H \cdot \frac{l_H}{c_{MAC}}} = \frac{1.44}{-0.5 \cdot 0.9 \cdot \frac{14.63}{2.2345}} = -0.4887 \quad (9.1.20)$$

$$b = \frac{C_{M,W} + C_{M,E}}{C_{L,H} \cdot \eta_H \cdot \frac{l_H}{c_{MAC}}} = b = \frac{-0.359 - 0.2529}{-0.5 \cdot 0.9 \cdot \frac{14.63}{2.2345}} = 0.20768 \quad (9.1.21)$$

### 9.1.2 Sizing According to Stability Requirement

An aircraft is stable around the lateral axis if the gradient of the pitching moment over the angle of attack,  $\frac{\partial C_{M,CG}}{\partial \alpha}$ , is negative and  $C_{M,CG}(\alpha = 0)$  is positive.

For sizing the empennage we choose the limit  $C_{M,CG} = 0$ , indifferent stability, and rewrite the equation:

$$0 = C_{L,\alpha,W} \cdot \overline{x_{CG-AC}} - C_{L,\alpha,H} \cdot \eta_H \cdot \frac{S_H}{S_W} \cdot \left(1 - \frac{\partial \varepsilon}{\partial \alpha}\right) \cdot \left(\frac{l_H}{c_{MAC}} - \overline{x_{CG-AC}}\right) \quad (9.1.22)$$

The condition for sizing the horizontal tailplane according to stability requirements becomes:

$$\frac{S_H}{S_W} = \frac{C_{L,\alpha,W} \cdot \overline{x_{CG-AC}}}{C_{L,\alpha,H} \cdot \eta_H \cdot \left(1 - \frac{\partial \varepsilon}{\partial \alpha}\right) \cdot \left(\frac{l_H}{c_{MAC}} - \overline{x_{CG-AC}}\right)} \quad (9.1.23)$$

Because  $\frac{l_H}{c_{MAC}} \gg \overline{x_{Cg-AC}}$ , we can assume that the variation is very close to a linear variation.

In order to obtain this equation, we need to estimate the parameters within. The lift-curve-slope was also approached in the Wing Design chapter, having the value of  $C_{L,\alpha,W} = 5.89$ .

In order to illustrate the straight variation of the non-dimensional area of the horizontal tailplane, we first need to estimate the downwash gradient,  $\frac{\partial \varepsilon}{\partial \alpha}$ .

$$\frac{\partial \varepsilon}{\partial \alpha} = 4.44 \cdot \left[ k_A \cdot k_\lambda \cdot k_H \cdot \sqrt{\cos \varphi_{25}} \right]^{1.19} \cdot \frac{(C_{L,\alpha})_M}{(C_{L,\alpha})_{M=0}} \quad (9.1.24)$$

The factors from the formula have the following signification:

$k_A$ , accounting for wing aspect ratio

$$k_A = \frac{1}{A} - \frac{1}{1-A^{1.7}} = \frac{1}{12} - \frac{1}{1-12^{1.7}} = 0.0981 \quad (9.1.25)$$

$k_\lambda$ , accounting for wing taper ratio

$$k_\lambda = \frac{10-3\lambda}{7} = \frac{10-3 \cdot 0.5927}{7} = 1.1745 \quad (9.1.26)$$

$k_H$ , accounting for the position of the horizontal tailplane

$$k_H = \frac{1 - \frac{|z_H|}{b}}{\sqrt[3]{\frac{2l_H}{b}}} = \frac{1 - \frac{3.4}{27.32}}{\sqrt[3]{\frac{2 \cdot 14.63}{27.32}}} = 0.856 \quad (9.1.27)$$

The last term from the equation (9.1.24) represents the lift curve slopes for  $M=0$  and  $M \neq 0$ . In the Wing Design chapter we have already calculated the lift curve slope, in cruise conditions, using the formula:

$$C_{L,\alpha} = \frac{2\pi A}{2 + \sqrt{A^2(1 + \tan^2 \varphi_{50} - M^2) + 4}} \quad (9.1.28)$$

For  $M=0$ , the lift curve slope becomes:

$$C_{L,\alpha} = \frac{2\pi \cdot 12}{2 + \sqrt{12^2(1+0-0)+4}} = 5.323 \quad (9.1.29)$$

$$C_{L,\alpha} = \frac{2\pi \cdot 12}{2 + \sqrt{12^2(1+0-0.191^2)+4}} = 5.406 \quad (9.1.30)$$

This gives the value of:  $\frac{(C_{L,\alpha})_M}{(C_{L,\alpha})_{M=0}} = \frac{5.406}{5.323} = 1.015$ . When inserting the results into the equation, we obtain the downwash gradient:

$$\frac{\partial \varepsilon}{\partial \alpha} = 4.44 \cdot \left[ 0.0981 \cdot 1.1745 \cdot 0.856 \cdot \sqrt{\cos(3^\circ \cdot \pi / 180^\circ)} \right]^{1.19} \cdot 1.015 = 0.286 \quad (9.1.31)$$

The  $C_{L,\alpha,H}$  can be approximated with the same formula (9.1.28):

$$C_{L,\alpha} = \frac{2\pi A_H}{2 + \sqrt{A_H^2(1 + \tan^2 \varphi_{50,H} - M^2) + 4}} = \frac{2\pi \cdot 6}{2 + \sqrt{6^2(1+0-1.191^2)+4}} = 4.586 \quad (9.1.32)$$

The linear variation of the area, summed by the equation:  $S_H / S_W = a \cdot \overline{x_{CG-AC}}$ , has the following slope, which should be positive:

$$a = \frac{C_{L,\alpha,W}}{C_{L,\alpha,H} \cdot \eta_H \cdot \left(1 - \frac{\partial \varepsilon}{\partial \alpha}\right) \cdot \left(\frac{l_H}{c_{MAC}}\right)} = \frac{5.89}{4.586 \cdot 0.9 \cdot (1 - 0.286) \cdot \left(\frac{14.63}{2.2345}\right)} = 0.305 \quad (9.1.33)$$

### 9.1.3 Conclusions - Horizontal Tailplane

The two equations we determined represent the boundaries for the horizontal tailplane area: equation (9.1.9) is the control boundary and equation (9.1.23), with the respective simplification, is the stability boundary. However, for obtaining these equations, we assumed a case situated on the limit of stability, which means we have to consider a safety margin when determining the minimum area for the horizontal tail.

In chapter 10 we established the location of the CG between the range of 14%-27% of the mean aerodynamic chord. According to the diagram from Fig. 9.3, this range requires a minimum area for the tail.

The static margin is a percentage from the mean aerodynamic chord; for regional turbopropellers the value is 5% of mean aerodynamic chord.

The pitching moment coefficient is given by:

$$C_{M,\alpha} = \frac{\partial C_{M,CG}}{\partial \alpha} = -C_{L,\alpha} \cdot (\text{static margin}) = -C_{L,\alpha} \cdot 5\% \cdot c_{MAC} = -5.89 \cdot 0.05 \cdot 2.2345 = 0.658$$

As we calculated in the CG chapter, the position of the center of gravity, relative to the mean aerodynamic chord is:  $\frac{x_{CG,LEMAC} - x_{AC}}{c_{MAC}} = 0.5586 - 0.25 = 0.3086$

When we read the corresponding value on the resulting diagram, taking into account also the safety margin of 5%, we obtain the value of  $\frac{S_H}{S_W} = 0.156 \Rightarrow S_H = 0.156 \cdot 62.187 = 9.701m^2$  for the horizontal tail area.

In chapter 7, we assumed a horizontal tailplane area of  $S_H = 10.756m^2$ . If the difference between the two is greater than 10%, we should recalculate the  $m_H$ , and obtain a new position of the CG. The difference is of 9.8%.

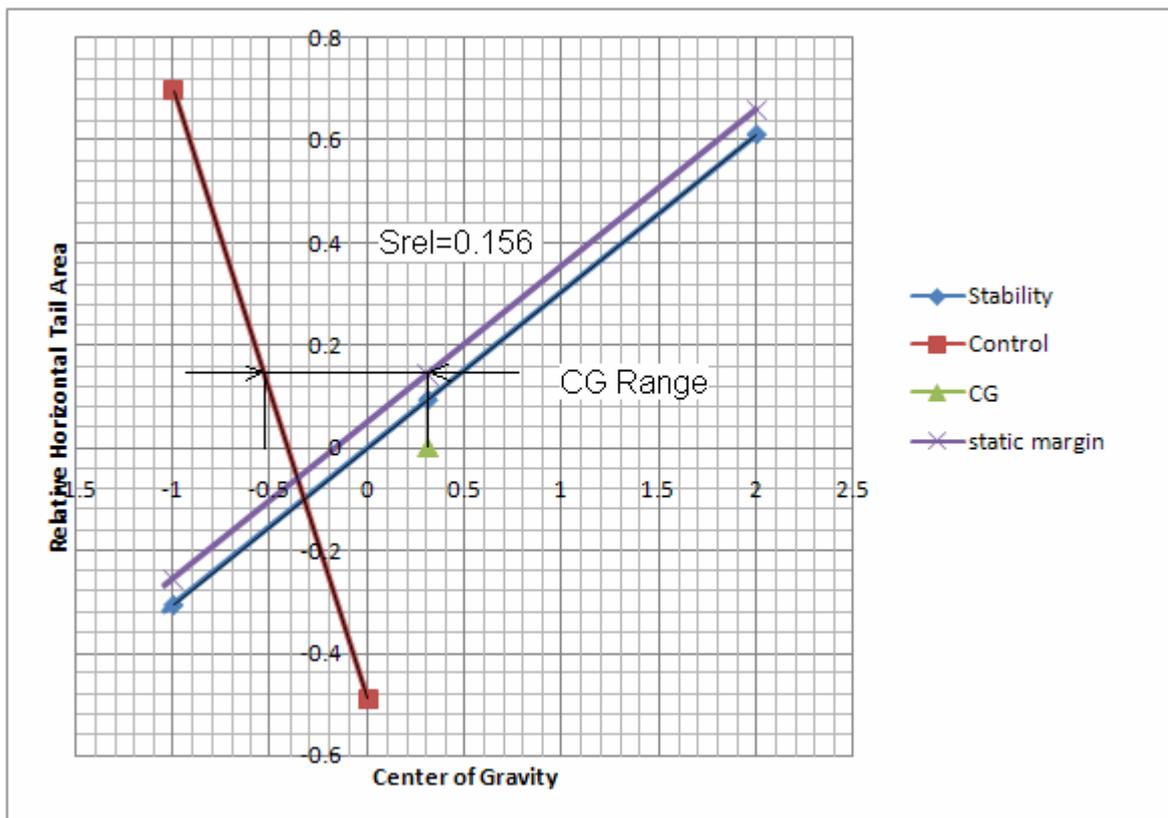


Fig. 9.3 Diagram to determine the minimum required relative horizontal tail area

## 9.2 Vertical Tailplane Sizing

### 9.2.1 Sizing According to Control Requirement

The flight phase we should consider for dimensioning in our case is the case when one engine fails during take-off. If one engine fails, the active engine produces the following moment:

$$N_E = \frac{T_{TO}}{n_E} \cdot y_E = \frac{\eta \cdot P}{V \cdot n_E} \cdot y_E \quad (9.2.1)$$

In our case the number of engines is  $n_E = 2$  and  $y_E$  is the distance between the failed engine and the plane of symmetry of the aircraft.

The rudder has to produce a compensating moment, accounting also for the drag produced by the failed engine:

$$N_E + N_D = N_V \quad (9.2.2)$$

The  $N_D$  moment has a value of  $N_D = 0.25 \cdot N_E$  (**Scholz 2008**) for propeller driven aircrafts with variable pitch propeller.

The compensating moment is given by **DATCOM 1978**:

$$N_V = \frac{1}{2} \rho V_{MC}^2 \cdot \delta_F \left[ \frac{c_{L,\delta}}{(c_{L,\delta})_{theory}} \right] \cdot (c_{L,\delta})_{theory} \cdot K' \cdot K_\Lambda \cdot S_V \cdot l_V \quad (9.2.3)$$

According to **JAR 25.149 (b)**: “ $V_{MC}$  is the calibrated airspeed, at which, when the critical engine is suddenly made inoperative, it is possible to maintain control of the aeroplane with that engine still inoperative, and maintain straight flight with an angle of bank of not more than  $5^0$ ” and **JAR 25.149 (c)**: “ $V_{MC}$  may not exceed  $1.2 \cdot V_S$ ”.

When combining the equations, we obtain the required vertical tail area:

$$S_V = \frac{N_E + N_D}{\frac{1}{2} \rho V_{MC}^2 \cdot \delta_F \left[ \frac{c_{L,\delta}}{(c_{L,\delta})_{theory}} \right] \cdot (c_{L,\delta})_{theory} \cdot K' \cdot K_\Lambda \cdot l_V} \quad (9.2.4)$$

The  $V_{MC}$ , the minimum control speed with one engine inoperative, according to CS 25, is:

$$V_{MC} = 1.2 \cdot V_S \tag{9.2.5}$$

In the same time, the approach speed is  $V_{APP} = 1.3 \cdot V_S$ . This gives the result for the  $V_{MC}$ :

$$V_{MC} = (1.3/1.2) \cdot V_{APP} = (1.3/1.2) \cdot 60.4 = 65.43m/s \tag{9.2.6}$$

The moment produced by the engine is:

$$N_E = \frac{\eta \cdot P}{V \cdot n_E} \cdot y_E = \frac{0.73 \cdot 2051 \cdot 10^3 W}{65.43m/s \cdot 2} \cdot 3.608m = 41280.8 \tag{9.2.7}$$

This gives:

$$N_D = 0.25 \cdot N_E = 0.25 \cdot 41280.8 = 10320.2 \tag{9.2.8}$$

The required rudder deflection should be no more than:  $\delta_F \leq 25^0$ .

The term  $\delta_F \left[ \frac{C_{L,\delta}}{(C_{L,\delta})_{theory}} \right] \cdot (C_{L,\delta})_{theory}$  represents the increase in lift  $\Delta C_{L,Flap}$  because the rudder is actually a plain flap and should be correspondingly calculated.

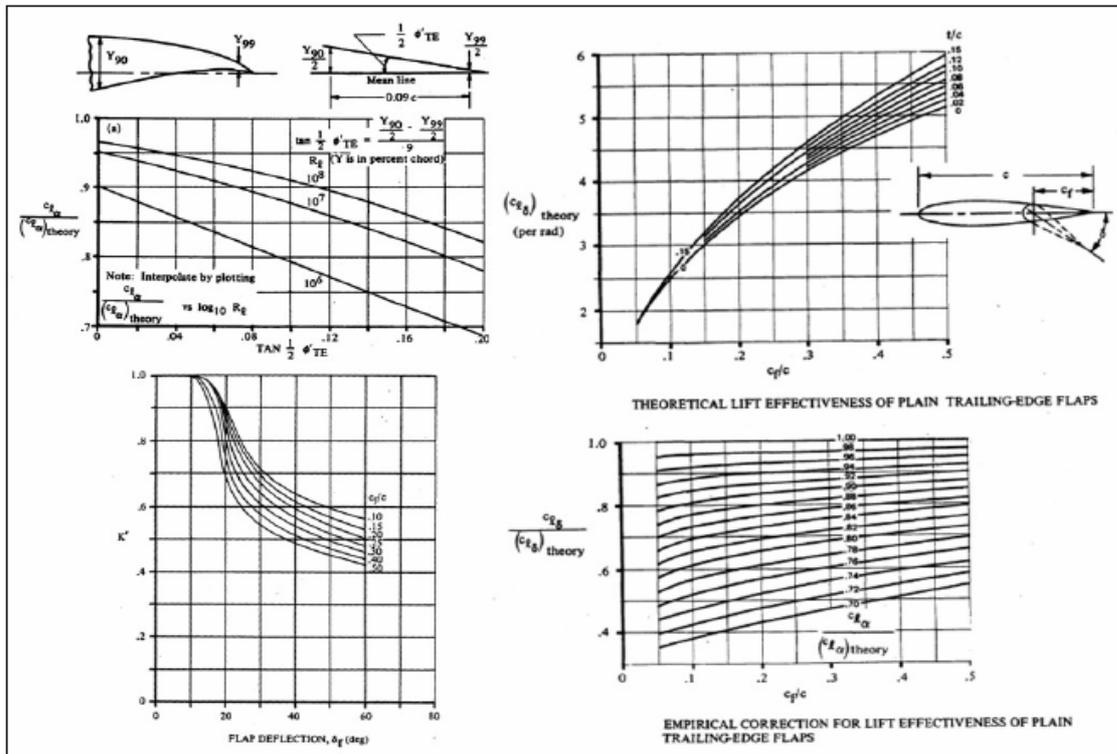


Fig. 9.3 Parameters for the Vertical Tailplane Sizing (DATCOM 1978)

The  $c_f/c$  we assume to be not less than 30%, so that the rudder effectiveness would be sufficient. With this value we can read a value of  $K' = 0.675$  and the value of  $4.52 \cdot 1/\text{rad}$  for the  $(c_{L,\delta})_{theory}$ , from Fig. 9.3.

For estimating the other coefficients we need the trailing edge angle for the airfoil, which is NACA 0012:  $\phi_{TE} = 13^\circ$ ;  $\tan(\frac{1}{2}\phi_{TE}) = 0.114$ . For a Reynolds number of  $10^8$ , we can read the value of 0.9 for the  $\frac{c_{L,\alpha}}{(c_{L,\alpha})_{theory}}$ , which allows us to read a value of 0.85 for  $\frac{c_{L,\delta}}{(c_{L,\delta})_{theory}}$ .

The  $K_\Lambda$  factor accounts for the sweep; for estimating it we can use the formula (6.1.13) from the High Lift Systems chapter. This gives the value of for a sweep angle of  $25^\circ$ , as we assumed in chapter 8.

$$K_\varphi = (1 - 0.08 \cos^2 \varphi_{25}) \cdot \cos^{3/4} \varphi_{25} = [1 - 0.08 \cos^2 (25^\circ \cdot \pi / 180^\circ)] \cdot \cos^{3/4} (25^\circ \cdot \pi / 180^\circ) = 0.862 \quad (9.2.9)$$

We now have all the data for calculating the required empennage surface; we make the following observation: the ATR 72 is the stretched version of the ATR 42; the manufacturer took into account for the stretched version the same type of configuration for the major components like wing or empennage, so to keep under control the production costs; new configurations would lead to a new design, so a new aircraft, and the costs would be correspondingly much higher; if we calculate the vertical empennage area of the ATR 72 with the estimated lever arm, this would lead to a smaller surface, which would not be sufficient to satisfy the stability requirements; this means we have to consider for our calculations the lever arm of the ATR 42, which is about 5 meters smaller; the  $2\pi$  comes from the angular speed of the propeller.

$$S_V = \frac{(41280.8 + 10320.2) \cdot 2 \cdot \pi}{\frac{1}{2} \cdot 1.225 \cdot 65.43^2 \cdot 25^\circ \cdot \pi / 180^\circ \cdot 0.85 \cdot 4.52 \cdot 0.675 \cdot 0.862 \cdot 9.13} = 14.085m \quad (9.2.10)$$

## 9.2.2 Sizing According to Stability Requirement

We have to write the sum of the moments around the vertical axis:

$$N_{CG} = N_W + N_F - L_V \cdot l_V \quad (9.2.11)$$

In the same time:

$$N_{CG} = C_{N,\beta,F} \cdot \beta \cdot q \cdot S_W \cdot b \quad (9.2.12)$$

The  $N_F$  is the moment reduced by the fuselage, and has a destabilizing effect:

$$N_F = C_{N,\beta,F} \cdot \beta \cdot q \cdot S_W \cdot b \quad (9.2.13)$$

The  $N_W$  is the wing moment and has a stabilizing effect if the wing has an aft sweep. The sweep in our design case is very small, due to the fact that our aircraft flies at lower speeds:

$$N_W = C_{N,\beta,W} \cdot \beta \cdot q \cdot S_W \cdot b \quad (9.2.14)$$

This is the reason for which the influence of the wing will be neglected.

The  $L_V$  is the transverse force caused by the vertical tailplane:

$$L_V = C_{Y,\beta,V} \cdot \beta \cdot q \cdot S_V \quad (9.2.15)$$

When introducing all the data in the equation (9.2.11) and dividing by  $\beta$ ,  $q$ ,  $S_W$  and  $b$ , we can determine the non-dimensional area of the vertical tailplane, for matching the static stability:

$$\frac{S_V}{S_W} = \frac{C_{N,\beta} - C_{N,\beta,F}}{-C_{Y,\beta,V}} \cdot \frac{b_W}{l_V} \quad (9.2.16)$$

**Roskam II 1997** gives an assumption for the  $C_{N,\beta}$  factor that should be sufficient for the stability requirements:

$$C_{N,\beta} \geq 0.001 \frac{1}{\text{deg}} = 0.0571 \frac{1}{\text{rad}} \quad (9.2.17)$$

The stability coefficient  $C_{N,\beta,F}$  can be calculated with **DATCOM 1978** methods; this coefficient provides a yawing moment,  $N$ , caused by the sideslip angle  $\beta$  due to the aerodynamic impact on the fuselage. The **DATCOM 1978** method accounts for this interference:

$$C_{N,\beta,F} = -\frac{360}{2\pi} \cdot k_N \cdot k_{R,J} \cdot \frac{l_F^2 \cdot d_F}{S_W \cdot b} \quad (9.2.18)$$

The  $k_N$  factor for cylindrical fuselages has the following form:

$$k_N = 0.01 \cdot \left[ 0.27 \cdot \frac{x_M}{l_F} - 0.168 \cdot \ln\left(\frac{l_F}{d_F}\right) + 0.416 \right] - 0.0005 \quad (9.2.19)$$

The  $x_m$  is the distance from the nose of the aircraft until the center of gravity, which is  $x_m=11.5586\text{m}$ . this gives the value of:

$$k_N = 0.01 \cdot \left[ 0.27 \cdot \frac{11.5586}{27.13} - 0.168 \cdot \ln\left(\frac{27.13}{2.77}\right) + 0.416 \right] - 0.0005 = 9.769 \cdot 10^{-4} \quad (9.2.20)$$

The  $k_{R,J}$  has the following form:

$$k_{R,J} = 0.46 \cdot \log\left(\frac{\text{Re}}{10^6}\right) + 1 \quad (9.2.21)$$

The Reynolds number of the fuselage is calculated having as reference values the length of the fuselage, the cruise speed and a kinematic viscosity of  $\nu = 5.4603 \cdot 10^{-5} \text{ m}^2 / \text{s}$ :

$$\text{Re} = \frac{V_{CR} \cdot l_F}{\nu} = \frac{141.944 \cdot 27.13}{5.4603 \cdot 10^{-5}} = 7.05 \cdot 10^7 \quad (9.2.22)$$

This gives the value of:

$$k_{R,J} = 0.46 \cdot \log\left(\frac{7.05 \cdot 10^7}{10^6}\right) + 1 = 1.85 \quad (9.2.23)$$

The stability coefficient becomes:

$$C_{N,\beta,F} = -\frac{360}{2\pi} \cdot 9.769 \cdot 10^{-4} \cdot 1.85 \cdot \frac{27.13^2 \cdot 2.77}{62.187 \cdot 27.32} = -0.124 \quad (9.2.24)$$

The second coefficient we need to determine is  $C_{Y,\beta,V}$  which accounts for a side force,  $Y$ , caused by the sideslip  $\beta$ , due to the aerodynamic impact on the vertical fin. A simplified version can be applied by means of **DATCOM 1978**:

$$C_{Y,\beta,V} = -(C_{L,\alpha})_V \quad (9.2.25)$$

$$C_{L,\alpha} = \frac{2\pi A_V}{2 + \sqrt{A_V^2 (1 + \tan^2 \varphi_{50,V} - M^2) + 4}} \quad (9.2.26)$$

The new  $\tan(\varphi_{50,V})$  for the vertical tailplane, calculated with the formula (5.11.3) from the Wing Design chapter, is  $\tan(\varphi_{50,V}) = 0.31$ .

The final result for the coefficient, when inserting all the additional values, becomes:

$$C_{L,\alpha} = \frac{2\pi \cdot 1.6}{2 + \sqrt{1.6^2 (1 + 0.31^2 - 0.44^2) + 4}} = 2.228 \quad (9.2.27)$$

This gives:  $C_{Y,\beta,V} = -2.228$ .

We are now able to obtain the non-dimensional area of the vertical tailplane:

$$\frac{S_V}{S_W} = \frac{0.0571 - (-0.124)}{-(-2.228)} \cdot \frac{27.32}{14.43} = 0.1539 \quad (9.2.28)$$

The area of the vertical tailplane becomes:

$$S_V = \left( \frac{S_V}{S_W} \right) \cdot S_W = 0.1539 \cdot 62.187 = 9.57 m^2 \quad (9.2.29)$$

For sizing the vertical empennage we have to choose between the area resulted from control requirements and the one resulting from stability requirements; a safe sizing implies choosing the larger area of the two, which is:  $S_V = 14.085 m^2$ .

## 10 Landing Gear Conceptual Design and Integration

The main functions of the landing gear are related to take-off, landing (by absorbing the bumps and by braking) and taxi (for allowing the aircraft to move on the ground); for cruise the landing gear has to be retractable.

Our design case has a high wing; this makes the integration of the landing gear difficult. The integration into the wing is completely inefficient; this means that large fairings are required to integrate the landing gear into the fuselage. These fairings, however, bring an increase of the drag, which is to be compensated by the high lift systems, for example.

### 10.1 Position of the Landing Gear

The position of the landing gear is strictly correlated with the CG aft position of the airplane, in order to prevent tail tipping. Also, the side tipping has to be prevented by the wide width of the gear, allowing the full envelope of ground maneuvers.

Because the wing is high-positioned the track of the landing gear will be limited which gives a very low ground line and this means we have to pay a special attention to the tail clearance. For the track the value of  $y_{track} = 4.10m$  corresponds to the real design.

The three parts of the landing gear form a triangle, like in Fig. 10.1. The position of the landing gear relatively to the center of gravity has to insure that the airplane does not tip over the lateral axis:

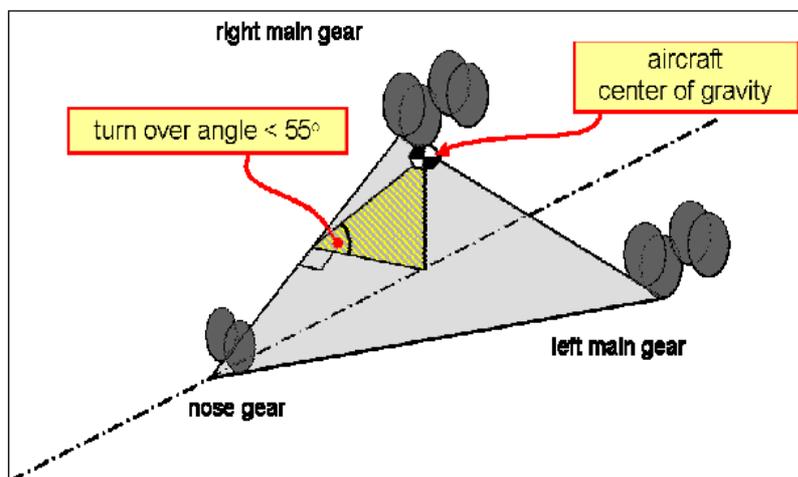


Fig. 10.1 Landing gear position, so to prevent lateral turn-over (Trahmer 2008)

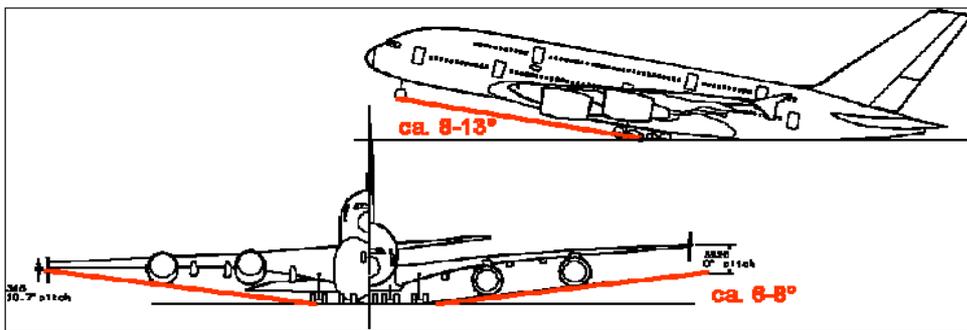
The distance between the nose gear and the nose of the fuselage is 1.75 m and the distance between the main gear and the fuselage aft part is 14.61 m. This gives the distance of:  $x_{LG,N-LG,M} = 27.13 - 1.75 - 14.61 = 10.77\text{m}$  between the nose gear and the landing gear.

The tail clearance is controlled by means of two angles (**Trahmer 2008**):

*static tail clearance angle*: landing case, fully compressed damper  
between  $6-8^\circ$

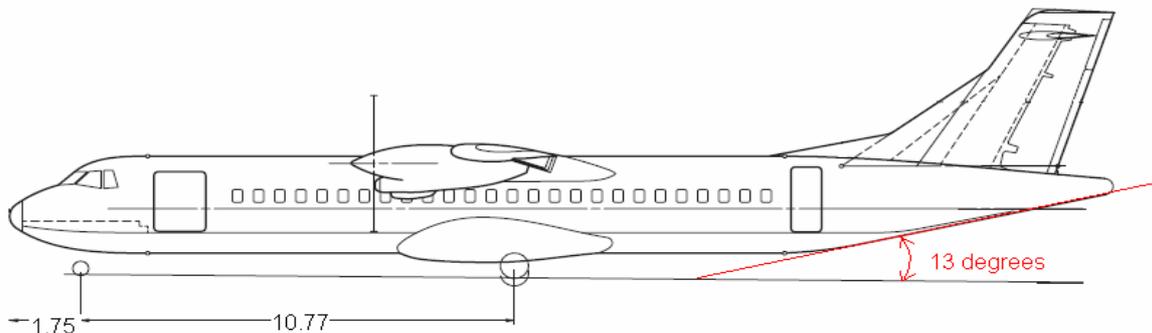
*extended tail clearance angle*: take-off, fully extended damper  
between  $8-13^\circ$

*chosen angles according to task*:  $8^\circ$  for static clearance;  $8^\circ$  for extended clearance (Fig 10.2)

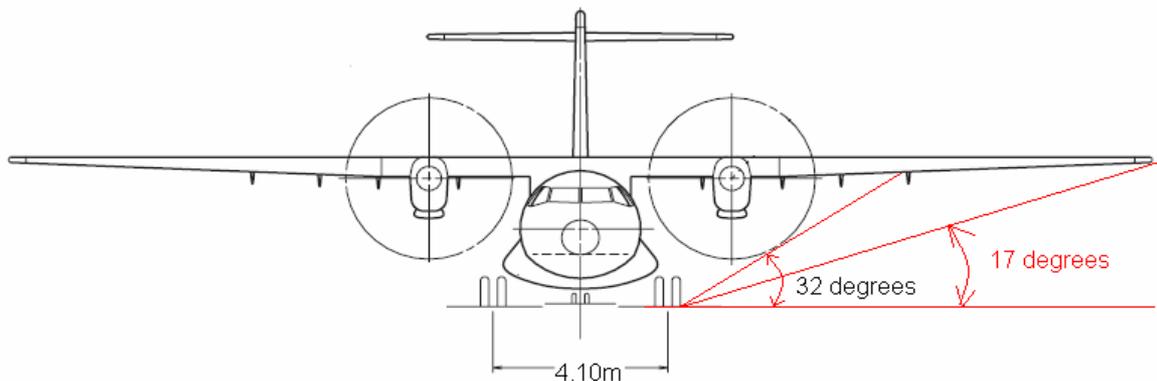


**Fig. 10.2** Tail clearance and bank clearance angle (**Trahmer 2008**)

The ATR 72 has the following layout:



**Fig. 10.3** Tail clearance during take-off, for the ATR 72



**Fig. 10.4** Lateral clearance when one engine fails, for the ATR 72

It is necessary to evaluate the CG location on the vertical axis of the aircraft. For this estimation we use the same method as in chapter 8 (see Table 10.1):

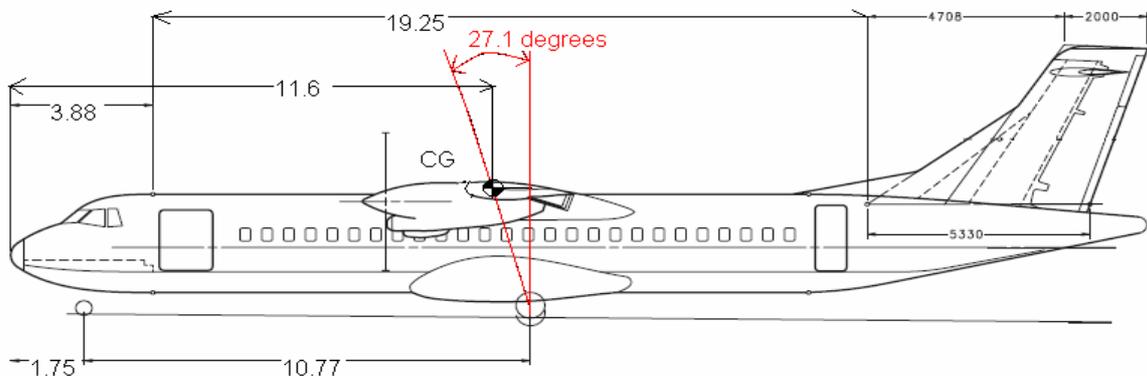
**Table 10.1** Estimation of the CG location over the vertical axis

Massgroup	Mass [kg]	CG [m]	Mass·CG[kg·m]
Fuselage	3373.612	0.6	2024.1672
Nacelle	164.568	1.56	256.72608
Engines	1532.466	1.56	2390.64696
Systems	3113.865	-0.51	-1588.07115
Wing	3309.432	1.5	4964.148
Vertical Tail	178.607	$0.55 \cdot h = 1.265^7$	225.937855
Horizontal Tail	124.445	5.03	625.95835
Nose Gear	172.285	-1.38	-237.7533
Main Gear	787.044	-1.38	-1086.12072
$m_{OE}$	$\Sigma = 12758.325$	-	$\Sigma = 7575.639$
	Estimation of CG	$z_{CG} = \frac{\sum m_i \cdot z_i}{\sum m_i}$	0.59378

### Turn over angle in x-axis direction

In order to prevent the tip over of the aircraft, the position of the main landing gear has to be accordingly behind the CG, so that the angle between the vertical axis and the line meeting the most aft position of the CG and the landing gear is not smaller than  $15^\circ$ , according to **Roskam II**. This angle is calculated with the following formula (10.1). The  $z_{f,u}$  indicates the length of the main gear, between the fuselage and the floor, when the aircraft is empty. In our case  $z_{f,u} = 0.7m$ .

$$\psi_L = \arctan \frac{x_{LG,M} - x_{CG,most,aft}}{z_{f,u} + z_{CG}} = \arctan \frac{(10.77 + 1.75) - 11.86}{0.7 + 0.59} = 0.473 \Leftrightarrow \psi_L = 27.1^\circ \quad (10.1)$$



**Fig. 10.5** Turn over angle in x-axis direction

<sup>7</sup>

According to Fig. 8.2, when  $h$  is the span of the vertical tail, with the value of  $4.3m$

### Turn over angle in y-axis direction

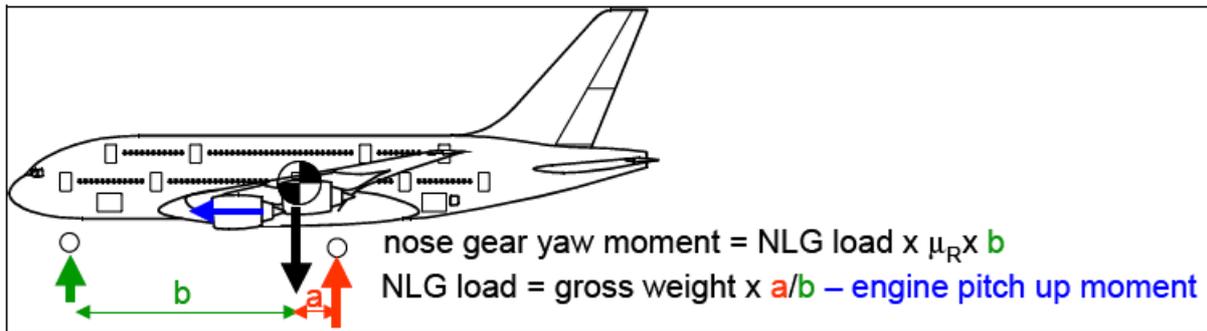
In order to prevent the airplane from rolling over when in a turn, when it is maneuvered on ground, another angle has to be calculated. It is the angle from **Fig. 10.1**, which, according to the international certification requirements, has to be less than 55°:

$$\psi_Q = \arctan \frac{z_{f,u} + z_{CG}}{y_{track} \cdot (x_{CG,most,aft} - x_{NG})} = \arctan \frac{0.7 + 0.59}{4.1 \cdot (11.86 - 1.75)} = 0.591 \Leftrightarrow \psi_Q = 33.862^\circ \quad (10.2)$$

$$2 \cdot x_{LG,N-LG,M} \quad 2 \cdot 10.77$$

### Nose landing gear load

The airplane has to be controllable also when an engine fails. The nose gear has to bear the lateral load which is dependent on the relative position of the CG and the main landing gear. The maximum load that the tires can generate is the vertical load multiplied by a friction coefficient; so the distance between the main and nose gear has to be sufficient to provide enough load on the nose gear, so that the airplane does not tip over when an engine fails (see Fig. 10.6).



**Fig. 10.6** Nose gear load drivers (Trahmer 2008)

Accordingly, the nose gear load is:

$$NLGload = 23296.272kg \cdot \frac{0.96}{10.77} - (-90825.03W \cdot s \cdot \frac{1}{m}) = 92901.58kg \cdot W \cdot s \quad (10.3)$$

The controllability on ground, when fast maneuvers are required, depends on the nose gear lateral load multiplied by the wheel base, and not so much on the nose gear position.

## 10.2 Choosing the Wheels

The load per wheel has to meet the airport capability of sustaining this load. In our case, it is assumed that short range aircrafts use regional airports with weaker surfaces; an average of 20 tones per wheel, as a maximum, comes out from statistics (see Fig. 10.7).

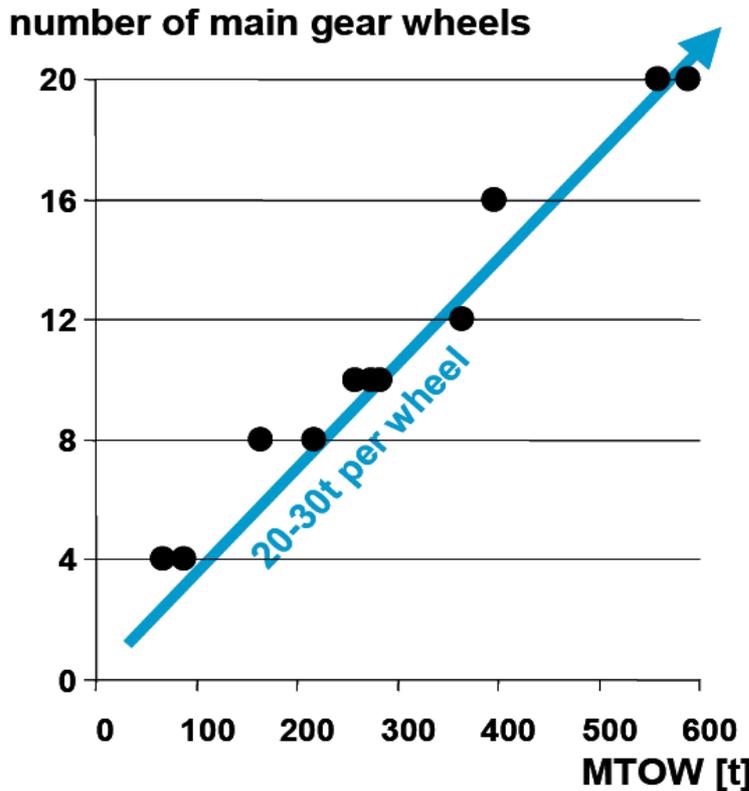


Fig. 10.7 Empirical diagram of number of wheels vs MTOW (Trahermer 2008)

In our case, the maximum take-off weight is almost 23 tones, which means a number of two wheels per each gear is suitable.

We can confront this assumption with Roskam IV's 1997 opinion, who gives the following Table:

Type	$W_{TO}$ lbs	$D_t \times b_t$ in. x in.	Main Gear			Nose Gear			
			$2P_m/W_{TO}$	PSI	$n_{mt}$	$D_t \times b_t$ in. x in.	$P_n/W_{TO}$	PSI	$n_{nt}$
Regional Turbo-propeller Driven Airplanes	12,500	18x5.5	0.89	105	2	22x6.75	0.11	57	1
	21,000	24x7.25	0.90	85	2	18x5.5	0.10	65	2
	26,000	36x11	0.92	40	1	20x7.5	0.08	40	1
	44,000	30x9	0.93	107	2	23.4x6.5	0.07	77	2

Fig 10.8 Typical landing gear wheel data (Roskam II 1997)

The weight of the ATR 72 in pounds is approximately 51000 lbs, which means at least the values from the last row should be valid.

### 10.3 Aircraft Classification Number (ACN)

The ACN is a number between 40 and 80 which is indicated in the manual of an aircraft and in the manual of the airfield. The signification of this number is related to the impact of the loads to the ground when an aircraft uses the runways of an airfield. This classification was required due to the costly damage which aircrafts can produce when they frequently use the runways or taxi routes. If the ACN of an aircraft is larger than the ACN of an airfield, the aircraft needs a special approval to operate on the respective airfield. This number drives the layout and sizing of the landing gear.

This parameter is calculated for two sorts of pavements: flexible surfaces (tarmac, asphalt) and rigid surfaces like concrete; in this last case the tire pressure and size have no impact, the damage in the case of rigid surfaces results from fatigue. The ACN is:

$$ACN = 2 \times \text{equivalent single wheel loading [ in tonnes]} \quad (10.3.1)$$

**Torenbeek 1978** gives an estimation method for the ESWL (equivalent single wheel load):

“ The ESWL of a group of two or more wheels which are relatively close together, is equal to the load of an isolated wheel, having the same inflation pressure, and causing the same stresses in the runway material as those due to the group of wheels (...) a given loading, spread over a number of contact areas, causes lower stresses in the runway material than would be the case when the same load is concentrated on a single wheel”.

For dual wheel layouts, as it is our case, a typical reduction factor for a rigid type of pavement is  $4/3$ .

If for our design we would impose an minimum ACN number of 40, then the wheel loading would be 20 tones, which is almost the maximum take-off weight; this means that the ATR will be suitable for landing on even worse conditions, on not so well maintained airfields.

### 10.4 Breaking Capability and Free Fall Capability

The necessary breaking force is proportional with the weight of the aircraft:

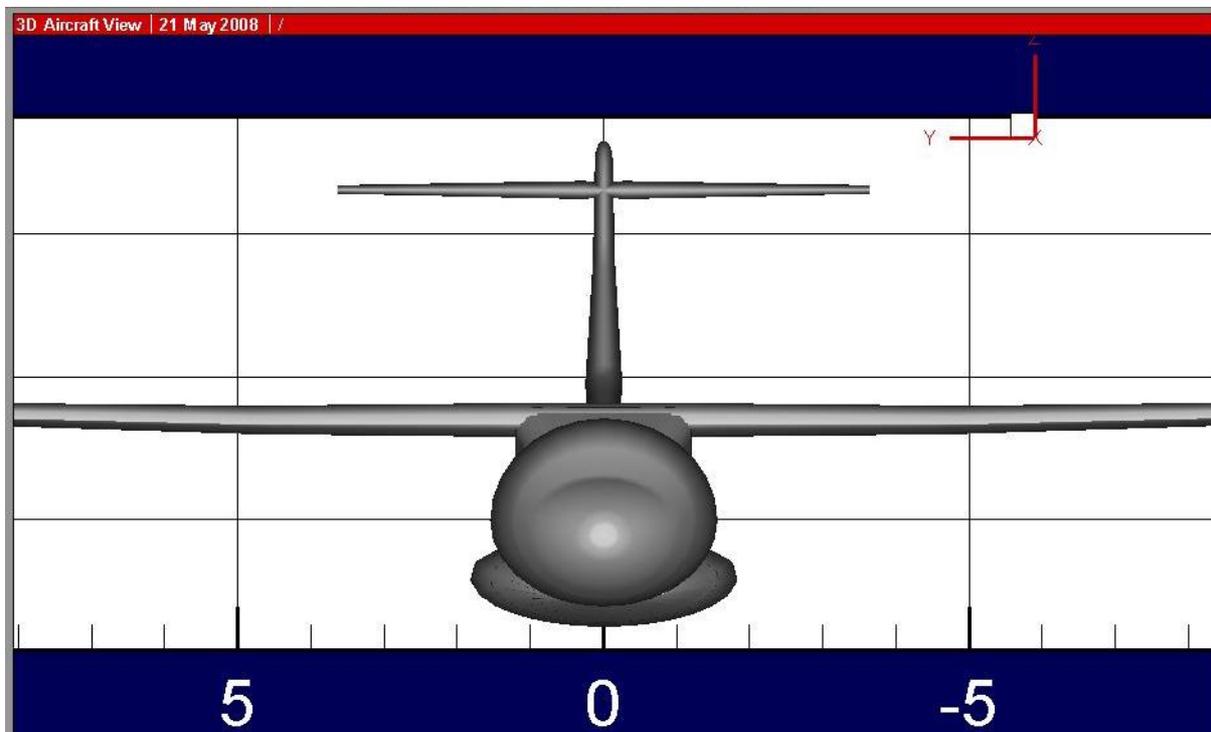
$$\text{breaking force} = MTOW \times g \times 0.8 = 23.3t \times 9.8 \times 0.8 = 182.672tf \quad (10.4.1)$$

According to **FAR** regulations, when the systems fail and the power supply is interrupted, the landing gear has to be capable to free fall, due to its weight and with the help of the dynamic pressure; however, the dynamic pressure helps only if the gear retracts against the airflow, as

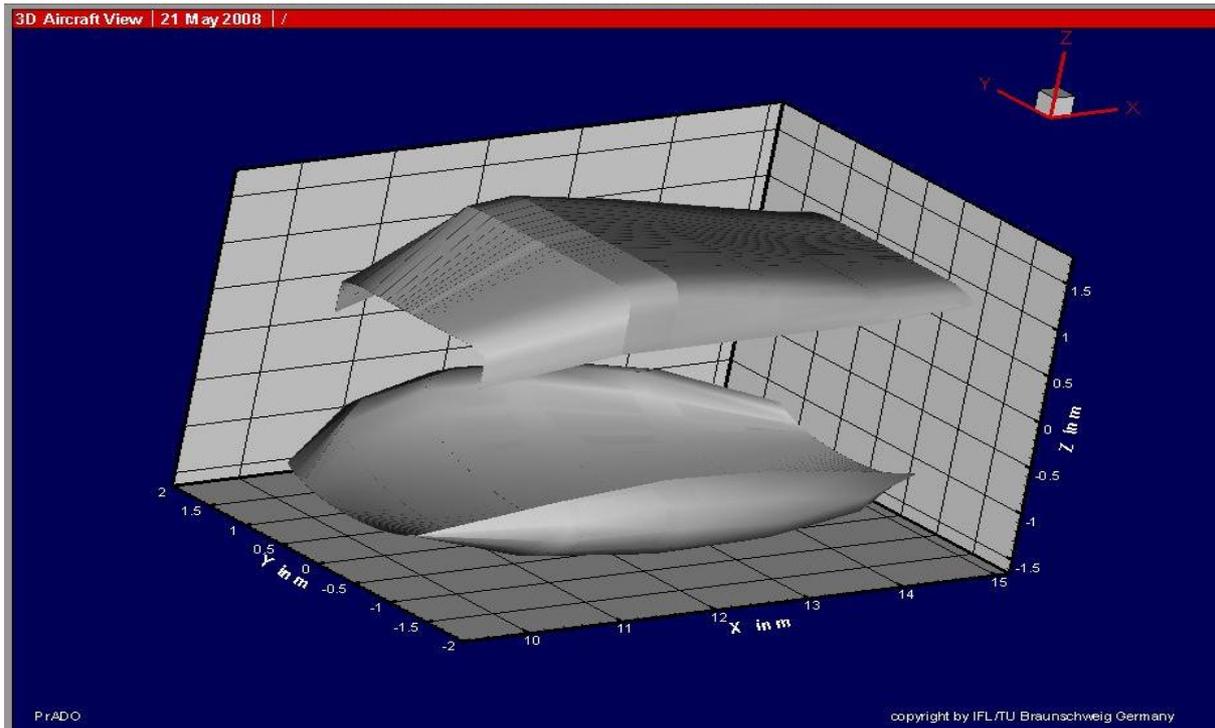
it is the case for the nose gear of the ATR. The main gear also has to ensure a free fall in the case of an emergency.

## 10.5 Integration of the landing gear

The ATR 72 is designed with a high wing, which means the landing gear has to be integrated either into the fuselage or, as it is the case of *Bombardier Dash 400*, into the nacelle of the engine. An advantage for the first variant is that a smaller track between the legs of the landing gear is required, which allows the aircraft to land on narrow runways, but in the same time this solution requires large fairings with large effects on drag increase. These effects have to be compensated by, for example, a large area of the wing and proper high lift devices.



**Fig. 10.9** Undercarriage fairings for the landing gear integration and wing fairings for wing integration designed with PRADO – 1 (Domingues 2008)



**Fig. 10.10** Undercarriage fairings for the landing gear integration and wing fairings for wing integration designed with PRADO – 2 (Domingues 2008)

# 11 Drag Prediction

At this stage of our design a proper estimation of the drag allows us to verify if we have correctly estimated the flight performances of our aircraft. The required lift was a known parameter, but once the configuration of the aircraft is designed, the drag and all its components must be determined.

$$C_D = C_{D,0} + \frac{C_L^2}{\pi \cdot A \cdot e} \quad (11.1)$$

The equation (11.1) represents the polar of the aircraft. Practically the aerodynamic efficiency of an aircraft can be expressed by the amount of drag which is produced by the aircraft for a required lift at a given altitude and speed.

The drag has three sources:

- zero lift drag
- lift dependent drag
- Mach drag

We can ignore from the beginning the third form of drag, as it is not the case of our design; our aircraft flies at lower speed, so no wave drag is produced.

## 11.1 Zero Lift Drag

Even if a body does not produce lift when moving through a fluid, the viscosity of the air produces drag. The zero lift drag depends on the surface and the quality of the surface moving through the air, on the shape and on the air viscosity. Summing up, these effects give the following equation for the drag coefficient:

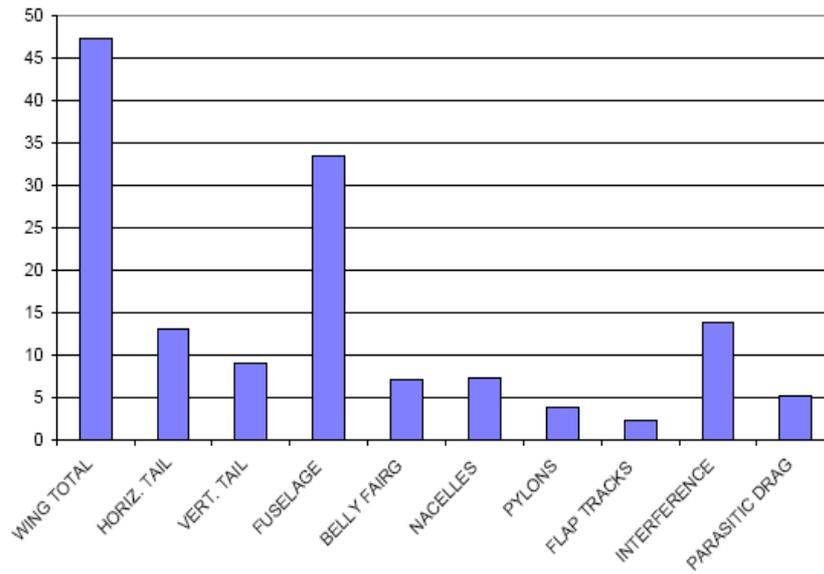
$$C_{D,0} = \sum C_f \cdot FF_c \cdot Q_c \cdot S_{wett} / S_{ref} \quad (11.1.1)$$

The sign  $\sum$  accounts for the parts of the aircraft which play an important role in estimating the drag: *wing, fuselage, empennage, nacelles and pylons, belly fairings, flap tracks, landing gear, interference and parasitic drag.*

With courtesy of Airbus, the influence of the various parts is showed in the next figure.

The rest of the terms have the following signification:

- $C_f$  friction coefficient  
 $Q_c$  accounts for interference between the above mentioned parts  
 $FF_c$  form factor  
 $S_{wett} / S_{ref}$  wetted area is the area 'touched' by the airflow



**Fig. 11.1** Zero lift drag caused by various parts of a typical transport aircraft (Böttger 2008)

For estimating the zero lift drag, we have to first express the wetted areas for each component. The reference area will be the wing area. The result would be:

$$S_{wett} = S_{wett,F} + S_{wett,W} + S_{wett,V} + n_E \cdot S_{wett,N} + n_E \cdot S_{wett,pylons} \quad (11.1.2)$$

### Fuselage

According to **Torenbeek 1986**, for a fuselage with a cylindrical middle part:

$$S_{wett,F} = \pi \cdot d_F \cdot l_F \cdot \left(1 - \frac{2}{\lambda_F}\right)^{2/3} \left(1 + \frac{1}{\lambda_F^2}\right) = \pi \cdot 2.77 \cdot 27.13 \cdot \left(1 - \frac{2}{10}\right)^{2/3} \left(1 + \frac{1}{10^2}\right) = 205.492 m^2 \quad (11.1.3)$$

### Wing

**Torenbeek 1986:**

$$S_{wet,W} = 2 \cdot S_{exp} \cdot \left( 1 + 0.25 \cdot (t/c)_r \cdot \frac{1 + \tau \cdot \lambda}{1 + \lambda} \right) = 2 \cdot 62.187 \cdot \left( 1 + 0.25 \cdot 0.18 \cdot \frac{1 + 13/18 \cdot 0.59}{1 + 0.59} \right) = 129.394 m^2 \quad (11.1.4)$$

where  $\tau = (t/c)_i / (t/c)_r = 0.13 / 0.18 = 0.722$  and  $\lambda = c_i / c_r = 0.59$

### Horizontal Tail

**Torenbeek 1988:**

$$S_{wet,H} = 2 \cdot S_{exp} \cdot \left( 1 + 0.25 \cdot (t/c)_r \cdot \frac{1 + \tau_H \cdot \lambda_H}{1 + \lambda_H} \right) = 2 \cdot 10.756 \cdot \left( 1 + 0.25 \cdot 0.09 \cdot \frac{1 + 0.7 \cdot 0.6}{1 + 0.6} \right) = 21.942 m^2 \quad (11.1.5)$$

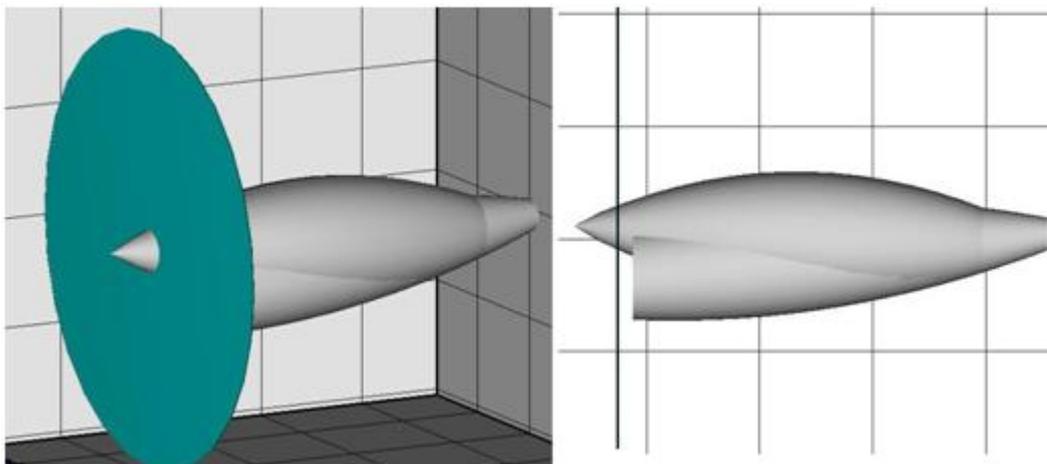
### Vertical Tail

**Torenbeek 1988:**

$$S_{wet,V} = 2 \cdot S_{exp} \cdot \left( 1 + 0.25 \cdot (t/c)_r \cdot \frac{1 + \tau_V \cdot \lambda_V}{1 + \lambda_V} \right) = 2 \cdot 14.085 \cdot \left( 1 + 0.25 \cdot 0.12 \cdot \frac{1 + 0.7 \cdot 0.6}{1 + 0.6} \right) = 28.92 m^2 \quad (11.1.6)$$

### Nacelle

$$S_{wet,nacelle} = 18.802 m^2 \quad (11.1.7)$$



**Fig. 11.2** Engine nacelle, calculated with PRADO, for estimating wetted area (Dielbandhoesing 2008)

### 11.1.1 Calculating $C_{D,0}$ for Each Component

According to (11.1.1) we need estimations for the friction coefficient  $C_f$ , form factor  $FF$  and interference factor  $Q$ . However, other components, like landing gear, antennas, doors, are more difficult to calculate with exact formulas, therefore we include these influences in a new term, as it is shown in (11.1.8).

$$C_{D,0} = \sum_{c=1}^n C_{f,c} \cdot FF_c \cdot Q_c \cdot S_{wett,c} / S_{ref} + C_{D,misc} + C_{D,L+P} \quad (11.1.8)$$

For each component we have to calculate all the factors from the above formula:

$$C_{f,laminar} = 1.328 / \sqrt{\text{Re}}$$

$$C_{f,turbulent} = \frac{0.455}{(\log \text{Re})^{2.58} \cdot (1 + 0.144 \cdot M^2)^{0.65}} \quad (11.1.9)$$

The form factor has a different expression for each component and the interference factor comes out from statistics.

### Fuselage

#### $C_{f,F}$ Estimation

The Reynolds number for the fuselage can be calculated having as reference values the cruise speed, the length of the fuselage and the density of the air at cruise level. We have already calculated this value in *Empennage Sizing* chapter, formula (9.2.22):

$$\text{Re}_F = 7.05 \cdot 10^7 \quad (11.1.10)$$

In a laminar flow, the friction coefficient, according to (11.1.9) is:

$$C_{f,laminar,F} = 1.328 / \sqrt{7.05 \cdot 10^7} = 1.582 \cdot 10^{-4} \quad (11.1.11)$$

In a turbulent flow, the formula (11.1.9) contains the Mach number. If the Mach number is smaller than 0.9, as it is our case, we need to calculate the *cut-off Reynolds number* with the formula:

$$\text{Re}_{cut-off} = 38.21 \cdot \left( \frac{l}{k} \right)^{1.053} \quad (11.1.12)$$

Afterwards we have to compare the values obtained with the formulas (11.1.12) and (11.1.10). If the resulting  $Re$  number, obtained with (11.1.12) is smaller than the initial  $Re$  number, then we proceed with our calculations with the  $Re_{cut-off}$ . The  $k$  factor accounts for the type of surface, and has a value of 0.00013 for polished metal surfaces.

This gives a smaller value, so we use the new  $Re$  value for further calculations:

$$Re_{cut-off} = 38.21 \cdot \left( \frac{27.13}{0.00013} \right)^{1.053} = 1.526 \cdot 10^7 \quad (11.1.13)$$

we obtain the following:

$$C_{f,turbulent,F} = \frac{0.455}{(\log 1.526 \cdot 10^7)^{2.58} \cdot (1 + 0.144 \cdot 0.44^2)^{0.65}} = 2.76 \cdot 10^{-3} \quad (11.1.14)$$

We assume for the fuselage that only 20% of the surface has a laminar flow, and the rest is turbulent (**Raymer 2006**); the overall friction coefficient results to be:

$$C_{f,F} = 20\%C_{f,laminar,F} + 80\%C_{f,turbulent,F} = 0.2 \cdot 1.582 \cdot 10^{-4} + 0.8 \cdot 2.76 \cdot 10^{-3} = 2.24 \cdot 10^{-3} \quad (11.1.15)$$

### FF<sub>F</sub> Estimation

**DATCOM 1978:**

$$FF_F = 1 + \frac{60}{(l_F / d_F)^3} + \frac{(l_F / d_F)}{400} = 1 + \frac{60}{(27.13 / 2.77)^3} + \frac{(27.13 / 2.77)}{400} = 1.088 \quad (11.1.16)$$

### Q Estimation

In the case of the fuselage this factor does not make sense; we talk about interference between the fuselage and other components, but not the other way around. So this factor is  $1.0$  for the fuselage.

### C<sub>D,0</sub> Estimation

We have now all the data to make the  $C_{D,0}$  calculation, according to (11.1.1):

$$C_{D,0,F} = C_{f,F} \cdot FF_F \cdot Q_F \cdot S_{wet,F} / S_{ref} = 2.24 \cdot 10^{-3} \cdot 1.088 \cdot 1 \cdot 205.492 / 62.187 = 8.053 \cdot 10^{-3} \quad (11.1.17)$$

## Wing

### $C_{f,W}$ Estimation

The reference values for the  $Re$  number estimation are the mean aerodynamic chord of the wing and the cruise speed. This gives:

$$Re_w = \frac{V_{CR} \cdot c_{MAC}}{\nu} = \frac{141.944 \cdot 2.2345}{5.4603 \cdot 10^{-5}} = 5.809 \cdot 10^6 \quad (11.1.18)$$

$$C_{f,laminar,W} = 1.328 / \sqrt{5.809 \cdot 10^6} = 5.51 \cdot 10^{-4} \quad (11.1.19)$$

$$Re_{cut-off} = 38.21 \cdot \left( \frac{2.2345}{0.00013} \right)^{1.053} = 1.101 \cdot 10^6 \quad (11.1.20)$$

The  $Re_{cut-off} < Re$  which means we use the  $Re_{cut-off}$  in our calculations.

$$C_{f,turbulent,W} = \frac{0.455}{(\log 1.101 \cdot 10^6)^{2.58} \cdot (1 + 0.144 \cdot 0.44^2)^{0.65}} = 4.314 \cdot 10^{-3} \quad (11.1.21)$$

$$C_{f,W} = 20\% C_{f,laminar,W} + 80\% C_{f,turbulent,W} = 0.2 \cdot 5.51 \cdot 10^{-4} + 0.8 \cdot 4.314 \cdot 10^{-3} = 3.561 \cdot 10^{-3} \quad (11.1.22)$$

### $FF_w$ Estimation

**DATCOM 1978:**

$$FF_w = \left[ 1 + \frac{0.6}{x_t} \left( \frac{t}{c} \right) + 100 \left( \frac{t}{c} \right)^4 \right] \cdot \left[ 1.34 \cdot M^{0.18} \cdot (\cos \varphi_m)^{0.28} \right] \quad (11.1.23)$$

$x_t$  is the distance between the leading edge and the point on the chord where the maximum thickness is achieved and  $\varphi_m$  is the corresponding sweep angle. The reference chord is the mean aerodynamic chord, for which the maximum thickness is 14%. Our wing airfoil is a modified NACA 43014 airfoil, with the maximum thickness situated at 15% of the chord. This gives:  $x_t = 0.15$ .

$$\tan \varphi_{15} = \tan \varphi_{25} - \frac{4}{A} \left[ \frac{25-15}{100} \cdot \frac{1-\lambda}{1+\lambda} \right] = \tan(3^\circ \cdot \frac{\pi}{180^\circ}) - \frac{4}{12} \left[ \frac{10}{100} \cdot \frac{1-0.59}{1+0.59} \right] = 0.044 \quad (11.1.24)$$

$$\Rightarrow \varphi_{15} = 2.509^\circ$$

The form factor becomes:

$$FF_W = \left[ 1 + \frac{0.6}{0.15} (0.14) + 100 (0.14)^4 \right] \cdot \left[ 1.34 \cdot 0.44^{0.18} \cdot (\cos 2.509^0 \cdot \pi / 180^0)^{0.28} \right] = 1.847 \quad (11.1.25)$$

### Q Estimation

The value of the interference factor for a high wing with fairings is 1. (Scholz 2008)

### C<sub>D,0</sub> Estimation

$$C_{D,0,W} = C_{f,W} \cdot FF_W \cdot Q_W \cdot S_{wet,W} / S_{ref} = 3.561 \cdot 10^{-3} \cdot 1.84 \cdot 1 \cdot 129.394 / 62.187 = 0.014 \quad (11.1.26)$$

## **Horizontal Tail**

### C<sub>f,H</sub> Estimation

This estimation is similar to the one for the wing, except that we use the mean aerodynamic chord of the horizontal tail for calculating the *Re* number:

$$Re_H = \frac{V_{CR} \cdot c_{MAC}}{\nu} = \frac{141.944 \cdot 2.839}{5.4603 \cdot 10^{-5}} = 7.38 \cdot 10^6 \quad (11.1.27)$$

$$C_{f,laminar,H} = 1.328 / \sqrt{7.38 \cdot 10^6} = 4.888 \cdot 10^{-4} \quad (11.1.28)$$

$$Re_{cut-off} = 38.21 \cdot \left( \frac{2.839}{0.00013} \right)^{1.053} = 1.417 \cdot 10^6 \quad (11.1.29)$$

For further calculations we use  $Re_{cut-off}$  as it is smaller than the initial *Re* number.

$$C_{f,turbulent,H} = \frac{0.455}{(\log 1.417 \cdot 10^6)^{2.58} \cdot (1 + 0.144 \cdot 0.44^2)^{0.65}} = 4.118 \cdot 10^{-3} \quad (11.1.30)$$

$$C_{f,H} = 20\% C_{f,laminar,H} + 80\% C_{f,turbulent,H} = 0.2 \cdot 4.888 \cdot 10^{-4} + 0.8 \cdot 4.118 \cdot 10^{-3} = 3.392 \cdot 10^{-3} \quad (11.1.31)$$

### FF<sub>H</sub> Estimation

#### **DATCOM 1978:**

We use the same formula as for the wing, having in mind that the horizontal empennage has a NACA 0009 profile. The 4 digit series have, by default, the maximum thickness 30% of the chord (0.3 chords) from the leading edge. This means that the position of the maximum thickness is  $x_t = 0.3$ , while the maximum thickness is 9% of the chord. Correspondingly the sweep angle we need to calculate is  $\varphi_{30}$ .

$$\tan \varphi_{30} = \tan\left(8^\circ \cdot \frac{\pi}{180^\circ}\right) - \frac{4}{6} \left[ \frac{30-25}{100} \cdot \frac{1-0.6}{1+0.6} \right] = 0.132$$

$$\Rightarrow \varphi_{15} = 7.531^\circ \quad (11.1.32)$$

$$FF_H = \left[ 1 + \frac{0.6}{0.3} \cdot 0.09 + 100 \cdot 0.09^4 \right] \cdot \left[ 1.34 \cdot 0.44^{0.18} \cdot \left( \cos(7.531^\circ \cdot \pi / 180^\circ) \right)^{0.28} \right] = 1.368 \quad (11.1.33)$$

### Q Estimation

For the conventional empennages, the value is:  $Q_H = 1.04$  (Scholz 2008).

### $C_{D,0}$ Estimation

$$C_{D,0,H} = C_{f,H} \cdot FF_H \cdot Q_H \cdot S_{wett,H} / S_{ref} = 3.392 \cdot 10^{-3} \cdot 1.368 \cdot 1.04 \cdot 10.756 / 62.187 = 8.347 \cdot 10^{-4} \quad (11.1.34)$$

## **Vertical Tail**

### $C_{f,V}$ Estimation

$$\text{Re}_V = \frac{V_{CR} \cdot c_{MAC}}{\nu} = \frac{141.944 \cdot 1.389}{5.4603 \cdot 10^{-5}} = 3.611 \cdot 10^6 \quad (11.1.35)$$

$$C_{f,laminar,V} = 1.328 / \sqrt{3.611 \cdot 10^6} = 6.989 \cdot 10^{-4} \quad (11.1.36)$$

$$\text{Re}_{cut-off} = 38.21 \cdot \left( \frac{1.389}{0.00013} \right)^{1.053} = 6.675 \cdot 10^5 \quad (11.1.37)$$

Again, the cut-off Reynolds number is smaller, so we use it for further calculations.

$$C_{f,turbulent,V} = \frac{0.455}{(\log 6.675 \cdot 10^5)^{2.58} \cdot (1 + 0.144 \cdot 0.44^2)^{0.65}} = 4.741 \cdot 10^{-3} \quad (11.1.37)$$

$$C_{f,V} = 20\% C_{f,laminar,V} + 80\% C_{f,turbulent,V} = 0.2 \cdot 6.989 \cdot 10^{-4} + 0.8 \cdot 4.741 \cdot 10^{-3} = 3.933 \cdot 10^{-3} \quad (11.1.38)$$

### $FF_V$ Estimation

#### **DATCOM 1978:**

The airfoil for the vertical tail is NACA 0012. The position of the 12% maximum thickness is also at 30% of the chord.

$$\tan \varphi_{30} = \tan\left(25^\circ \cdot \frac{\pi}{180^\circ}\right) - \frac{4}{6} \left[ \frac{30-25}{100} \cdot \frac{1-0.6}{1+0.6} \right] = 0.458$$

$$\Rightarrow \varphi_{15} = 24.607^\circ \quad (11.1.39)$$

$$FF_V = \left[ 1 + \frac{0.6}{0.3} \cdot 0.12 + 100 \cdot 0.12^4 \right] \cdot \left[ 1.34 \cdot 0.44^{0.18} \cdot \left( \cos(24.607^\circ \cdot \pi / 180^\circ) \right)^{0.28} \right] = 1.419 \quad (11.1.40)$$

### Q Estimation

The value  $Q_H = 1.04$  (Scholz 2008) is also valid for the vertical tail.

### C<sub>D,0</sub> Estimation

$$C_{D,0,V} = C_{f,V} \cdot FF_V \cdot Q_V \cdot S_{wet,V} / S_{ref} = 3.933 \cdot 10^{-3} \cdot 1.419 \cdot 1.04 \cdot 14.085 / 62.187 = 1.315 \cdot 10^{-3} \quad (11.1.41)$$

## Engine nacelle

### C<sub>f,N</sub> Estimation

For calculating the *Reynolds number* we take into account as reference value the length of the engine, which is approximately  $l_N = 3.3m$ , having in mind that the length of the engine is  $2.134m$ . This gives:

$$Re_N = \frac{V_{CR} \cdot l_N}{\nu} = \frac{141.944 \cdot 3.3}{5.4603 \cdot 10^{-5}} = 8.579 \cdot 10^6 \quad (11.1.42)$$

$$C_{f,laminar,N} = 1.328 / \sqrt{8.579 \cdot 10^6} = 4.534 \cdot 10^{-4} \quad (11.1.43)$$

$$Re_{cut-off} = 38.21 \cdot \left( \frac{3.3}{0.00013} \right)^{1.053} = 1.66 \cdot 10^6 \quad (11.1.44)$$

For further calculations we use  $Re_{cut-off}$ .

$$C_{f,turbulent,N} = \frac{0.455}{(\log 1.66 \cdot 10^6)^{2.58} \cdot (1 + 0.144 \cdot 0.44^2)^{0.65}} = 4.002 \cdot 10^{-3} \quad (11.1.45)$$

$$C_{f,N} = 20\% C_{f,laminar,N} + 80\% C_{f,turbulent,N} = 0.2 \cdot 4.534 \cdot 10^{-4} + 0.8 \cdot 4.002 \cdot 10^{-3} = 3.292 \cdot 10^{-3} \quad (11.1.46)$$

### FF<sub>N</sub> Estimation

**Raymer 2006:**

$$FF_N = 1 + \frac{0.35}{(l_N / d_N)} = 1 + \frac{0.35}{(3.3 / 0.68)} = 1.072 \quad (11.1.47)$$

The diameter of the engine is  $660\text{mm}$ ; accordingly we have the nacelle diameter of  $680\text{mm}$ .

### Q Estimation

For engines situated directly on the wing or fuselage, as it is the case for our design, the value of the interference factor is  $1.5$ .

### $C_{D,0}$ Estimation

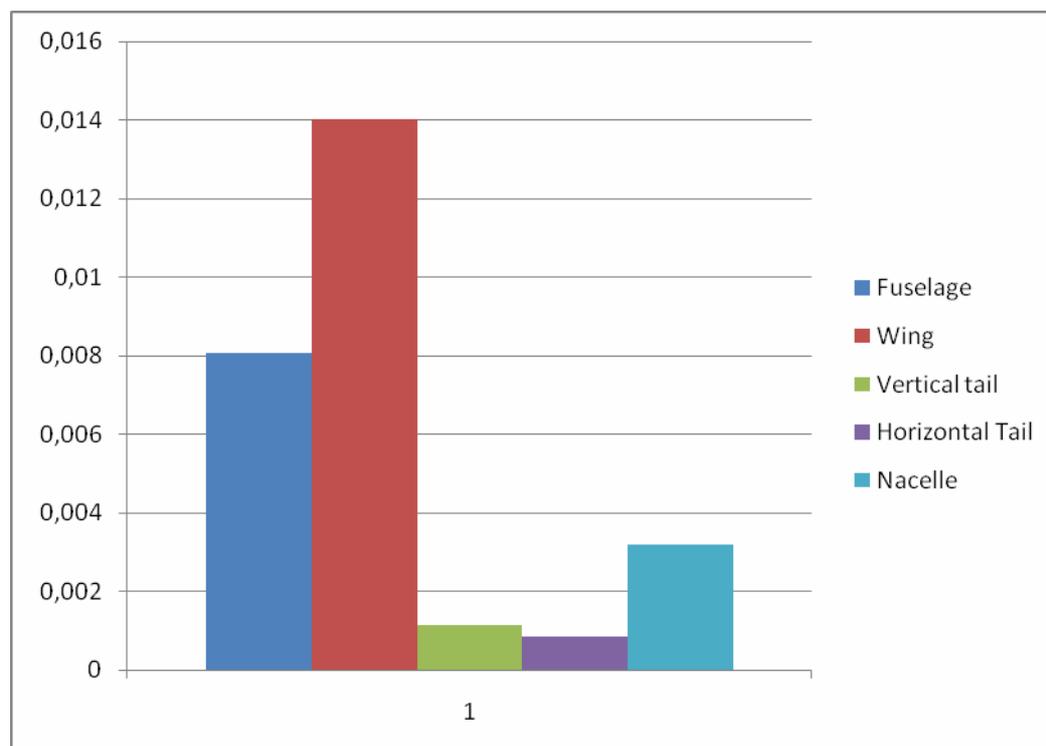
$$C_{D,0,N} = C_{f,N} \cdot FF_N \cdot Q_N \cdot S_{wet,N} / S_{ref} = 3.292 \cdot 10^{-3} \cdot 1.072 \cdot 1.5 \cdot 18.802 / 62.187 = 1.6 \cdot 10^{-3} \quad (11.1.48)$$

## 11.1.2 The Total Zero Lift Drag

The total zero lift drag is given by the sum of all components:

$$\begin{aligned} C_{D,0} &= C_{D,0,F} + C_{D,0,W} + C_{D,0,H} + C_{D,0,V} + 2C_{D,0,N} \\ &= (8.053 + 14 + 0.8347 + 1.315 + 2 \cdot 1.6) \cdot 10^{-3} = 0.027403 \end{aligned} \quad (11.1.49)$$

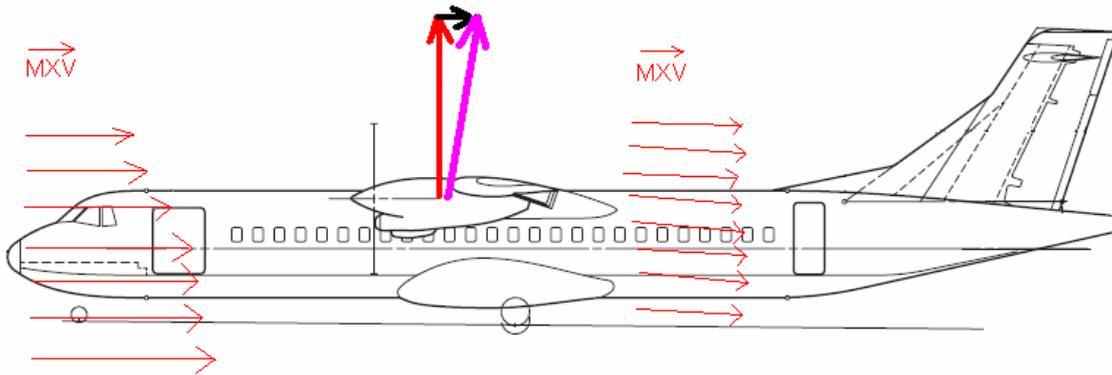
We can graphically illustrate the zero lift drag, as it is in Fig. 11.1, but for the specific case of the ATR 72:



**Fig. 11.3** Zero lift drag per component for the ATR 72

## 11.2 Lift Dependant Drag

The lift dependant drag is the induced drag. A simple explanation of what causes the induced drag is illustrated in Fig. 11.3:



**Fig. 11.4** The lift and drag on the ATR 72

In order to produce lift, the angle of attack is increased; as a consequence the air flow is diverted downward, and a change in the impulse of the aircraft takes place. In order for the impulse to be conserved, a reaction force is created. This is the induced drag. The more the air flow is diverted, the higher the drag. This means it is more efficient to obtain lift by increasing the wing span, rather than the angle of attack; the wing span plays an important role in creating induced drag. This is underlined in the parameter  $A$ , the aspect ratio, which is:  $A = \frac{b^2}{S}$ .

This influence can be noticed in the formula:

$$C_{D,i} = \frac{C_L^2}{\pi \cdot A \cdot e} \quad (11.2.1)$$

The Oswald factor accounts for the shape of the lift distribution; when equal to 1, the distribution is elliptic. **Howe 2000** gives the following equation for subsonic and transonic flow:

$$e = \frac{1}{(1 + 0.12M^6) \left\{ 1 + \frac{0.142 + f(\lambda)A(10t/c)^{0.33}}{(\cos \varphi_{25})^2} + \frac{0.1(3N_e + 1)}{(4 + A)^{0.8}} \right\}} \quad (11.2.2)$$

$$f(\lambda) = 0.005(1 + 1.5(\lambda - 0.6)^2) \quad (11.2.3)$$

$N_e$  is the number of engines situated on the wing.

When calculating according to the formulas, we obtain the following results:

$$f(\lambda) = 0.005(1 + 1.5(0.59 - 0.6)^2) = 0.005 \quad (11.2.4)$$

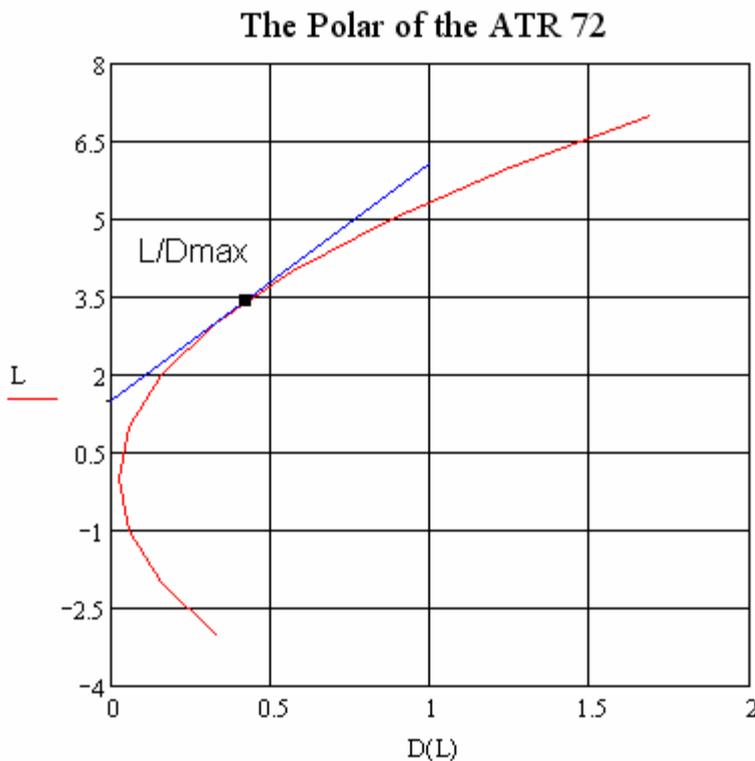
$$e = \frac{1}{(1 + 0.12 \cdot 0.44^6) \left\{ 1 + \frac{0.142 + 0.005 \cdot 12(10 \cdot 0.14)^{0.33}}{(1)^2} + \frac{0.1(3 \cdot 2 + 1)}{(4 + 12)^{0.8}} \right\}} = 0.777 \quad (11.2.5)$$

If we consider in the calculations the  $C_L$  obtained with the **DATCOM** method from the *High Lift Devices* chapter, for a clean wing, the induced drag becomes:

$$C_{D,i} = \frac{1.374^2}{\pi \cdot 12 \cdot 0.77} = 0.064 \quad (11.2.6)$$

In order to represent the polar of the aircraft, we have to keep  $C_L$  as a parameter:

$$C_D = C_{D,0} + \frac{C_L^2}{\pi \cdot A \cdot e} \Leftrightarrow C_D = 0.027403 + 0.034 \cdot C_L^2 \quad (11.2.7)$$



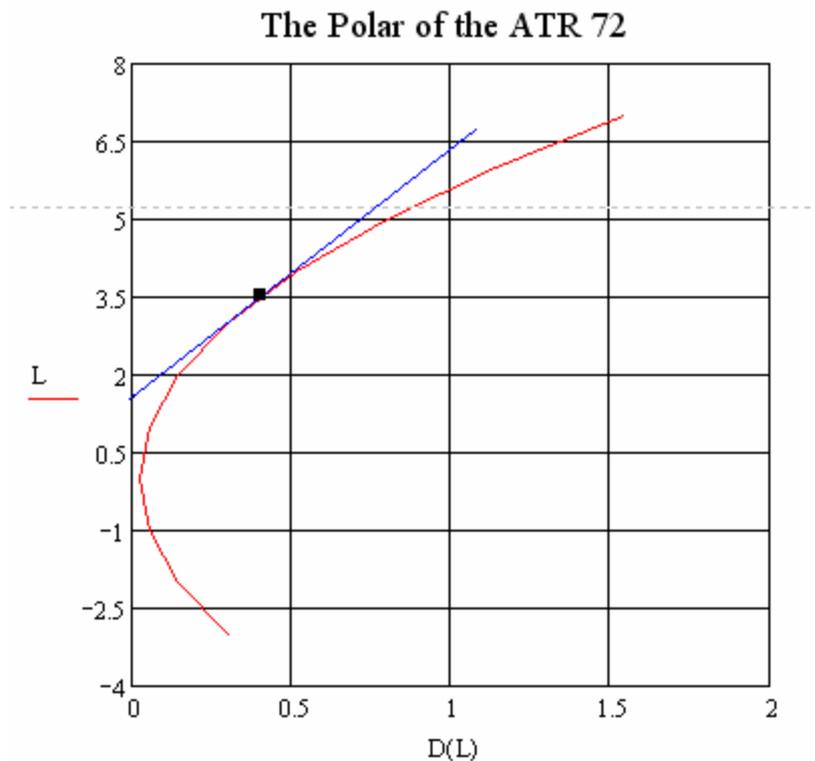
**Fig. 11.5** The polar of the ATR 72, representing equation (11.2.7), with calculated Oswald factor

For cruise conditions, the lift coefficient is  $C_{L,CR} = 0.81$ , as we estimated in the Preliminary Sizing chapter. We can now calculate the corresponding value of  $C_D$  from equation (11.2.7) and obtain a new value for the lift-to-drag ratio:

$$(L/D)_{CR} = 0.81/0.05 = 16.294 \quad (11.2.8)$$

If we consider the statistical value of 0.85 instead of the calculated value of 0.77 for the Oswald factor, then the coefficient of  $L^2$  becomes  $\frac{1}{\pi \cdot 12 \cdot 0.85} = 0.031$ , and correspondingly,

the polar moves to the left, and the  $L/D$  becomes more realistic:  $L/D = \frac{0.81}{0.048} = 16.996 \approx 17$ .



**Fig. 11.6** The polar of ATR 72 calculated with statistical Oswald factor

## 12 Design Evaluation

In this chapter we will choose a method in order to carry out a cost analysis of our design. This is an important aspect when it comes to selling an aircraft which has been not yet produced; in reality an aircraft is first sold and if a sufficient number of clients is found, the development from the project status until the final product begins. Therefore the importance of a correct evaluation of the costs is very high, as it brings high consequences on the profits.

Another important aspect is that the costs eventually drive the solutions the engineers choose, and the costs are always the cross point between the project on the paper and the effective production. Therefore an optimization of the parameters towards the costs is expected.

### 12.1 Different perspectives

The *Aircraft Manufacturer* has a different perspective over the costs than the *Operator*. The first one divides the costs into *fixed* and *variable* costs, as we can notice in the next table:

**Table 12.1** Fixed and variable costs from the Aircraft Manufacturer perspective (**Scholz 2008**)

Fixed Costs	<i>Development and design:</i> engineer's services (in-house and contracted out), equipping of development and design offices (computers);	Perspective of the Aircraft Manufacturer
	<i>Testing departments:</i> engineer's services, test setups (wind tunnel models, wind tunnel operation)	
	<i>Cooperation with suppliers</i>	
	<i>Analyzing reliability, maintainability, certification procedures</i>	
	<i>Analyzing manufacturing methods</i>	
	<i>Construction of jigs and fixtures</i>	
	<i>Building prototypes (if necessary)</i>	
	<i>Flight testing and certification</i>	
<i>Financing costs</i>		
Variable Costs	<i>Manufacturing support:</i> services connected with rectifying errors/faults, taking into account customers' wishes, instruction manuals	
	<i>Production costs:</i> wage costs, cost of materials, tool costs, quality assurance costs, costs of purchased parts: auxiliary power unit (APU), landing gear; ...	
	<i>Flight testing with production planes and customer service</i>	
	<i>Financing costs</i>	

The costs estimation from the above mentioned point of view cannot really allow an optimization of the parameters since even for the manufacturer, most of them derive from assumptions in the simple methods; so if these parameters vary, we cannot make correct estimations of how the costs will vary.

The perspective of the *Operator* has as consequence a large number of methods, brought up by different authors, which deal with models, like shown in the next table:

**Table 12.2** Models for cost analysis from the perspective of the operator

LCC	<i>Life Cycle Costs</i> - for military aircrafts	Perspective of the Operator
COO	<i>Cost of Ownership</i> - resulting only from ownership of the aircraft	
DOC	<i>Direct Operating Costs</i> - only aircraft related costs	
IOC	<i>Indirect Operating Costs</i> - passenger related costs (complement of DOC)	
TOC	<i>Total Operating Costs</i> - sum between DOC and IOC	
COC	<i>Cash Operating Costs</i> - DOC without depreciation	

From the perspective of the operator, the most widely accepted method is the DOC calculation. We choose the estimation of the DOC according to the *Association of European Airlines (AEA 1989 a* for short and medium range aircrafts and **b** for long range aircrafts).

## 12.2 DOC Method According to AEA

The direct operating cost is a sum of other specific costs, as follows:

$$C_{DOC} = C_{DEP} + C_{INT} + C_{INS} + C_F + C_M + C_C + C_{FEE} \quad (12.2.1)$$

The terms from the formula represent, correspondingly, the costs of:

Depreciation,  $C_{DEP}$

Interest,  $C_{INT}$

Insurance,  $C_{INS}$

Fuel,  $C_F$

Maintenance (airframe  $C_{M,AF}$  and power plant  $C_{M,PP}$ )

Crew (cabin  $C_{C,CA}$  and cockpit  $C_{C,CO}$ )

Fees and charges (landing fees  $C_{FEE,LD}$ , ATC or navigation charges  $C_{FEE,NAV}$ , ground handling  $C_{FEE,GND}$ )

The AEA method takes into account all these costs, while other methods include only a part of all this elemental costs.

In the calculations, we will consider the cost per one aircraft per one year, therefore the index changes:  $C_{DOC} = C_{a/c,a}$ .

## 12.3 Calculation of DOC Cost Elements

### 12.3.1 Depreciation

When an item is new, its value corresponds with the acquisition price,  $P_{total}$ . In the useful service time,  $n_{DEP}$ , the value reduces. When the aircraft is sold, the corresponding price is the residual value,  $P_{residual}$ . This reduction in value is called *depreciation*:

$$C_{DEP} = \frac{P_{total} - P_{residual}}{n_{DEP}} = \frac{P_{total} \left(1 - \frac{P_{residual}}{P_{total}}\right)}{n_{DEP}} \quad (12.3.1)$$

The total purchase price consists of:

*delivery price*: list price, discounts, surcharges for modifications, buyer furnished equipment (BFE), the interest on construction progress payment

*spares price*.

There are three possibilities to estimate the delivery price, accounting that:  $\frac{P_{delivery}}{m_{MTO}} \approx 500 \frac{US\$}{kg}$ ,

for short and medium aircrafts;  $\frac{P_{delivery}}{m_{OE}} \approx 860 \frac{US\$}{kg}$  for short to long range aircrafts and

$\frac{P_{delivery}}{n_{PAX}} = 265000US\$$ , also for short to long range aircrafts.

$$P_{delivery} = \frac{P_{delivery}}{m_{MTO}} m_{MTO} \approx 500 \frac{US\$}{kg} \cdot 23296.272kg = 11648136US\$ \quad (12.3.2)$$

$$P_{delivery} = \frac{P_{delivery}}{m_{OE}} m_{OE} \approx 500 \frac{US\$}{kg} \cdot 13231.874kg = 11379411.64US\$ \quad (12.3.3)$$

$$P_{delivery} = \frac{P_{delivery}}{n_{PAX}} n_{PAX} \approx 265000US\$ \cdot 70 = 18550000US\$ \quad (12.3.4)$$

The delivery price we choose is the medium value between the three:  
 $P_{delivery} = 13859182.55US\$ \approx 13.85 \text{ mil US\$}$ .

The price of the spare parts is calculated as proportions from airframe and engines price:

$$P_S = k_{S,AF} P_{AF} + k_{S,E} n_E P_E \quad (12.3.5)$$

The price of the engines, according to **Jenkinson 1999b** is:

$$P_E = 293US\$ \cdot \left( \frac{T_{T/O,E}}{N} \right)^{0.81} = 293US\$ \cdot \left( \frac{\eta \cdot P}{V} \right)^{0.81} = 293US\$ \cdot \left( \frac{0.85 \cdot 2051}{142} \right)^{0.81} = 606567.96US\$ \quad (12.3.6)$$

The price of the airframe is:

$$P_{AF} = P_{delivery} - n_E P_E = 13.85 \text{ milUS\$} - 2 \cdot 606567.96US\$ = 12646046.63 \approx 12.6 \text{ milUS\$} \quad (12.3.7)$$

The resulting price of the spares is calculated with the values of:  $n_{DEP} = 14$ ,  $k_{S,AF} = 0.10$  and  $k_{S,E} = 0.30$ , according to the **AEA 1989a** method; the results are:

$$P_S = 0.10 \cdot 12.6 \text{ milUS\$} + 0.30 \cdot 2 \cdot 606567.96US\$ = 1628545.44US\$ \approx 1.6 \text{ milUS\$} \quad (12.3.8)$$

The total price of the aircraft becomes:

$$P_{total} = P_{delivery} + P_S = 13.85 \text{ milUS\$} + 1.6 \text{ milUS\$} = 15487727.99 \approx 15.5 \text{ milUS\$} \quad (12.3.9)$$

The value of  $\frac{P_{residual}}{P_{total}} = 0.10$  in the **AEA** method. Accordingly, the depreciation cost is:

$$C_{DEP} = \frac{15.5(1-0.10)}{14} = 995639.66US\$/ \text{ year} \quad (12.3.10)$$

## 12.3.2 Interest

We can assume that the price of a new aircraft is covered by outside sources, as it is actually the case for larger aircrafts. The interest is calculated using an average interest rate, which in the case of AEA method, has the value  $p_{av} = 0.0529$ .

$$C_{INT} = p_{av} P_{total} = 0.0529 \cdot 15.5 \text{ milUS\$} = 732893.71 \text{ US\$} / \text{ year} \quad (12.3.11)$$

### 12.3.3 Insurance

The insurance is calculated as a percentage of the aircraft price; the  $k_{INS}$  has the value of 0.005 in the case of AEA method:

$$C_{INS} = k_{INS} P_{delivery} = 0.005 \cdot 13.85 \text{ milUS\$} = 69295.91 \text{ US\$} / \text{ year} \quad (12.3.12)$$

### 12.3.4 Fuel Costs

The fuel costs per year are given by:

$$C_F = n_{t,a} P_F m_F \quad (12.3.13)$$

$n_{t,a}$	number of flights per year
$P_F$	fuel price
$m_F$	fuel consumed during one flight

The fuel mass consumed during one flight has to be recalculated, due to the fact that the range is smaller than the one we used in the Preliminary Sizing Chapter. With the Breguet factor calculated in chapter 3,  $B_s = 23021043.748m$ , the range between the departure and destination airport, according to the AEA method should be covered by a fuel mass that:

- assures a 5% fuel reserve
- an alternative airfield at a distance of 250NM could be reached
- 30 minutes flight in a holding pattern at 1500 ft with minimal drag

The range becomes:

$$S_{CR} = (0.5 \cdot R) \cdot 1.05 + 250NM \cdot \frac{1852m}{NM} = 1241 \times 10^6 m \quad (12.3.14)$$

The fuel mission segment mass fraction, cruise, becomes:

$$M_{ff,CR} = e^{-\frac{S_{CR}}{B_s}} = 0.948 \quad (12.3.15)$$

The loiter time also changes, according to the AEA method:  $t_L = 1800s$ . The new fuel friction with the Breguet factor from Preliminary Sizing chapter of  $B_s = 162183.859s$  becomes:

$$M_{ff,loiter} = e^{-\frac{t_s}{B_s}} = 0.989 \quad (12.3.16)$$

The new mission fuel fraction becomes:

$$M_{ff} = 0.948 \cdot 0.989 \cdot 0.995 \cdot 0.990 \cdot 0.985 \cdot 0.995 = 0.905 \quad (12.3.17)$$

The fuel mass consumed during one flight becomes:

$$m_F = m_{MTO}(1 - M_{ff}) = 23296.272(1 - 0.905) = 2213kg \quad (12.3.18)$$

The time necessary for one flight is:

$$t_f = \frac{S_{CR}}{V_{CR}} = \frac{1.241 \cdot 10^3}{141.944 \cdot 3.6} = 2.429hours \quad (12.3.19)$$

The number of flights per year according to the AEA method is:

$$n_{t,a} = \frac{k_{U1}}{t_f + k_{U2}} = \frac{3750h}{2.429h + 0.75} = 1180 \quad (12.3.20)$$

We can finally calculate the fuel costs per year, applying formula (12.3.13):

$$C_F = n_{t,a} P_F m_F = 1180 \cdot 0.91 \cdot 2213 = 2376319.4US\$ \approx 2.37milUS\$/year \quad (12.3.21)$$

### 12.3.5 Maintenance Cost

All the actions necessary for maintaining an aircraft in service reflect on two types of costs:

$C_{M,L}$  labor costs

$C_{M,M}$  material costs

The costs can be calculated per hour, and by multiplying with the flight time and number of flights per year we obtain:

$$C_M = (t_{M,f}L_M + C_{M,M,f})t_f n_{t,a} \quad (12.3.22)$$

If we differentiate between the maintenance of the airframe and the maintenance of the engine, we obtain:

$$C_M = ((t_{M,AF,f} + t_{M,E,f})L_M + C_{M,M,AF,f} + C_{M,M,E,f})t_f n_{t,a} \quad (12.3.23)$$

According to the AEA method for short range aircrafts, the equation for estimating the maintenance man hours per flight hour (MMH/FH), for the airframe is:

$$t_{M,AF,f} = \frac{1}{t_f} \left( 9 \cdot 10^{-5} \frac{1}{kg} \cdot m_{AF} + 6.7 - \frac{350000kg}{m_{AF} + 75000kg} \right) (0.8h + 0.68t_f) \quad (12.3.24)$$

$$C_{M,M,AF,f} = \frac{1}{t_f} (4.2 \cdot 10^{-6} + 2.2 \cdot 10^{-6} \frac{1}{h} \cdot t_f) P_{AF} \quad (12.3.25)$$

The airframe mass is given by:

$$m_{AF} = m_{OE} - m_{E,inst} = 13231.874 - 1532.46 = 11699.414kg \quad (12.3.26)$$

The value of  $m_{E,inst}$  comes from the CG estimation chapter.

The price of the airframe is:

$$P_{AF} = P_{delivery} - n_E P_E \approx 12.6milUS\$ \quad (12.3.27)$$

This gives the following results:

$$t_{M,AF,f} = \frac{1}{2.429h} \left( 9 \cdot 10^{-5} \frac{1}{kg} \cdot 11700kg + 6.7 - \frac{350000kg}{11700 + 75000kg} \right) (0.8h + 0.68 \cdot 2.429h) = 4.16MMH / FH \quad (12.3.28)$$

$$C_{M,M,AF,f} = \frac{1}{2.429} (4.2 \cdot 10^{-6} + 2.2 \cdot 10^{-6} \frac{1}{h} \cdot 2.429) \cdot 12.6 \text{milUS\$} = 49.688 \text{US\$} / h \quad (12.3.29)$$

For the engine AEA gives a method for propfans. The engine labor is divided in time dependant labor and cycle dependant labor; the same dividing is valid for the engine material.

*time dependent labor*

$$L_t = 0.172 \cdot R \cdot k_3 \cdot (1 + N)^{0.4} \quad (12.3.30)$$

$$k_3 = 0.032n_c + K \quad (12.3.31)$$

The  $L_t$  is the time labor spent for the core. The  $n_c$  is the number of compressor stages, which for the PW 127F is 2, as it has two centrifugal impellers in series, each driven by its own turbine and the  $K$  factor is 0.64 for an engine with 3 shafts, like the PW 127 F; correspondingly the  $k_3 = 0.032 \cdot 2 + 0.64 = 0.704$ .

$$L_p = 0.072R \cdot B(1 + N)^{0.4} \quad (12.3.32)$$

$$B = K_{D,P} + K_{n,B}$$

$$K_{D,P} = 0.4D / A + 0.6$$

$$A = 8.5 \sqrt{\frac{N}{3P + 28}} + 0.9 \quad (12.3.33)$$

$$K_{n,B} = 0.05P + 0.6$$

The  $N$  represents the take-off power in 1000 SHP. Our PW engine has a 2750 SHP for take-off with one engine, which means that  $N=2.75$ .

The  $R$  factor is the *labor rate* which has the value of 63US\$/h.

The  $D$  is the propeller diameter, and the  $P$  is the number of propellers. The factor  $A$  has the value of  $A=2.978$ .

This gives the values of:  $K_{D,P} = 0.4 \cdot 3.93 / 2.978 + 0.6 = 1.128$ ;  $K_{n,B} = 0.05 \cdot 6 + 0.6 = 0.9$ ;  $B = 1.128 + 0.9 = 2.028$ .

The results for the labor time are:

$$L_t = 0.172 \cdot 63 \cdot 0.704 \cdot (1 + 2.75)^{0.4} = 13.34h \quad (12.3.36)$$

$$L_p = 0.072 \cdot 63 \text{US\$} / hh \cdot 2.028(1 + 2.75)^{0.4} = 16.08h \quad (12.3.35)$$

*cycle dependant labor*

$$L_{p,c} = 0.5 \cdot L_p = 0.5 \cdot 16.08 = 8.04h \quad (12.2.36)$$

*time dependant material*

$$M_t = 2.16(1 + N)^{0.8}(k_2 + k_3) \quad (12.2.37)$$

$$M_p = 0.56(1 + N)^{0.8} \cdot B \quad (12.3.38)$$

$$k_2 = 0.4 \cdot \left( \frac{OAPR}{20} \right)^{1.3} + 0.4 \quad (12.3.39)$$

The overall pressure ratio for our engine is, according to **Jane's** 14.7., so the  $k_2$  has the value of  $k_2 = 0.4 \cdot \left( \frac{14.7}{20} \right)^{1.3} + 0.4 = 0.668$ . The values obtained are:

$$M_t = 2.16(1 + 2.75)^{0.8}(0.668 + 0.704) = 8.532h \quad (12.3.40)$$

$$M_p = 0.56(1 + 2.75)^{0.8} \cdot 2.028 = 3.269h \quad (12.3.41)$$

*cycle dependent material*

$$M_{p,c} = 0.5M_p = 0.5 \cdot 3.269 = 1.57h \quad (12.3.42)$$

$$M_{t,c} = 1.3 \cdot M_t = 1.3 \cdot 8.532 = 11.09h \quad (12.3.43)$$

The final direct engine maintenance cost is given by:

$$\begin{aligned} C_{M,E} &= n_E \left[ (L_t + M_t) \frac{t_f + 1.3}{t_f + 0.25} + (L_p + M_p) \frac{t_f + 0.5}{t_f + 0.25} \right] \cdot k_{inf} \\ &= 2 \left[ (13.34 + 8.532) \frac{2.429 + 1.3}{2.429 + 0.25} + (16.08 + 3.269) \frac{2.429 + 0.5}{2.429 + 0.25} \right] \cdot 1.85 = 190.917US\$/h \end{aligned} \quad (12.3.44)$$

The final value of the maintenance costs is:

$$\begin{aligned} C_M &= (t_{M,AF,f} L_M + C_{M,M,AF,f} + C_{M,E}) t_f n_{t,a} = (4.16 \cdot 63 + 49.688 + 190.91) \cdot 2.429 \cdot 1180 \\ &= 1440785.74 \approx 1.44 \text{milUS}\$/\text{year} \end{aligned} \quad (12.3.45)$$

The inflation rate was calculated with the formula:

$$k_{INF} = (1 + p_{INF})^{n_{year} - n_{method}} = (1 + 0.033)^{19} = 1.85 \quad (12.3.46)$$

where the  $p_{INF}$  has a value of 0.033, according to **AI 1989** and  $n_{method}$  is the year 1989, which is the year of the AEA publication.

### 12.3.6 Staff Costs

The cockpit crew,  $n_{CO}$ , is paid at a mean hourly rate,  $L_{CO}$ , and the cabin crew,  $n_{CA}$ , at the rate  $L_{CA}$ :

$$C_C = (n_{CO}L_{CO} + n_{CA}L_{CA})t_b n_{t,a} \quad (12.3.47)$$

The AEA method, for short and medium range aircrafts gives the following values:

$$\begin{aligned} L_{CO} &= 246.5 \text{US\$} / h && \text{for each crew person} \\ L_{CA} &= 81 \text{US\$} / h \\ n_{CO} &= 2 && \text{one cabin crew for each 35 passengers} \\ n_{CA} &= 2 \end{aligned}$$

The results are:

$$C_C = (2 \cdot 246.5 + 2 \cdot 81) \cdot (2.429 + 0.25) \cdot 1180 = 2070599.1 \approx 2.07 \text{milUS\$} / \text{year} \quad (12.3.48)$$

In the formula we used the block time,  $t_b = t_f + 0.25h$ ; according to AEA method the 15 minutes from the formula include 10 minutes of start-up and taxi-out and 5 minutes for taxi-in.

### 12.3.7 Fees and Charges

There are three types of charges:

*landing fees*

$$C_{FEE,LD} = k_{LD} m_{MTO} n_{t,a} k_{INF} \quad (12.3.49)$$

*navigation fees*

$$C_{FEE,NAV} = k_{NAV} R \sqrt{m_{MTO} n_{t,a}} k_{INF} \quad (12.3.50)$$

ground handling fees

$$C_{FEE,GND} = k_{GND} m_{PL} n_{t,d} k_{INF} \quad (12.3.51)$$

The three factors are, according to the AEA method:

**Table 12.3** Parameters for calculating Fees and Charges (AEA 1989a)

$k_{LD} [US\$/kg]$	$k_{NAV} \left[ \frac{US\$}{nm\sqrt{kg}} \right]$	$k_{GND} [US\$/kg]$
0.0078	0.00414	0.10

The results are as follows:

$$C_{FEE,LD} = 0.0078 US\$/kg \cdot 23296.272 kg \cdot 1180 \cdot 1.85 = 397346.28 US\$/year$$

$$C_{FEE,NAV} = 0.00414 \frac{US\$}{nm\sqrt{kg}} \cdot 670 NM \cdot \sqrt{23296.272 kg} \cdot 1180 \cdot 1.85 = 925777.19 US\$/year \quad (12.3.52)$$

$$C_{FEE,GND} = 0.10 US\$/kg \cdot 6650 kg \cdot 1180 \cdot 1.85 = 1454151.85 US\$/year \approx 1.45 milUS\$/year$$

The total fees comprise of the sum:

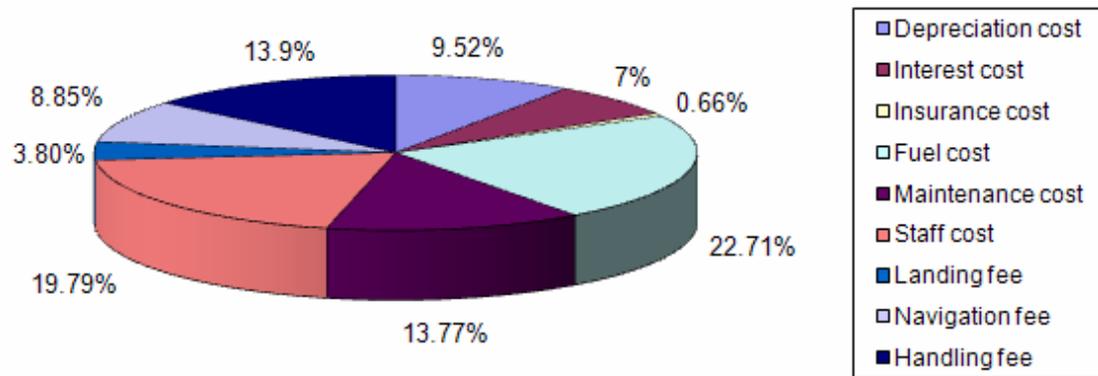
$$C_{FEE} = C_{FEE,LD} + C_{FEE,NAV} + C_{FEE,GND} = 2777275.32 US\$/year \approx 2.77 milUS\$/year \quad (12.3.53)$$

## 12.4 Summing up DOC Contributions

The sum of the elements calculated above give the final DOC value:

$$\begin{aligned} C_{DOC} &= 995639.66 + 732893.71 + 69295.91 + 2376319.4 + 1440785.74 + 2070599.1 + 2777275.32 \\ &= 10462808.84 US\$/year \approx 10.5 milUS\$/year \\ &\quad (12.3.54) \end{aligned}$$

The graphic representation of the elemental costs is shown in the next figure:



**Fig. 12.1** DOC graphic representation

## 13 Summary

The redesign process of the ATR 72 covered the envelope of all important parameters for defining the aircraft in the preliminary phase.

These parameters are now compared with the data of the real aircraft (see Table 13.1).

<i>Aircraft Components</i>	<i>Redesign</i>	<i>Original</i>
<b>Fuselage</b>		
Length	27.13m	27.166m
Diameter	2.77m	2.57m
Cabin length	19.25m	19.21m
<b>Wing</b>		
Span	27.32m	27.05m
Surface	62.187m <sup>2</sup>	61m <sup>2</sup>
Wing loading	374.317kg/m <sup>2</sup>	373.77kg/m <sup>2</sup>
High lift devices	Double slotted flap with slats	Double slotted flap
<b>Horizontal Tail</b>		
Span	4.747m <sup>2</sup>	
Surface	14.085m <sup>2</sup>	11.73m <sup>2</sup>
<b>Vertical Tail</b>		
Span	7.629m <sup>2</sup>	
Surface	9.701m <sup>2</sup>	12.48m <sup>2</sup>
<b>Other Parameters</b>		
Maximum take-off weight	23296.272kg	22800kg
Maximum operating empty weight	13231.874kg	12950kg
Take-off power	4189.59kW	4102kW

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