Projekt

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Optimal Boarding Methods for Airline Passengers

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Abstract

In this document, currently applied boarding policies are introduced. Their main characteristics are shown and briefly analyzed. Due to the variety of existing research works concerning the boarding problem that have already been done, some selected works are introduced. An overview of what has been done in these works, as well as a short outlook of their findings and results is given. This document is mainly built up by using these references whereby it can be treated as a literal research work.

The fundamental characteristics of the boarding process are herewith stated, as well as with the help of general understanding gathered by personal observations. The understanding of the process enables to generate formulations that can be used to be represented by mathematical models or simulation. Based on this understanding, a comparison study of two different policies, representing two major popular applied methods, is being undertaken. The result of this investigation shall give insights to the influence of the cabin width on boarding time; it shows furthermore that the often criticized back-to-front method could be preferred under given circumstances.

The results and findings that were done by other studies are stated and discussed. By summarizing these results, the major accordance of the 'Window-Middle-Aisle' policy to be the most efficient one is found. An outlook of the impact of some innovative policies in reality is given by the example of two market leading airlines.
Optimal Boarding Methods for Airline Passengers

Task for a project

Background
Boarding methods have a great influence on turnaround time and direct operation costs of an airplane. The processes of boarding and de-boarding take part in the critical path of a turnaround. Therefore, a reduction in boarding time has a direct impact on the total turnaround time. As a result, several airlines currently apply boarding policies to optimize turnaround processes, while there is no clear identification of the best method. As an example, EasyJet uses a free seating policy, British Airways uses the Back-to-Front method and the Window-Middle-Aisle (WMA) method is used by United Airlines. There exist also combinations of different boarding policies, such as the Block Boarding, developed by Delta Airlines. Some software tools and mathematic models have been developed in order to simulate and analyze the boarding processes. These boarding policies and tools lead to an extensive literature with multiple results. This project is part of the aircraft design research project "ALOHA" (http://ALOHA.ProfScholz.de).

Task
The task of the project is to summarize and describe existing boarding policies as well as to identify the most suitable boarding policy for selected cabin layout configurations.

The task includes:

- Literature research on current boarding policies and tools for simulation.
- Detailed description and analysis of boarding processes and their characteristics.
- Identification and brief explanation of the main mathematical models used to describe boarding processes.
- Identification of the most suitable boarding policy for different cabin layout configurations.
- Listing of current available software tools that are able to simulate boarding processes.

The report has to be written in English based on German or international standards on report writing.
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Nomenclature

$T$  total (boarding) time
$n$  number of seat rows
$s$  number of seats abreast
$g$  group of passengers
$i$  position within a group of passengers
$x$  number of passengers
$z$  number of zones
$k$  number of rows where seats will be getting occupied by group $g$
$P$  probability
$D$  total delay

Subscript

$(i)$  position within a group of passengers
$(g)$  group of passengers
$(wma)$  window-middle-aisle
$(btf)$  back-to-front

List of Abbreviations

Pax  passengers
BTF  back-to-front
WMA  window-middle-aisle

Note: Where certain authors used other specific determinations, they can particularly be found in the corresponding chapter!
Terms and Definitions

**Turnaround**
Turnaround is the period of time that the aircraft is on the airport ramp, from blocks on at the aircraft arrival to blocks off at the aircraft departure. This includes the positioning of the pushback tractor and tow bar in preparation for the pushback process.  
* (Airbus GH 1995)

**Boarding**
The act of going on board of a ship, aircraft, bus, etc.  
* (YourDictionary 2009)

**Simulation**
Simulation is the initiation of a dynamic process in order to derive insights which can be applied to reality  
* (VDI 1997)

**System**
A system is a subset of reality which we study to answer a question; i.e. its boundary to the environment in which it is embedded will be determined by the questions we wish to ask. A system must have a number of distinct and clearly identifiable components, which may themselves be considered as systems at a „lower“ level.  
* (Niemeyer 1977, p.57)

**Models**
Models are material or immaterial systems which represent other systems in such a way that experimental manipulation of the modelled structures and stated becomes possible  
* (Niemeyer 1977, p.57)

**Heuristic**
Helping to discover or learn; specif., designating a method of education or of computer programming in which the pupil or machine proceeds along empirical lines, using rules of thumb, to find solutions or answers  
* (YourDictionary 2009)

**Boarding strategy or policy**
A boarding strategy is a group of rules that aim at accommodating all passengers by using as little time as possible.  
* (Capelo 2008)
1 Introduction

1.1 Motivation

A variety of different research studies dealing with the boarding problem were done in the past. Although the authors of the particular studies refer to each other, there is no document yet that summarizes basic findings and general common basis that were stated in these studies. The findings include results of the fastest boarding methods as well as general insights and assumptions that were made. It is of interest, whether there were made similar or different assumptions and/or similar results were found.

1.2 Objectives

The main objectives of this project are to summarize and describe existing boarding policies as well as to identify the most suitable boarding policy for selected cabin layout configurations.

This includes Literature research on current boarding policies in order to describe in detail the boarding processes and analyze its characteristics.

The main mathematical models that are used to describe the boarding process need to be found and briefly described. Furthermore, a list of current available software tools that are able to simulate boarding processes shall be created.

A secondary objective is to investigate the influence of different fuselages and according seat plans.
1.3 Literature

The basic literature that was used to create this document is research studies that deal with the boarding problem. These studies were mostly done with cooperation of universities and/or industry. More detailed descriptions of these documents can be found in chapter 2.7.

For the description of simulation techniques, Page 2005 was mainly used as reference.

Kolonko 2008 provided some inspiration and idea finding for the analytical analysis in chapter 5. This book mainly deals with stochastical methods of simulations.

Your Dictionary 2009 was used as help to define some terms.

A variety of projects have been done to do investigations on the boarding method problem. In the following lines, some authors will be shortly introduced who have taken up the work. These study contents and results have basically been used to create this document.

Research study by Jason H. Steffen (2008)

J.H. Steffen, an astrophysicist at the ‘Fermi lab Centre for Particle Astrophysics’ published his work „Optimal Boarding Method for Airline Passengers“ in February 2008. He uses a Markov Monte Carlo optimization algorithm applied to a computer simulation in order to find the optimal passenger order that minimizes the time for the boarding process. He uses a model of a single aisle cabin with 6 seats abreast and 20 rows in order to investigate the problem. In this work, none of the popular known boarding logics has been applied.

The result is based on iterating passenger orders and can be later on classified with one of these methods.

Research study by Stolyarov et al. (2007)

Tom Caswell, Kyle Story, and Rafael Frongillo participated on a mathematical contest in modelling for COMAP\textsuperscript{1}. The task was to devise and compare procedures for boarding and deplaning with varying numbers of passengers. They developed two models: one analytical and another computational simulation. With the highly idealized analytical model, they find that the WMA variant is basically the most efficient one. The computational model is based on iterations. It tracks single passengers boarding the airplane while interacting with other people.

\textsuperscript{1} Consortium for Mathematics and Its Applications - an award-winning non-profit organization whose mission is to improve mathematics education for students of all ages
Research study by Marelli et al. (Boeing corp. study) (1998)

BOEING developed a computer simulation model called PEDS (Passenger Enplane/Deplane Simulation). The purpose for this project was to prevent airlines from costly in-service experiments. Next to the examination of different loading procedures, a remarkable feature of this software tool is dynamic interior configurations and the consequential ability of their evaluation. The mathematical model is based on discrete event simulation, which analyzes the process as a set of interrelated elements. BOEING conducted in service observation as well as passenger loading tests in order to validate the simulation results.

Research study by Landeghem and Beuseling (2000)

Landeghem and Beuseling defined different boarding classes which they investigated within the simulation. These classes are potentially similar to given boarding logics, although they were not namely linked to them in this study. The classes are for example called “by block” or “by half block” meaning that the boarding is separated into a set of passengers belonging to this class enters the plane. There is no clear explanation which type of simulation they used to model the process itself.

Research study by Van Den Briel (2005)

The Arizona State University worked on a joint project with America West, where the goal was to cut passenger boarding times for the Airline’s narrow body passenger airplanes (such as A320 & B737). The project included gathering data, developing and solving mathematical programming and simulation models and validating and implementing the results. Van den Briel, the leader and author of this study developed a model based on MINLP, a mixed integer nonlinearly constrained optimization solver. He introduced the terms aisle/- and seat- interference and included them into the model.

Research study by Bachmat et al. (2009)

‘Analysis of Airplane Boarding Times’ is the name of the work that uniquely deals with an analytical approach to investigate the boarding time problem. The authors criticise that in other works the deeper understanding of the boarding process is missing through the usage of simulations and describe their work as a natural framework for modelling analytically the airplane boarding process. They use the Lorentzian geometry to model the process and apply various existing boarding policies to it. In the end of the document, the results are being compared with earlier works.
Research study by Ferrari & Nagel (2005)

A research work based on *discrete event simulation* was done by Ferrari and Nagel. In contrasts earlier works by putting special emphasis on disturbances, such as a certain number of passengers not following their boarding group but boarding earlier or later. One result that is discovered is that the typical Back-to-Front boarding strategy becomes improved when passengers do not board with their assigned group.

Research study by McFadden (2008)

David C. Nyquist and Kathleen L. McFadden published an article in the Journal of Air Transport Management in 2008 that summarizes the strategies and results of various existing studies (e.g. Landeghem 2000, Ferrari 2005, etc.). They discuss the impacts of different findings of turn time improvements on financial aspects.
1.4 Report Structure

Chapter 2    Introduction of ALOHA and into the general topic by explaining the ground handling process and the importance of time saving. Furthermore, the main currently applied boarding policies are listed as well as existing research studies.

Chapter 3    Analysis of the process characteristics. Research in order to gather general understanding. Furthermore, major assumptions of current research works are stated here

Chapter 4    Explanation of analytical and computational models used to describe and investigate the boarding problem. Furthermore explanation of existing simulation software.

Chapter 5    Analytical approach in order to investigate the effect of varying cabin diameter on boarding time at the example of Back-to-Front compared to Outside-In

Chapter 6    Provides alternative ideas and possibilities (than boarding strategies) that can potentially decrease boarding time

Chapter 7    Summation of major findings in public research works and conclusion
2 Introduction to Boarding

2.1 ALOHA

This work is part of the ALOHA project. ALOHA is a science research project with duration of 2 years and 2 month (01.11.07-31.03.10).
Partners are

- University Of Applied Sciences Hamburg (HAW) – “Federführer”
- Airbus GmbH, Hamburg - Future Project Office (FPO)
- Airport Research Centre GmbH (ARC)
- Hamburg Airport GmbH (Ground Handling Division)

The aim is to preliminary design conventional and unconventional passenger aircraft configurations. These configurations shall finally decrease the DOC. The work is focused on low cost airlines (LCA), where a high percentage of the costs are generated by the ground service.

Technical features that potentially can lower ground costs are also taken into consideration. The creation of a program that calculates ground costs depending on aircraft parameters will be accomplished. On this matter, research of procedures and costs on the ground needs to be done.

LCA mostly use airplanes of the Boeing 737 and Airbus A320 family. Both of these airplane manufactures are currently working on a follower of these types of aircraft. By research works like ALOHA, the chance is given that the new developments are supported by knowledge based on results of these works.

As this work is part of the ALOHA project, the will focus will be– when necessary – on seat layouts with a single aisle and six seats abreast, like it is common for this aircraft class.
2.2 The Ground Handling Process

In order to make an aircraft ready to leave the gate for its next turn, it requires a lot of single tasks. The main tasks are listed below. The order in reality of the single steps is not reflected by this table:

- Passengers debarring
- Passengers boarding
- Cargo unloading
- Cargo loading
- Catering service
- Cleaning
- Refuelling
- Lavatory service
- Potable water service
- Push back
- Ground power
- Ground air preconditioning
- Pre-flight inspections
- De-/anti-icing

(Sanchez 2009)

Although the boarding of passengers is only one single step of the whole process, it is yet the most critical one that determines the total ground handling time:

Figure 2.1 shows the generic time consumption of the ground service. On this illustration can be schematically seen which procedures are running simultaneously and which ones are starting as soon as another process is finished. It should additionally be said that on this image there is no information that illustrates which process requires another one to be finished. But it can generally be understood, that in order to speed up one process while another independent one is not finished, would not decrease the total turnaround time of an aircraft. The boarding of passengers requires all activities in the cabin to be finished, so that the boarding process starts as one of the last element. Thus, an improvement in boarding time can shorten the total ground service time. It is the critical procedure of the ground handling process that ends last before the plane is being prepared for taxiing by removing passenger bridges, closing doors and starting engines. There are no parallel activities running anymore at that time (see Figure 2.1).

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1 General case
On the following figure, the turnaround time of an aircraft is illustrated on a split up flow-chart. On this chart can be seen the dependence of different services as well as the approximated durations. This chart furthermore confirms the enplane time to be the most time consuming part of the process.

**Figure 2.1** Ground handling process (Marelli 1998)

**Figure 2.2** Elements and disturbances of airplane turnaround time (Landeghem 2000)
2.3 Importance of Time Saving

“Time is money”

(Benjamin Franklin)

Besides the effort of keeping airline passenger satisfied, the saving of time on the ground handling process has two main important reasons for the airline: if it is possible to gain sufficient time, this can potentially allow one more turn for an aircraft per day, resulting directly in more revenue. Of course this requires the route distance to be significantly short enough as well as the need on the market to be realized and so on. But nevertheless, for some airlines this could turn the scale when considering one or maybe even more legs per day.

But it is not only important when there is the potential for another trip. What is more - taking a major US airline as an example – it accrues around $30 for every active airplane per ground minute. (McFadden 2008)

2.4 Historical Trends

Turnaround times of short/midrange and wide body aircraft have experienced a gradual increase. Since 1975, the passenger flow rate (determined by the number of passengers that enplane per minute [pax/min] see chapter 3.5) has slowed down from 20 to 9. The main reasons for this trend of retardation by nearly 55% are stated by heavier carry-on luggage and airline service strategies that focus on passenger’s convenience. Unless new processes for boarding are developed, these trends will continue. (Marelli 1998)

2.5 Applied Boarding Policies

Today, there are five major boarding logics which are mostly being used - whether if they are most often being applied by airlines or popular for being considered within research. Several airlines use priority boarding for passengers travelling with small children, first class passengers, business class passengers, frequent flyers, certain card holders, and passengers who check in online. This could be treated as a boarding policy as well, but obviously does not follow a logic that shall increase boarding time. That is why advantages or disadvantages of this policy will not be discussed in this document. The list in chapter 2.6 gives an overview of the existing boarding logics being used by several airlines.
2.5.1 Seat Layouts

In this chapter is shown a typical layout of a short and mid range aircraft and its basic features are explained. This shall help to understand further topics of this work.

![Cabin layout definition](image)

During the boarding process, commonly only one boarding door is being used on such a layout. On larger airplanes, there are often used two or more boarding doors to serve premium classes and economy class separately.

2.5.2 Back-to-Front

The Back-to-Front logic implies that the passengers are boarding the airplane from the back row of the aircraft and continue with the next rows up to the front. This certainly requires the active boarding door to be in the front.

With this method, the cabin is being divided into zones, leading from the back to the front. The zones can notionally be any number reaching from two till to the number of actual rows in the cabin. In practice, around 4 to 5 zones of equal size are being used. The zones can furthermore be either a set of rows (block) or likewise a set of rows that is divided into left and right hand side (half block). In the case of a six abreast seat layout, this would particularly be the window, middle, and aisle seats of each side. If the zones are equal to the number of rows that are present in the cabin, the additional term “by row” is being used.

The Back-to-Front method is comparatively easy to implement for use in reality. It is therefore often in use and also known as the “traditional” boarding method.
For years US airlines have used traditional (by block, by row, by half-block, by half-row) enplaning methods in the boarding of most flights.

(McFadden 2008)

By common sense, the “Back-To-Front” policy provides good efficiency as one possibly assumes that a group of humans behave just similar like a liquid that is filled into a bottle, while the bottle naturally fills up from the bottom to the top.

Apparently, the simple logic of this method must be the reason for it being the most common one. As it will be mentioned later in this document, it does not reveal true efficiency:

“The worst method is, indeed, to board the plane from the front to the back. (...) boarding the airplane from the back to the front is very likely the second worst method”

(Steffen 2008)

Steffen 2008 describes a Front-to-Back logic as the worst one. However, since this one only exists on paper and possibly never has been realised, it can read out from this statement that the popular ‘Back-To-Front’ can be considered as the worst applied method.

The main disadvantage of the Back-to-Front method is that only a small section (particularly the area of one zone) of the airplane is being boarded at a time. When taking as an example a six abreast layout with 5 rows being part of one block, the result are 30 passengers trying to get seated simultaneously in only 5 rows at the same time. It does not require good faculty of imagination how this produces massive congestion in the particular part of the cabin - while the rest of the seats are not being considered for boarding at that time.

2.5.3 Rotating-zone

This method is similar to ‘Back-to-Front’. The seating layout is being divided into the same class of zones (longitudinal among the cabin). The boarding starts with the last zone in the back to be finished - then continuing with the first row in the front. After this, the order continues again with the furthest not yet occupied zone in the back, then in the front, and so on. The advantage of this method versus Back-to-Front is, that as soon as the queue in the aisle - caused by incoming passengers waiting to board the back zone - is short enough so that the aisle alongside the next to be occupied front rows is completely free. These particular passengers can start significantly earlier to board than if they would board the next zone in the back. Besides this, the passengers that are boarding the front zone simultaneously with the back
zone will not interfere each other. This lasts until the last two zones, which are together in the middle. This method possibly requires more coordination by the ramp agents.

As the efficiency of this method is implied by simultaneous boarding, a ramp agent\(^1\) or a cabin crew member is required in order to communicate from inside the airplane with the ticket counter. The agent needs to provide information of when the aisle is sufficiently cleared in order to allow the next group to continue boarding. This is basically not meaningless for other policies using zones as well. This method is currently used by Air Tran and Delta airlines. The main drawback of this method - analogue to the Back-to-Front method - is that basically all passengers belonging to a particular zone need to sit down in a relatively dense space.

2.5.4 Random

The random boarding method is actually not a true boarding logic – because there is no given algorithm that determines in which order passengers are boarding the airplane. In reality, this means there is no service agent calling respective passenger groups belonging to a zone to en-plane.

The random seating pattern generally has low aisle interference since numerous people can be stowing bags and seating themselves simultaneously. In general, it also increases seat interferences. The term ‘general’ is here being used, since the distribution of seat assignments can potentially be everything - even a perfect Back-to-Front. Solely the possibility of this to occur is obviously negligible.

The random method needs to be separated into 1) normal random boarding and 2) free seating.

At normal random boarding, passengers are assigned to specific seats but line up at the ticket counter and are admitted in the order that they arrive in line. While at the free seating policy, passengers can choose any non occupied seat that they spot as soon as they are onboard. As people obviously will start to rush through the air bridge into the plane in order to get their favoured seats at the free seating policy, this is possibly the faster method than if they had already assigned seats (normal random boarding). Everyone who has ever flown with Ryanair might have made personal experience of a real efficient boarding like this. However, for this reason this method will probably stay unique for low cost airlines that do not put their emphasis on passengers comfort.

\(^1\) Respectively ground staff
Previous studies suggest that random boarding is not a ‘customer friendly’ alternative

(McFadden 2008)

As it can be seen in Table 2.1, this assumption reveals as likely, as the mentioned airlines that use ‘Random’ can be classified to represent this market share. For some of these airlines however, this side effect turned out to be a good opportunity to gain some extra revenue: they demand an extra booking fee for a prioritised boarding, letting a certain group of passengers who are willing to pay this fee entering first.

Airlines are offering "first to board" perks to their frequent fliers or premium seat holders.

(Travel Sentry 2009)

But on the other hand, this possibly can be an annoyance for these passengers when they find themselves back on an almost empty flight. Now, leaving out the assumption that passengers compared to another boarding policy will walk or run faster, all investigations on this matter showed that the random policy is however still faster than “Back-To-Front”; it can be even faster than some innovative methods.

2.5.5 Outside-In

This method is also called ‘WMA’ – Window Middle Aisle. As the name already implies, passengers assigned to window seats will board first. When this finished, middle and aisle seats are following. At layouts with only four seats abreast, the middle seats will accordingly be left out from the order.

As it will be shown later in this document, this method reveals as very efficient. The reason for the efficiency of WMA is the long distribution of incoming passengers among the aisle. This causes the passengers potentially to less interfere each other while they load their baggage and then sit down. In theory, there are no passengers that stow their carry-on luggage in the same position of an overhead stowage bin, given that they all use exactly the one that is placed over their assigned seat. In practice this is possibly hard to reproduce, although this method is designed to achieve exactly this – so there can technically be done no more improvement on this matter.

The second (and very significant) advantage is that this method completely eliminates passengers who interfere each other among a row. This means that a passenger will never experience the situation of the necessity to over climb another already seated passenger, or ask
someone to get up in order to free his/her legroom. This case certainly occurs in theory only. But again, this feature significantly participates making this method very efficiently. Certainly, the order within window, middle and aisle seats is not determined by this method. This is a major drawback of WMA. Because of this fact, it is rather likely that people in practice will interfere each other as soon as one passenger’s seat of the boarding groups is assigned in front of his follower. And this is quite likely to occur, since the assigned rows are spread over the whole cabin.

Thus, on layouts with fewer seats in a row, this negative effect could dominate over the advantages and could possibly cause WMA to fail versus e.g. the Back-to-Front method. In chapter 5, the simple algorithm of this policy is used to do analytical investigations on this special problem.

The outside-in method is relatively easy to apply in reality, as there are no extra row-assignments necessary to be done. This provides more clarity and can potentially confuse passengers less. A zone could for example be determined by containing seat rows with the letters “A”, “F” or “A & F”. The next zone would be “B & E” and “C & D”. There is no need for extra information to be printed on the boarding passes in order to fully declare the zones.

2.5.6 Reverse-Pyramid

This method was introduced by America West Airlines and was developed by a team of the Arizona State University leaded by Van Den Briel. It combines two methods: Back-to-Front and WMA. The name of this method is determined by the way the passenger order is built up: from the outer back till to the inner front of the cabin. The best way to understand this strategy is to study its algorithm visualized by numbers (see Appendix A).

In so doing, basically the mentioned remaining aisle interfering of the WMA method is reduced by implementing a second, the Back-to-Front constraint, while the major advantages remain. This idea of combining Back-to-Front with WMA is providing very high efficiency. The drawback (mentioned in chapter 3.4) of WMA caused by aisle interference is herewith reduced.

The finding of this method has been applied by this American West Airlines. It proved in practice to provide faster boarding times and helped the Airline to have significant less gate-related delays since the implementation (see chapter 7.6.1).

This method certainly possibly needs more effort considering the designation of zones as well as the monitoring and steering of the boarding process. This is possibly a reason why it has so far only been applied by one airline.
### 2.6 Policies used by Airlines

The following table gives an overview which boarding policy is used by which airline:

<table>
<thead>
<tr>
<th>Back to front</th>
<th>Rotating zone</th>
<th>Random</th>
<th>Outside-in</th>
<th>Reverse Pyramid</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air Canada</td>
<td>Air Tran</td>
<td>Jet2</td>
<td>Ted</td>
<td>America West</td>
</tr>
<tr>
<td>Alaska</td>
<td>Delta Airlines</td>
<td>JetBlue</td>
<td>United Airlines</td>
<td>U.S. Airways</td>
</tr>
<tr>
<td>American Airlines</td>
<td></td>
<td>Maxjet</td>
<td></td>
<td></td>
</tr>
<tr>
<td>British Airways</td>
<td></td>
<td>Northwest</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Continental</td>
<td></td>
<td>Southwest</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Frontier</td>
<td></td>
<td>US Airways</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Midwest</td>
<td></td>
<td>Easy Jet</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Spirit</td>
<td></td>
<td>Ryanair</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Virgin Atlantic</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lufthansa</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

---

1. Van Den Briel 2005 said he believes that U.S. Airways will adopt this policy as they merged with America West.
3  Boarding Process Analysis

In order to do a simulation of the boarding process, its characteristics need to be analyzed and fragmented. The process as it happens in real can be written down by observations. Solely sorting out the negligible elements reveals as the significant part of this procedure. Concerning simplifications, different authors of boarding studies apply likewise different conditions on their simulations. Finally, a simulation is based on a certain amount of assumptions that model the real world process. There can be added an optional amount of parameters to the simulation that refine the model. In so doing, the model gets closer to reality. The main characteristics of a boarding process are determined by human behaviour, as human beings are the actors in this model.

In order to investigate the consequences of human activities, the system must be analyzed - where great care must be taken to ensure that all relevant aspects of the real system are preserved. System analysis is important to increase the understanding, where science has made good use of a range of techniques for abstraction and aggregation. Models are always abstractions (simplifications) of reality. (Page & Kreutzer)

3.1  Model System Identification

As passengers will interact with each other (for example lining up in the queue), the boarding process is a Cybernetic system.

*Cybernetic systems contain feedback connections between their components*

(Page & Kreutzer 2005)

As the process takes places within a certain amount of time, where various parameters are time-depending, the boarding process is also a Dynamic system.

*Dynamic systems contain feedback connections between their components*

(Page & Kreutzer 2005)
The system boundary of the actual boarding process is the aircraft cabin. Nevertheless, some authors mention the ticket counter as part of the system as well. This is rather a question of definition – the ticket counter can also be seen as a generator for the system input.

*System boundary is a border separating a system from its environment (from everything not included as part of the system)*

(Page & Kreutzer 2005)

Since passengers are boarding the airplane from outside the system boundary (see chapter 3.2 process breakdown, the boarding of an airplane is an open system.

*Open systems have at least one interaction with their environment*

(Page & Kreutzer 2005)

### 3.2 Process breakdown

Beyond, an overview of single steps that a passenger experiences when boarding an aircraft are listed. These steps are determined according to (Stolyarov 2007) as well as by confirmation and addition based on personal experience:

1) **Ticket counter**
   queuing and wait for ticket to be scanned

2) **Passenger boarding bridge**
   proceeding to airplane door

3) **Airplane aisle**
   entering boarding door and proceeding to assigned row

4) **Stowing bags**
   stowing luggage into the bins – will be also denoted as clearing time

5) **Sit down**
   sitting down at the assigned seat

In principle, steps 1) and 2) do not have any effect on boarding time, as long as people are passing the ticket counter faster than actually boarding the airplane. Yet, not all studies highlighted in this work are only taking the steps into consideration starting at point 3). The virtual breezeway is rather being used to put the passengers into the specific order - given by the simulation rules. Now, it is a matter of opinion if this can be seen as a simulated breezeway or just a simulation constraint. The airplane door is nevertheless the system border, from where “objects” (passengers) are being sent into the system.
When considering the application of gathered simulation result later on, the boarding request of passengers with particularly assigned seats would be the equivalent part of this.

The time to sit down is negligible as the aisle is immediately being cleared after step 4). Solely in case of seat interference, the time to get up and sit down again needs to be considered.

### 3.3 Assumptions for Modelling

For the essential boarding process reaching from step 3) to 5), basic assumptions need to be made. These assumptions will be e.g. parameters and constraints on a simulation. The basic parameters are the walking speed of a passenger and the time that one passenger needs to sit down respectively to stow his/her luggage. Another essential parameter is the pax flow rate (number of passengers that enter the airplane in a certain amount of time, see more details further down in this chapter). All these parameter assumptions vary in different studies. When there is a work focusing on the comparison of different policies rather than finding a reality-related time, the association of a time to this parameters can be left out. The parameters must therefore have a factor relation or a unit that fits the simulation clock (e.g.: *Clearing time* = 3 times the *Sit down time*, *Walking speed* = 0.3 grid units/simulation step and so on). The following table lists the range of these parameter assumptions used in various studies. As these parameters can depend on others, the range can occasionally underlie high factors:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Range</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Walking speed</td>
<td>0.27...0.44</td>
<td>[m/s]</td>
</tr>
<tr>
<td>Clearing time</td>
<td>6.00...30.00</td>
<td>[s]</td>
</tr>
<tr>
<td>Get up out of seat</td>
<td>3.00...4.20</td>
<td>[s]</td>
</tr>
<tr>
<td>Pax flow rate</td>
<td>0.20...1.00</td>
<td>[pax/s]</td>
</tr>
</tbody>
</table>

These assumptions can possibly only be found empirically.

Other assumptions that are necessarily to be made (especially for simulations) are edge constraints. The basic edge constraints are the *system boarders* and the *behaviour* of passengers. The system boarder is basically the aircraft cabin, limited to the boarding door as the system entrance and the seats on the other hand as the final position of passengers. The assumptions that need to be made for the human behaviour are a bit more complex, since one cannot consider the passengers to be robots that follow a strict statement: “go and sit down!”’. Nevertheless, they need to be done by entirely the developer of the model. Due to this fact, the simpli-
Simplification can be a sensitive element of modelling. **Page & Kreutzer 2005** describe the simplification process by occasionally following steps:

- Removing elements and actions of no importance to model goals
- Aggregating elements and actions of little importance to model goals
- Restricting the number of values state variables may take
- Replacing detailed causal scenarios by mathematical functions

Below, essential assumptions for the behaviour of passengers respectively the boarding process model itself are listed. This can be treated as the realization of the points mentioned above. These were made by authors of certain studies. Thus, the particular author will be mentioned. The assumptions are sorted by the steps that were determined above.

When there is a source by an author missing, it is still possible (in some cases rather likely) that the assumption was still made. There are only sources put in where there was clear evidence in the particular document.

**Step 3**

The aisle is only wide enough for one person\(^1\)

*(Stolyarov 2007), (Bachmat 2009), (Landeghem 2000)*

On airplanes with more than one aisle, passengers choose the one which is the closest to their seat. In the event of a tie, the aisle is being chosen at random

*(Stolyarov 2007)*

On airplanes with two decks\(^2\), passengers always choose the correct deck

*(Stolyarov 2007)*

**Step 4 & 5**

People always choose the correct seat.

(There is no indication that someone did another assumption; although Landeghem 2000 included: when a passenger has taken a wrong seat, he will be bumped when the right passenger arrives. When the mistaken passenger has to take his seat more towards the front of the airplane, he/she will have to wait for (part of the) the aisle to clear.)

When passengers arrive at their assigned seats, they must stow their carry-on luggage

*(Stolyarov 2007), (Bachmat 2009)*

The time for already seated passengers to get up in order to let someone pass is negligible

*(Steffen 2008)*

---

\(^1\) Though Marelli 1998 found that *in real*, some passing does in fact occur

\(^2\) Currently Airbus A380 and Boeing B747
Until a passenger has taken seat, the aisle will be occupied by: the space in front of his/her seat + an additional specific space value
(Bachmat 2009)

Passengers stow their bags always in the bins in front of their seats (bins never fill up)
(Stolyarov 2007)

Passengers stow their bags always in the bins in front of their seats, but the loading time increases as the bins fill up
(Landegehem 2000)

If there is already someone present between him/her and the assigned seat, the seated passenger needs a specific time to gets up and sit down again
(Stolyarov 2007)

This movement time only affects the prolonging time of the incoming passenger
(Landegehem 2000)

3.4 Passenger flow rate

The passenger flow rate is determined to be the number of passengers that pass a system boarder per time \( \frac{\text{pass}}{\text{sec}} \). In some cases of boarding policies, the flow needs to be interrupted in order ensure the logic constraints. For example: when passengers board in groups, the following set of passengers will not directly pass the ticket counter until the first group has finished or nearly finished seating. In other words: passengers are not necessarily constantly passing the ticket counter.

As it can be of interest which value to consider for a computational or analytical model, Marelly 1998 discovered the average passenger flow rate on the deplane and enplane process over a number of decades from the 1960’s until to the late 1990’s:
From the linearization line on this graph can be read out a value of around
\[ 11 \frac{\text{pax}}{\text{min}} \approx 0.18 \frac{\text{pax}}{\text{sec}} \] for the latest results. This value could be assumed for an actual approach of a boarding process model.

### 3.5 Passenger Flow Rate Influence

As passenger interference in the cabin only occurs when passengers block each other because they are boarding at the same time, it is manifest that by a lower passenger flow rate, the applied policy will less influence the boarding time. Van Den Briel 2005 plotted two boarding method times over the passenger flow rate and showed exactly this very clearly: at a passenger flow rate greater than 10, the two methods result in approximately equal times.
One can now assume that other boarding methods than the two exampled ones show similar behaviour. It is furthermore interesting to see that between a passenger flow rate of 1 and 5 in one case and between 1 and 7 in the second case, there is no difference in boarding time over increasing pax/flow rate at all.

The curve that indicates lower boarding times shows that its quality decreases earlier.

### 3.6 Impediments of Boarding Process

There are two basic elements that interfere the boarding process:

**Aisle interference**
One passenger loads his luggage and blocks the aisle for other passengers that are lining up behind him.

**Seat interference**
An already seated passenger blocks another one because his assigned seat is located in the same row and in front of his follower. Either the passenger needs to get up or be over climbed. In both cases the aisle will stay blocked for a certain amount of time. Since the passenger that gets up in order to enable the incoming passenger to sit down needs to escape into the aisle, **seat interference always causes aisle interference** by logical constraint (unless it is being assumed that the interfering person is being over climbed with the same speed than
normal sitting down process). In the following figure, the two kinds of interferences are visualized:

![Figure 3.3](image)

**Figure 3.3** Seat interference: A passenger (1A) tries to get to a seat near the window (1B) but is obstructed by another passenger already seated near the aisle.
Aisle interference: A passenger (2A) tries to reach his seat further down the aisle (2B) but is obstructed by other passengers trying to find their seats or stow their luggage. (Van Den Briel 2005)

It needs to be mentioned here, that due to JAR 25.817 the seat interference does not force more than two passengers to get up in order to let an incoming one sit down.

Following JAR 25.817:

\[
\begin{align*}
n_{SA} & \leq 6 \quad \text{for single aisle} \\
n_{SA} & \geq 6 \quad \text{for wide body aircraft}
\end{align*}
\]

The assumption of having these two factors as boarding obstructers requires assuming constant parameters. Loitering passengers could definitely slow down the process as well, but this does not play a role when comparing different policies which are determined by the order of assigned seats to be filled. When changing the policy, only the resulting difference in number of aisle and seat interferences is significant for the quality. Van Den Briel 2005 and Stolyarov 2007 mention this most clearly in their documents) although there can be found statements in all other works that indicate general accordance of the particular authors).

**Mathematical Approach Seat Interference**

In order to calculate the seat interference time, the following approach can be used:

\[
T = t_{p1} + \cdots + t_{pk} + t_p
\]

Where \( t_p \) is the aisle passenger’s passing time, and \( t_{p1}, \ldots, t_{pk} \) are being passing times for the \( k \) seated passengers and \( t_p \) is the time to get up and sit down again. (Stolyarov 2007).
Mathematical Approach Aisle Interference
The time a passenger needs to wait in the aisle is not determined by a constant time, rather than by the constraint that he/her can only continue when the passenger in front continues to walk or clears the aisle. The same can be applied for the next passenger, and so on. Within a simulation, this could be realized by a do-while loop. In any event, the origin of the aisle interference is always a passenger that has not yet cleared the aisle. The following approach for the clearing time has also been done by Stolyarov 2007:

\[
\text{clearing time} = t_B \cdot \left( \frac{\text{num\_seated}}{10} \right) \tag{3.2}
\]

While \(t_B\) is a determined time factor [sec] and \(\text{num\_seated}\) the number of passengers that have already sit down. This causes the clearing time to be a function of the occupancy of the airplane, representing the time increase to stow the hand luggage as the bins start to fill up. This assumption is not essential for the boarding process simulation and has therefore not been made in all studies (see assumptions chapter 3.2).

3.7 Implementation in Reality

A reader of this topic could ask him/herself how realistic it would be to implement a result that has been discovered by-seat assignments as the optimum. A problem in reality would be in fact to ask every single passenger to board the airplane. The result of a group, where within this group passengers enplanes randomly, is much easier to realize. To influence the boarding sequence of passengers, call-off systems are currently often used. With this method, the particular zone that is next entering the plane is called by the ground staff.

“By-group” call of systems are being used by airlines applying all policies where the cabin is separated into certain zones determined by the according policy. Passengers always must be requested when it is their turn to enter the airplane. One way to realize this, is giving announcements via a loudspeaker.

An alternative to the acoustic call off system is a visual system which is already being applied in reality: a coloured card indicating the block that the passenger will board in is being handed out. When boarding starts, a lamp of the same colour at the gate entrance indicates which block can board. The coloured cards are being handed over by ground staff.
“This system causes more confusion than it solves: the colours are not always unambiguous (e.g. blue next to violet) and passengers forget to return the colour card”

(Landegehem 2000)

An idea of improvement on this matter could be to print or stick a colour bar directly on the boarding pass. The more zones are used in any boarding method, the smaller the according groups become. The best sequences all require calling off individual passengers by their row and seat number (Landegehem 2000). This generally makes the process more complicated to be implemented in reality, as the accuracy with which the passengers need to board will increase. In other words: the less instruction given to passengers, the easier the implementation.

“Passengers do not appreciate too complicated call systems. A compromise has to be found between simplicity of the call system and velocity of boarding.”

(Landegehem 2000)

A solution, where not only groups but even single passengers can be guided to a specific seat would be to distribute the seat assignments dynamically. In this case, the seats could be assigned not till the passenger passes the ticket counter. The next to be occupied seat could be printed on the ticket when the passenger or the ramp agent pulls the ticket into the counter. The disadvantage is clearly that passengers do not have an opportunity to choose their favourite seat. Furthermore, people travelling together (families, business travellers etc) will probably hardly accept to be placed certain rows apart from each other. A resulting re-arrange chaos by passengers is likely and could potentially delay the boarding even more.

3.8 Premium Classes Priority

On short haul and midrange flights conducted by aircraft types like Airbus 320 and Boeing 737, many airlines use a cabin layout that compromises a premium class (business and/or first class) in the first front rows. Now, some of the airlines using this cabin configuration use to prior board premium class passengers, meaning they are accepted to board separated (commonly earlier) from the remainder. Due to the small percentage of cabin coverage, a general boarding method for this part of the cabin would be meaningless here. However, when including the premium class into the whole process, its additional boarding time consumption would be constant in any case. Thus, for the comparison of different boarding method this can be left out. Van den Briel nevertheless included the business class priority assumption in his studies.
4 Models for Boarding Process

Models help to understand the behaviour of a system and the effects of interactions among its components. In the scope of boarding process modelling, the understanding of the effects of the boarding policy needs to be understood and investigated.

There are several ways to model natural processes. The main methods that come into question for the boarding process are simulations or analytical models.

4.1 Analytical Models

Analytical models are practically very limited in terms of complexity. At least since the implementation of non-linear processes, analytical models need to be split up into steps or solved by a differential equation. This easily becomes too complex as it would be still worth avoiding a simulation. Only a few boarding policies can be realized by the analytical method, requiring however a strongly simplified system model.

Analytical models provide a set of equations for which closed-form solutions can be obtained. For example:

Analytical queuing models; e.g. an M/M/1 waiting system (single server queue model) with expected value $ET = \frac{1}{\mu(1-\rho)}$ of residence time $T$ for service rate $\mu$ and utilisation $\rho$.

(Page & Kreutzer 2005)

Optimization models for maximizing or minimizing an objective function under constraints

$$g(x) \geq \leq \text{for } i = 1...m$$

Where $x$ here is a vector of variables with $n$ components.

---

1 Derived by Domschke & Drexl (1998), found in Page & Kreutzer (2005)
4.1.1 Simple Approach for Analytical Models of Boarding Process

Stolyarov 2007 designed models which allow expressing boarding times linearly. Every boarding policy requires its own formulation. Their models require four variables:

\[ t_w := \text{the time it takes 1 passenger to walk 1 row} \]
\[ t_B := \text{the time it takes 1 passenger to load a bag and vacate the aisle} \]
\[ n := \text{the number of rows on the airplane} \]
\[ s := \text{number of seats abreast} \]
\[ T := \text{total boarding time} \]

Due to the few variables, the analytical models require some major assumptions that enable their validity. Stolyarov 2007 used the following declarations in order to define the model:

- \( t_w \) is constant for all passengers
- \( t_B \) is constant (bins do not fill up)
- All people enter the plane in a pre-set order
- The seating floor plan has one central aisle with \( k \) seats in each row
- All passengers stow equivalent carry-on bags
- All passengers at a time can stow a bag above a given row
- Only one passenger at a time can stow a bag above a given row
- There cannot be more passengers in the aisle than there are rows

Now, this model can be used to analytically calculate boarding times of various logics. The simplest boarding method to be reflected within the model is the WMA method. As many people here can board the plane as there are rows \( (n) \) at one go, these people need the time:

\[ n \cdot t_w \] \hspace{1cm} (4.1)

to enter the cabin.

The next step is stowing the carry-on luggage. This requires an additional time \( t_B \). Due to assumption at point 3.), the process of stowing can be done by all of them at the same time. This needs to be repeated \( s \) times for each seat abreast: in case of a 6-abreast seat plan, two times for window seats, two times for middle seats and two time for aisle seats. The total boarding time for the WMA method now reads as:

\[ T = s(n t_w + t_B) \] \hspace{1cm} (4.2)

Within the same way of simple analyzing of the boarding methods, other policies can be described within this model as follows:
Back to Front (by row):
\[ T = sn t_w + (sn - n + 1) t_B \]  \hspace{1cm} (4.3)

\[ s^{th} \text{ row parallel boarding:} \]
\[ T = s (nt_w + kt_B) \]  \hspace{1cm} (4.4)

In the \( s^{th} \) row parallel boarding calculation, for simplicity reasons the number of rows is a multiple of \( s \). A brief explanation to this logic reads as follows: the first \( s \) people that enter the plane are all sitting in the back row. The next \( s \) people all sit in the \( s^{th} \) row up from the back, and so on. The logic behind this policy is that the passengers are loading their luggage at the same time with a maximum distance. This is possibly only meaningful in airplanes with relatively short cabins.

### 4.1.2 Lorentzian Geometry

Bachmat 2009 showed that the asymptotic behaviour of the boarding process is captured by a 2-dimensional space-time structure on a domain in the unit square. This is also known as the Lorentzian Geometry. The Lorentzian Geometry was initially designed to model the relativity theory.

Passengers are represented by pairs \((q,r)\) in the unit square \([0,1]^2\). The passengers are sorted into rows by the \(r\)-coordinate. The \(q\)-coordinate determines the location in the queue. They introduce the following constant parameters for this model:

- \( D \) – A delay distribution. The delay values are sampled from this distribution.
- \( h \) – number of passengers per row
- \( l \) – Distance between rows

Furthermore
- \( W \) – A width distribution
- \( F \) – An airline policy represented by the function \( F(r) \) that indicates the first time at which passengers from row \( r \) are allowed to join the boarding queue
- \( \Omega \) - A passenger’s reaction model. This model represents the human’s natural reactions to the boarding policy – reaching from no attention to full attention to the instructions are given to them.

Bachmat 2009 generally associate a Lorentzian metric on a unit square to the airplane boarding process with these parameters.
4.1.3 MINLP

MINLP (Mixed Integer Nonlinear Programming) models are models that combine combinatorial aspects with nonlinearities. MINLP is a mixed integer nonlinearily constrained optimization solver. The MINLP technique uses a branch-and-bound algorithm in which each node corresponds to a continuous nonlinearily constrained optimization problem. This algorithm is a heuristic approach (see heuristic term definition); thus it does not give any guarantee of finding a global solution. But on the other hand, it is nevertheless effective in solving non-convex MINLP problems (Van Den Briel 2005).

The MINLP solver was used by Van Den Briel 2005 in order to find a solution that minimizes the boarding process time problem. He, together with his team, compared different boarding patterns for various numbers of boarding groups.

In practice, they considered a single aisle cabin model with six seats abreast. In this model, they let $N$ represent the set of rows and $M$ the set of the seats $A, B, C, D, E, F$ among each row. $A + F$ are window seats, $B + E$ middle seats and $C + D$ represent aisle seats. The row number is determined to be $i \in M$ and the seat position among the row by $i \in N$.

In so doing Van Den Briel 2005 created different boarding patterns by assigning seats to groups. For the airplane-boarding problem, he assigned each seat $i,j$ to a boarding group $k, k \in G$ with $G$ representing the set of groups. He then defined a decision variable $x_{ijk} = 1$ if seat $i,j$ is assigned to group $k$ and $x_{ijk} = 0$ if not. This was done for all $i \in N$, $i \in M$ and $k \in G$. In so doing, he defined the boarding policy within the mathematical model.

Van Den Briel 2005 furthermore defined a seat-interference penalty factor represented by $\lambda_s$ and aisle interference represented by $\lambda_a$. The penalties associated with the different types of interferences capture their relative contributions to the total delay of the boarding procedure. By this, he will not be able to calculate an actual boarding time rather than evaluate the quality of a specific policy based on occurring aisle and seat interferences.

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1 An integer programming problem is any mathematical optimization or feasibility program in which some or all of the variables are restricted to be integral. In many settings the term integer program is used as short-hand for integer linear programming.

2 In mathematics, nonlinear programming (NLP) is the process of solving a system of equalities and inequalities, collectively termed constraints, over a set of unknown real variables, along with an objective function to be maximized or minimized, where some of the constraints or the objective functions are non-linear.
The MINLP formulation of Van Den Briel 2005 for the boarding process now reads as follows:

Minimize $z = \sum_{i \in N} \sum_{k_1 \in G} x_{I_{ik}} \cdot x_{I_{ibk}} \cdot x_{I_{ck}} + \sum_{i \in N} \sum_{k_1 \in G} x_{I_{ik}} \cdot x_{I_{ibk}} \cdot x_{I_{ck}}$

$+ \sum_{i \in N} \sum_{k_1, k_2 \in G: k_1 < k_2} x_{I_{ik}} \cdot x_{I_{ibk}} \cdot x_{I_{ck}}$

$+ \sum_{i \in N} \sum_{k_1 \in G: k_1 < k_2} x_{I_{ik}} \cdot x_{I_{ibk}} \cdot x_{I_{ck}}$

$+ \sum_{i \in N} \sum_{k_1 \in G: k_1 < k_2} x_{I_{ik}} \cdot x_{I_{ibk}} \cdot x_{I_{ck}}$

$+ \sum_{i \in N} \sum_{k_1 \in G: k_1 < k_2} x_{I_{ik}} \cdot x_{I_{ibk}} \cdot x_{I_{ck}}$

$+ \sum_{i \in N} \sum_{k_1 \in G: k_1 < k_2} x_{I_{ik}} \cdot x_{I_{ibk}} \cdot x_{I_{ck}}$

$+ \sum_{i \in N} \sum_{k_1 \in G: k_1 < k_2} x_{I_{ik}} \cdot x_{I_{ibk}} \cdot x_{I_{ck}}$

$+ \sum_{i \in N} \sum_{k_1 \in G: k_1 < k_2} x_{I_{ik}} \cdot x_{I_{ibk}} \cdot x_{I_{ck}}$

$+ \sum_{i \in N} \sum_{k_1 \in G: k_1 < k_2} x_{I_{ik}} \cdot x_{I_{ibk}} \cdot x_{I_{ck}}$

$+ \sum_{i \in N} \sum_{k_1 \in G: k_1 < k_2} x_{I_{ik}} \cdot x_{I_{ibk}} \cdot x_{I_{ck}}$

$+ \sum_{i \in N} \sum_{k_1 \in G: k_1 < k_2} x_{I_{ik}} \cdot x_{I_{ibk}} \cdot x_{I_{ck}}$

$+ \sum_{i \in N} \sum_{k_1 \in G: k_1 < k_2} x_{I_{ik}} \cdot x_{I_{ibk}} \cdot x_{I_{ck}}$

$+ \sum_{i \in N} \sum_{k_1 \in G: k_1 < k_2} x_{I_{ik}} \cdot x_{I_{ibk}} \cdot x_{I_{ck}}$

$+ \sum_{i \in N} \sum_{k_1 \in G: k_1 < k_2} x_{I_{ik}} \cdot x_{I_{ibk}} \cdot x_{I_{ck}}$

$+ \sum_{i \in N} \sum_{k_1 \in G: k_1 < k_2} x_{I_{ik}} \cdot x_{I_{ibk}} \cdot x_{I_{ck}}$
\[ + \lambda_4^g \sum_{i \in N} \sum_{u \in \text{vel}} \sum_{k < l} x_{iuk} \cdot x_{ivl} + \lambda_5^g \sum_{i \in N} \sum_{u \in \text{vel}} \sum_{k, l \in G; k < l} x_{iuk} \cdot x_{ivl} \]

\[ + \lambda_5^g \sum_{i \in N} \sum_{u \in L \in R} \sum_{k, l \in G; k < l} x_{iuk} \cdot x_{ivl} + \lambda_6^g \sum_{i \in N} \sum_{u \in R \in L} \sum_{k, l \in G; k < l} x_{iuk} \cdot x_{ivl} \]

\[ + \lambda_6^g \sum_{u, b \in N; a < b} \sum_{u \in \text{vel}} \sum_{k, l \in G; k < l} x_{auk} \cdot x_{bvl} \]

Subject to

\[ \sum_{i \in G} x_{ijk} = 1 \]

for all

\[ i \in N, j \in M \]

\[ \sum_{i \in G} \sum_{j \in M} x_{ijk} \geq c_{\min} \]

for all

\[ k \in G \]

\[ \sum_{i \in G} \sum_{j \in M} x_{ijk} \leq c_{\max} \]

for all

\[ k \in G \]
4.2 Simulation

Simulation is the modelling of dynamic processes in real systems based on real data and seeking predictions for a real system’s behaviour by tracing system’s changes of state over time starting from some initial state. Simulations are being used to explore real or imaginary systems. Simulations can explore models with unlimited complexity. When the solution of a problem is too complex in order to be solved by analytical methods, a simulation needs to be taken into consideration. The disadvantage of a simulation is that they can only be computed step-by-step. This does not guarantee complete satisfaction of a solution (Page & Kreutzer 2005). The developer of the simulation needs to decide about the step range - ergo the “accuracy” of the simulation.

In the scope of the boarding process, a computational simulation model can literally track passengers as they walk from the breezeway to their seats. Herewith, boarding methods that require passengers to interact with others can be implemented relatively easily compared to an analytical method. The increments would be the step size of the passengers (how precise they move) as well as the simulation clock (see next chapter ‘Discrete Event Simulation’).

4.3 Applied Simulation Methods for Boarding Process

4.3.1 Discrete Event Simulation

In discrete event simulation, a chronological sequence of events defines the system. Following the example of boarding process, an event could be “passengers belong to zone 3 now board the airplane”. The resulting system state could be for example to randomize the passenger queue within this group and start sending them into the cabin model.

Following Banks 1986, discrete event simulations include the following:

Clock
The simulation must keep track of the current simulation time, in whatever measurement units are suitable for the system being modelled. In discrete-event simulations, as opposed to real time simulations, time ‘hops’ because events are instantaneous – the clock skips to the next event start time as the simulation proceeds.

Events List
The simulation maintains at least one list of simulation events. An event is described by the time at which it occurs and a type, indicating the code that will be used to simulate that event.
Random-Number Generators
The simulation needs to generate random variables of various kinds, depending on the system model. This is accomplished by one or more ‘pseudorandom’ number generators. The use of pseudorandom numbers as opposed to true random numbers is a benefit should a simulation need a rerun with exactly the same behaviour.

Statistics
The simulation typically keeps track of the system's statistics, which quantify the aspects of interest. In the bank example, it is of interest to track the mean service times.

Ending Condition
Because events are bootstrapped, theoretically a discrete-event simulation could run forever. So the simulation designer must decide when the simulation will end.

4.3.2 Simulation Optimization
One way to find the shortest boarding time is to investigate given ideas of boarding policies, either analytically or by a simulation. Another way is to find an optimal boarding order within an optimization algorithm. Simulations cannot be solved for an absolute optimum like an analytical method, although you can approximate to an optimal solution. In order to do so, the simulation must be ran a certain amount of times with different parameters. On every run, the parameters need to be changed in a way that optimizes the result in terms of goal satisfaction. The result is a set of parameters by which an (almost) optimal solution can be achieved. In the scope of boarding method optimization, the most relevant factor is possibly the order in which the passengers enter the plane.

4.4.3 Monte Carlo Method
Using the Monte Carlo-Integration as an example, the area of a given function can be found by “throwing” random points into a square around the function and calculate the relative number of hits. This is also called the Hit-or-miss-method. Another Monte Carlo method is the Monte Carlo Marcov Chain, where a random change on a process or function is made. Depending of its result the change will be kept or replaced with another random change.

Steffen 2008 applied the Monte Carlo Marcov Chain algorithm to his boarding process simulation. With this strategy implemented, all seat assignments are being randomised for the first simulation. The simulation will be started with this seat distribution created and the consumed time is being noticed.
After this first iteration, two random seat assignments are being swapped and the boarding process calculation is being repeated with the remainder of the passengers having the same seats than in the previous process.

If the second configuration loads faster than before, it will be kept and two other seats will be swapped. In case the resulting configuration loads slower than before, the change is being rejected. In so doing, the time required to load the plane is being minimized. After the simulation ran around 10,000 iterations (case of Steffen’s work) the result converged to non remarkable changes for every following change. Steffen 2008 mentions that the ultimate optimal method for this model has a possibility of $10^{-100}$ to be experienced by a modern computer.

### 4.5 Software Tools

There is no commercial or freeware simulation tool for boarding simulation. Solely the developments that were made within various studies produced the particular software tools. These are certainly are not available online or commercially. Marelli 1998 is the only developer of such kind of a tool that is mentioned namely.

#### 4.5.1 PEDS

PEDS (Passenger Enplane/Deplane Simulation) is a computer simulation software tool developed by Marelli 1998. The tool is based on a discrete-event simulation model that is able to simulate the boarding process and record its time consumption. The emphasis of the development was to help airlines to identify the impact of interior configuration changes or alternative boarding procedures. Within the software, costly in-service experiments could be avoided.

Features of the PEDS software tool are:

- Calculating passenger loading and unloading time allowing the airline to conduct turn time trade studies analytically
- Allowing individual factors such as interior configuration, passenger mix, and boarding scenarios to be varied and then estimates the expected time savings
- Evaluating potential changes to interior configurations
- Evaluating the effect of passenger behaviours associated with different travelling populations
• Helping to quantify the effect of passenger behaviour variations that an airline may encounter over time

(Marelli 1998), (McFadden 2008)

PEDS is not available commercially or on the internet.
5 Analytical Approach for Investigation of different Cabin Diameters

5.1 Introduction

In this chapter, an analytical method is being derived and applied to the WMA and Back-to-Front policy in order to show the impact on the number of seats abreast.

The general idea of the WMA method is to eliminate seat interferences. Thus for this method, only aisle interference is notionally significant. But with decreasing number of seats abreast, the other important factor of seat interferences will accordingly decrease (for methods containing seat interference). Hence, one can suppose that for a deceasing number of seats the WMA efficiency gets weaker in comparison to for example the Back-to-Front policy, as its main advantage will be eliminated. As the Back-to-Front method is often considered as the worst and the WMA as the best method, this shall be an attempt to show that under given circumstances the result can possibly be flipped between these two methods.

In the analytical model of Stolyarov 2007, it was assumed so far that all passengers enter the airplane in a pre-set order within a boarding group. This is in fact generally far from reality; but only when considering smaller groups to board at once, this assumption gets less important for the result. In other words: the probability of aisle interference in reality to occur at WMA is unlike greater than at BTF.

5.2 Mathematical Assumptions

When wanting to investigate the impact of aisle interference within the analytical method, a few assumptions need to be made. This considers the probability of aisle interference depending on the size of a boarding group, as well as its impact on boarding time. The probability depends on the number of seat rows that are a possibility for the boarding group to be occupied.

Let \( i \) people belong to a group \( g \) and to a boarding zone \( z \) of \( n \) rows that belong to the zone:

In case of WMA:

\[
g_{wma} = \frac{x}{s} \quad (5.1)
\]

or

\[
g_{wma} = n \quad (5.2)
\]
In case of BTF:

\[ g_{btf} = \frac{x}{z} \]  \hspace{1cm} (5.3)

Let \( P_i \) be the probability of a passenger to experience aisle interference; where \( i \) is the position within the group:

\[ i \in g \in \mathbb{N} \]

and

\[ 1 < i < g \]

The further behind a group, the greater the probability of experiencing aisle interference. For the leader of the group, the probability is 0. Where \( k \) is the number of rows where seats will be getting occupied, for any follower \( i \) in this group, the probability reads as:

\[ P_i = 1 - \frac{k-1}{ki} \]  \hspace{1cm} (5.4)*1

When plotting \( P \) over \( i \), it can be seen that the probability shows up an asymptotic behaviour. The further behind in the group, the probability to be blocked approaches 100%.

\[ \text{Figure 5.1} \quad P \text{ over } i \text{ with the example of } k = 30 \]

*1 This formula was derived using generic analytical stochastical approaches. A simple stochastical simulation can be used in order to evaluate this formula. The result of the simulation revealed values for \( P_i \) that are close to the analytical approach (see Appendix B for simulation extract and results).
With a given number of rows \( n \), \( k \) stays constant over \( s \) for both WMA and BTF:

\[
k_{wma} = \frac{x}{s} \tag{5.5}
\]

\[
btf = \frac{x}{sz} \tag{5.6}
\]

When plugging (5.5) respectively (5.6) in (5.4), \( P_i \) now reads as:

\[
P_{i,wma} = 1 - \frac{u - 1}{u}
\]

and

\[
P_{i,btf} = 1 - \frac{u - 1}{us}
\]

Since the interference time will not only affect the passenger directly behind, but also the whole queue, the total delay time \( D \) for a group \( g \) can now be found by numerical integration:

\[
D_g = p \sum_{l=1}^{g} P_l \tag{5.7}
\]

where \( p \) is a single penalty time that it takes a passenger to load his/her luggage. The total extra delay time \( D_{total} \) for the particular policy is the number of groups that is necessary to fill the cabin multiplied by \( D_g \). The number of groups is \( \frac{u}{g} \). \( D_{total} \) then reads as:

\[
D_{total,wma} = D_{g,wma} \cdot \frac{x}{g_{wma}} = D_{g,wma} \cdot \frac{x}{n} = D_{g,wma} \cdot s \tag{5.8}
\]

and

\[
D_{total,btf} = D_{g,btf} \cdot \frac{x}{g_{btf}} = D_{g,btf} \cdot \frac{xz}{x} = D_{g,btf} \cdot z \tag{5.9}
\]

### 5.3 Sample Study

A defined seat layout with \( n = 30 \) rows will now be taken as a reference where the calculation of \( D_{g,wma} \) and \( D_{g,btf} \) will be repeated for \( 2 < s < 6 \in \mathbb{N} \). The number of zones will be determined to be \( z = 5 \).
The resulting numbers of seats for this layout is $x = (60, 90, 120, 150, 180)$. This represents the same fuselage length over a changing number of seats abreast (see Figure 5.2):

![Table showing sample seat layouts](image)

**Figure 5.2** Sample seat layouts
5.4 Results

The results for the sample layouts in chapter 5.2 are listed in the following table:

<table>
<thead>
<tr>
<th>Seats abreast</th>
<th>Pax</th>
<th>Penalty [s]</th>
<th>( P_g ) WMA [s]</th>
<th># of groups</th>
<th>( P_{total} ) WMA [s]</th>
<th># of groups</th>
<th>( P_g ) BTF [s]</th>
<th># of groups</th>
<th>( P_{total} ) BTF [s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>60</td>
<td>5</td>
<td>130,5</td>
<td>2</td>
<td>261,0</td>
<td>4</td>
<td>46,2</td>
<td>5</td>
<td>231,2</td>
</tr>
<tr>
<td>3</td>
<td>90</td>
<td>5</td>
<td>130,5</td>
<td>3</td>
<td>391,6</td>
<td>7</td>
<td>74,6</td>
<td>5</td>
<td>373</td>
</tr>
<tr>
<td>4</td>
<td>120</td>
<td>5</td>
<td>130,5</td>
<td>4</td>
<td>522,1</td>
<td>10</td>
<td>103,4</td>
<td>5</td>
<td>517,15</td>
</tr>
<tr>
<td>5</td>
<td>150</td>
<td>5</td>
<td>130,5</td>
<td>5</td>
<td>652,6</td>
<td>13</td>
<td>132,5</td>
<td>5</td>
<td>662,6</td>
</tr>
<tr>
<td>6</td>
<td>180</td>
<td>5</td>
<td>130,5</td>
<td>6</td>
<td>783,1</td>
<td>15</td>
<td>161,8</td>
<td>5</td>
<td>808,85</td>
</tr>
</tbody>
</table>

With this method, now the extra penalty when considering aisle interference for each method has been discovered. The analytical methods of Stolyarov 2007 can now be used to calculate the walking times. The two results could be super positioned afterwards. But since the delta total boarding times between the two methods by Stolyarov 2007 do not change over \( s \), it is sufficient for the purpose of comparison to only consider the interference times. Plotting the total boarding times over \( s \) now reveals the following figure:

![Figure 5.3](image)

**Figure 5.3** \( P_{total} \) of WMA and BTF over \( s \)

As it can be seen from Table 5.1 and Figure 5.3, the Back-to-Front method features less aisle interference on layouts with \( s < 5 \). Certainly, all assumptions explained above need to be respected here when considering the results. For example, seat interference is not respected. The assumption of leaving seat interference out could comprise that people can reach their window/middle seats without the interfering person getting up.
6 Alternative Methods for Boarding Process Improvement

The strategy to let all passengers enter the airplane in a specific order is possibly the most influential factor on boarding time. However, there are also other methods that are potentially able to influence it. While it would be a questionable method to cheer people up while they are enplaning, they rather need to be pushed by a natural habit: the wider a passage, the greater the flow can be. The two main bottlenecks after the breezeway are the aircraft door and the aisle, respectively the space between two seats (depending on the seat pitch). By changing the dimensions between these objects, a higher passenger flow rate could possibly be achieved.

6.1 Innovative Seat Configurations

The German company ‘AIDA’ has designed a foldable passenger seat, that significantly increases the seat pitch while no passenger is located on the seat. The principle how it works is just like ‘theatre-style’ seats. AIDA 2009 states that the flip-up seats could decrease aircraft boarding and turnaround times.

This type of seat provides a new situation of boarding. Passengers could move directly into their assigned seat row to clear the aisle and stow their hand luggage. The aisle interference would be reduced significantly.

---

1 In fact, in this case should be the talk of a “virtual pitch”
Additionally, the seat can reduce cases of deep-vein thrombosis on long-haul flights, as the seat provides nearly three times the room for a passenger to stand. (AIDA 2009)

6.2 Cabin Layout

The optimisation of greater armrest distance (increasing comfort) and a wider aisle (increasing passenger flow rate) is a never-ending one, just as the optimisation between seat pitch and number of seat rows (question of comfort versus revenue). As passengers seek for comfort, increasing the aisle width needs to be handled carefully. Therefore, a concept with a funnel-like ending of the aisle could be a useful cabin layout change. This means, that only the last or two last rows need to be changed in order to provide a physically boarding friendlier cabin.

Increased seat pitch generally would help passengers to load their hand luggage while they stand in the seat row. By this way, they can clear the aisle more early and reduce aisle interferences. There is no question that airlines would use this option very carefully, as a greater seat pitch decreases the number of possible seat rows which results in lower revenue. If the boarding time could be reduced significantly enough by this in order to justify one less seat row is rather questionable.
6.3 Overhead Bin Improvement

The overhead stowage compartments are generally already opened by the cabin crew before the boarding process begins. This because it would take extra time if the passengers would need to open them, respectively checking if it still free or another passenger has already put his carry-on luggage in it.

A way to improve boarding time within the overhead bins could be to increase their size. The result would possibly less required time to stow the bags into the bins and therefore less aisle interference time. But on the other hand, the space in the cabin must be used carefully. Therefore every gained space must be exploited. Larger overhead bins (given that their installation is possible on the certain aircraft type) would be used the same way as the smaller ones and consume even more time. The rather significant point where the stowing time could possibly be improved is the limitation of carry-on luggage by the airline. Passengers tend to put luggage from the cargo bags in their carry-on bags, since airlines limit the weight of costless-baggage.

“With all major airlines, except Southwest, charging for checked bags, passengers are taking everything as a carry-on except the kitchen sink. As the overhead bins are stuffed to capacity, the cargo hold packs lighter and lighter...

...Things have gotten so bad with excessive carry-ons, Congress is threatening to regulate the size and number of carry-ons stopping passengers at the security checkpoints.”

(Travel Sentry 2009)
7 Results

The results that are presented in this chapter are entirely based on the results that have been explored by other studies. These results will be discussed and put in contrast to each other in order to identify differences and similarities as well as the best boarding policy.

7.1 General Findings

The element on a boarding process that increases the boarding time most significantly is the time that a passenger needs to stow the hand luggage (Steffen 2008), (Landeghem 2000). Most passengers have at least one piece of hand luggage. This explanation can literally be found in all research documents. Based on this insight, the computed results according to different policies can be explained rationally. Some authors even treated the seat interference as negligible (Steffen 2008).

In Van Den Briel 2005, the quality of the boarding methods is solely indicated by the sum of aisle and seat interferences occurring on a specific policy. When comparing boarding time and the number of total interferences, this assumption is proved. As explained in the mathematical model that he uses, the minimization of interferences reduces the boarding time. In order to show the relation between the number of interferences and boarding time, a graphical method can be used. The values are simulation results by Van Den Briel 2005.

![Figure 7.1](image)

Figure 7.1 Boarding time – interferences scatter
By this graph (Figure 7.1) can clearly be seen the relation of amount of interferences and boarding time. There are roughly two sets: few and many interferences. The set of little interference belongs to the class of Outside-In, respectively combined with Back-to-Front. The set of the higher values are all Back-to-Front policies with different zone splits. Interestingly, the boarding time increases over number of zones. This can be explained by the insight that random boarding is more efficient than back-to-front: the fewer zones, the more the boarding process is randomized.

7.2 Best Strategy

There are two main different classes of results for an optimal boarding strategy. It can be either by seat, or by seat group. By seat group is more practically. By seat-strategies provide more flexibility and optimized results.

“The best result is reached by the Class ‘By Seat’”

(Landegehem 2000)

But on the other hand it is less feasible in reality (see problem of implementation in reality chapter 3.7).

In fact, a ‘by group result’ compromises a ‘by seat result’, but only by additionally respecting that there is no big difference in which order the boarding takes place within the group. In other words: the order within the group is always random. Now, the results that were discovered in the different studies are either by-seat or by-group.

By-group results follow strategies like WMA, Rotating Zones, etc. While by-seat results can possibly be matched with a named strategy.

The following Table 7.1 summarizes the best practical results that were found in research studies:
Table 7.1  Results of best strategy

<table>
<thead>
<tr>
<th>Author</th>
<th>Best strategy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Landeghem 2000</td>
<td>WMA</td>
</tr>
<tr>
<td>Van den Briel 2005</td>
<td>WMA/Reverse pyramid</td>
</tr>
<tr>
<td>Ferrari 2005</td>
<td>WMA</td>
</tr>
<tr>
<td>Marelli 1998</td>
<td>WMA</td>
</tr>
<tr>
<td>Steffen 2008</td>
<td>WMA²</td>
</tr>
<tr>
<td>Stolyarov 2007</td>
<td>WMA</td>
</tr>
<tr>
<td>Bachmat 2009</td>
<td>WMA</td>
</tr>
</tbody>
</table>

It can clearly be seen that for the class of group boarding, the WMA method is the most efficient one.

The results of Steffen 2008 need to be explained more detailed, since it doesn’t directly follow one of the namely mentioned policies:

The optimal way to board an airplane is to have adjacent passengers in line separated by two rows. For example: if you send 12 people into an aircraft, they would all be spread apart by two rows so that they can all put the luggage away and sit down at the same time. This matches the assumption with other study results, that the aisle-interference needs to be avoided.

(Steffen 2008)

7.3 Worst Strategy

It is obvious to understand that a Front-to-Back system and possibly also “Aisle-Middle-Window” strategy would slow the whole process most significantly down while people keep blocking each when while loading their luggage or sitting down. There has been done no investigation considering the question of “what is the worst strategy”. Solely Steffen 2008 mentions that the Back-to-Front method is “the second worst”. Interestingly, this strategy is known as to be the traditional one and most commonly used by many airlines. Against expectations, random boarding, which one would associate with chaos, is significantly faster than Back-to-Front (Steffen 2008).

---

¹ For the findings of Ferrari 2005 as well as Marelli 1998, the Journal of Air Transport Management was used as source (McFadden 2008 study)

² Steffen 2008 furthermore says that it is most efficient to have the window seats being boarded first, although this is less important having multiple people putting their luggage away at the same time.
A similar statement can be found in Landeghem 2000 saying:

(...) in taking a structured approach to boarding, one should beware of making things far worse by choosing a wrong way of sequencing. A “wrong” block method can result in times up to 40 minutes!

### 7.4 Influence of Airplane Size

The more seats an aircraft layout has, the longer the boarding will obviously take. The rather interesting question is, if there is a boarding policy that outperforms another one when increasing the number of seats (respectively having two aisles instead of one). The results of Stolyarov 2007 provide the following answer to this:

For very small planes (< 75 pax), all algorithms performed within 1.2 minutes of each other. The relative efficiency of the algorithms remains all sizes of planes and all types of seating layouts

One can conclude from this, that on a layout of an airplane with less than 75 seats, it is not worth applying a certain boarding policy. For airplanes with more than 75 seats it useful to apply an efficient boarding policy.

### 7.5 Financial Impact

The essential question for airline in terms of boarding time improvement is how they can potentially benefit from it. As described in chapter 2.3, the less time an active aircraft is on the ground, the less money will be needed be accrued for it.

When considering an average boarding time, the cost savings for an airline over a year can be approximated. By this, different policies can be compared in terms of their rentability\(^1\).

When considering results of Landeghem 2000, an average boarding time of 30,33 min can be found for traditional methods (by block, by half block,). Following study results of Marelli 1998, Ferrari 2005, Van Den Briel 2005 and Landeghem 2000, the same can be done for all major non-traditional methods (WMA, Reverse Pyramid), where an average boarding time of 19,78 min can be found. When additionally considering the case of passengers being allowed to take only one, respectively no hand luggage at all with them, a particular average time sav-

---

1 Extra costs for resulting costs of new policy implementation not considered
ing of 4.6 min (one carry-on bag allowed) and 11.6 min (no hand luggage allowed) can be achieved over the assumption that each passenger has two carry-on bags.²

Following McFadden 2008, the annual ground costs for an airline can be approximated by the following formula:

\[
C = B \cdot M \cdot D \cdot 365 \text{ days} \tag{7.1}
\]

Where

\( C \)...annual cost
\( B \)...average boarding time
\( M \)...Cost per 1[min] on ground
\( D \)...average number of daily flights

It is being assumed that the average number of flights over the year is approximately 1500. Following chapter 2.3, the cost per minute on ground can be approximated to be $30. The cost savings for an airline applying an innovative boarding policy can be now read out of Table 7.2.

(McFadden)

<table>
<thead>
<tr>
<th>Boarding method</th>
<th>Average boarding time ( B ) [min]</th>
<th>Annual cost ( C ) [$]</th>
<th>Cost savings over traditional method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Traditional</td>
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² Though in reality, most passengers have at least one piece of hand luggage (Steffen 2008)
¹ The two-door boarding times were derived from Marelli 1998; simulation results that found boarding through two doors saved 5 min (McFadden 2008)
7.6 Application of Results

A model must offer sufficiently close and valid representations of a real system under investigation, at least with regard to the goals of study in question. Although models are necessarily always abstractions, a close correspondence to all relevant aspects of their target systems (which may or may not yet exist) is a crucial prerequisite for their usefulness. Otherwise, as it has sometimes been said, – modelling may just become science fiction. From daily experience we are used to the fact that computer systems and their software contain errors, whose consequences may reach from mere nuisance (e.g. figures who magically disappear in a document) to utter disaster. Computer models and the simulation programs by which they are implemented are unfortunately just as error-prone as any other complex piece of software. (Page & Kreutzer 2005)

The verification in this case means the implementation and observation in reality. On figure 7.2, this is be represented by “conclusion”.

![Diagram](image)

**Figure 7.2** System, model, and application (figure principal overtaken from Page & Kreutzer 2005)

7.6.1 America West

As Van-Den-Briel’s work was done in cooperation with America West Airlines, he provides some data of the impact on turnaround times of this airline before and after the implementation of the reverse-pyramid method.

The following figure shows the total departure delays by America West Airlines (hours per month): 2003 after, and 2002 before the implementation of the new boarding policy.
7.6.2 Lufthansa

Considering the results of *Steffen 2008*, the German airline Lufthansa mentioned general interest but was on the other hand sceptically. They said that the boarding process, no matter how many variables are being used, is only hard to model by a computer simulation since it is dealing with human beings. The theoretical results like the ones from *Steffen 2008* therefore would need to be handled carefully.

In the year 2005, Lufthansa did a large field experiment in order to test various boarding strategies. They considered three different variants on 450 flights with about 85,000 passengers totally:

- Normal random boarding
- Window Middle Aisle
- Back-To-Front

Finally, they kept their traditional method of the Back-to-Front strategy.

*(Spiegel 2008)*
8 Conclusion

The problem of finding an optimal policy how to board an airplane requires understanding of the process in order to derive mathematical models that can help again to do further investigations. Independent research works delivered similar results considering basic characteristics. The most efficient strategy has been figured out to be WMA of all major research studies. This shows that roughly a strategy from outside (windows seats) to the aisle seats should be preferred in order to board the airplane most sufficiently. The strategy is still the best when considering larger cabins. At smaller cabins, the absolute delta times between different strategies tend to zero - therefore it can be concluded that it is not worth implementing a strategy here. For cabins with less than 5 seats abreast, the Back-to-Front method could be preferred.

But still, when applying a strategy where good time saving qualities can be expected, the implementation needs to be handled carefully as extra costs for e.g. call up systems or staff training, are likely to be one of the major drawbacks next to annoyed passengers who are not willing to attend somebody’s orders. The strategy must be chosen on the one hand to be efficient and on the other hand easy to be applied and to be accepted by airline passengers. When considering these points, significant cost saving can be achieved.
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(Niemeyer 1977)

Sanchez 2009


Spiegel 2009

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Steffen 2008


Stolyarov 2007


Van Den Briel 2005


VDI 1997


YourDictionary 2009

LOVETOKNOW CORP, Online dictionary 2009, - URL: http://www.yourdictionary.com
# Appendix A

## Boarding Policy Illustrations

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**Aisle**

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## Appendix B

### Simulation Extract / Results

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### Results

![Graph showing the comparison between simulation and analytical results](attachment:image.png)

- Simulation
- Analytical