Selected statistics in aircraft design

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Abstract

Trends and relationships have been extracted from aircraft design literature in order to comment on 10 relationships between aircraft parameters. Statistical research has been undertaken.

A data base was created from 53 passengers aircraft. Thanks to selected data and literature excerpts, trends have been discussed and relationships have been statistically analyzed using regression tools. Each time a regression curve was acceptable, an equation was written and interpreted. For one equation corresponding to literature, a coefficient could be optimized. For newly found equations, interpretation was only done after validation of the underlying physical reality. Sometimes, equations were mathematically feasible but had no physical meaning. These equations had to be discarded. In two cases, no relationship or trend was revealed by analysis. Many factors can be responsible for the failure of finding a correlation. A regression may simply not exist at all.

In a nutshell, this project confirms relationships among aircraft parameters from literature and tries to optimize related equations. The project created new equations which were up to now not available in literature or questions the validity of equations from literature.
Selected Statistics in Aircraft Design

Task for a Projekt 2

Background
In aircraft design, the configuration (or: three-view drawing) of an aircraft evolves out of the requirements assigned to an aircraft. In contrast to other subjects in aeronautical engineering that deal with analysis, aircraft design deals with synthesis. This very nature of aircraft design makes the subject rely also heavily on statistics and past knowledge. The interest of this project is in passenger aircraft.

Task
In order to support activities in preliminary design, selected parameters have to be put in relation to each other. General trends should be made visible. This can be achieved most easily by drawing diagrams, correlating data and finally by trying to describe the correlations by equations. Examples of relationships that could be investigated are given below. Further interesting correlations should be discovered.

- Number of passengers and number of seats abreast.
- Length of the cockpit, fuselage tail section and length of cabin in the fuselage tail section.
- Fuselage length and length of the cargo compartments.
- Geometric relationships for engine integration.
- Sweep of the wing, cruise Mach number and entry into service.
- Sweep of the wing, dihedral and position of the wing.
- Length of the winglet, wing span and entry into service.
- Sweep of the wing, sweep of the horizontal tail and sweep of the vertical tail.
- Aspect ratio of the wing, aspect ratio of the horizontal tail and of the vertical tail.
- Taper ratio of the wing, taper ratio of the horizontal tail and taper ratio of the vertical tail.

The report has to be written according to German DIN standards on report writing!
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\begin{itemize}
  \item \textit{AR} Aspect Ratio
  \item \textit{b} Wing span
  \item \textit{S} Wing area
  \item \textit{C_t} Chord at the tip of the wing
  \item \textit{C_r} Root chord
  \item \textit{M} Mach number
  \item \textit{V} True speed
  \item \textit{a} Speed of sound
  \item \textit{M_{Crit}} Critical Mach number
  \item \textit{M_{Div}} Drag divergence Mach number
  \item \textit{N_{SA}} Number of seats abreast
  \item \textit{N_{Pax}} Total number of seats
  \item \textit{L_{Heck}} Length of the fuselage tail section
  \item \textit{L_{Kab}} Length of cabin in the fuselage tail section
  \item \textit{L_{Equipment}} Length of equipment compartments in the fuselage tail section
  \item \textit{M_{eff}} Effective Mach number
  \item \textit{L_{Comp}} Length of the cargo compartments
  \item \textit{L_{Fus}} Length of the fuselage
  \item \textit{H} Gulley between engine and wing
  \item \textit{X_f} Penetration of engine compared to wing
  \item \textit{C} Chord of the wing
  \item \textit{M_{Cr}} Cruise Mach number
  \item \textit{A_{HorizTail}} Aspect ratio of the horizontal tail
  \item \textit{A_{Wing}} Aspect ratio of the wing
\end{itemize}

Greek symbols

\begin{itemize}
  \item \textit{\lambda} Taper ratio
  \item \textit{\Lambda_{25}} ¼ Chord sweep
  \item \textit{\Gamma} Dihedral of the wing
  \item \textit{\Lambda_{HorizTail}} ¼ Chord sweep of the horizontal tail
  \item \textit{\Lambda_{Wing}} ¼ Chord sweep of the wing
\end{itemize}
Acronyms

CFD  Computational Fluid Dynamics
List of Key Words and Definitions

Aspect ratio
The aspect ratio is defined as the square of the wing span, $b$, divided by the wing area, $S$.

$$AR = \frac{b^2}{S}$$

AR selection is basically a compromise between aerodynamic efficiency and wing structural weight.

Taper ratio
The taper ratio is defined as the ratio of the chord at the tip of the wing, $C_t$, to the chord at the airplane centerline, called the root chord, $C_r$:

$$\lambda = \frac{C_t}{C_r}$$

$\lambda$ is also a compromise between aerodynamic considerations (primarily span low distribution is important for cruise efficiency and stall characteristics) and structural considerations.

Sweep
The angle in plan view between a specified spanwise line along an aerodynamic surface and the normal to the plane of symmetry. For an aerodynamic surface as a whole, the quarter-chord line is preferred, but any other specified line, such as the leading edge or trailing edge, may be taken for a particular purpose. (AGARD 1980)

Figure 1.1 shows this sweep angle.
**Dihedral**
Dihedral is the wing upward angle from the vertical when seen from the front, or nose of the aircraft.

**Mach number**
The Mach number is the ratio of the speed of an object or flow relative to the speed of sound in the medium through which it is traveling.

\[ M = \frac{V}{a} \]

The cruise Mach number is the Mach number which allows achieving the lowest possible rate of fuel of usage in a prescribed condition.

**Critical Mach Number**
\( M_{\text{crit}} \) is the flight Mach number (<1) at which supersonic flow first appears over some part of the wing.

**Drag Divergence Mach Number**
The Mach number for Drag Divergence, where the wave drag becomes significant, is more important: this is a little higher than \( M_{\text{crit}} \).
1 Introduction

1.1 Motivation

The civil transport aircraft market has become highly competitive. Many factors such as aircraft performance determine the overall economic appeal of the aircraft to the market. A few of these factors are influenced by design.

Aircraft design requires a process often extremely complex with a very large number of parameters. So design methods use a combination of approximation, experience and statistical information on similar aircraft to reduce the number of variables to a manageable number.

The project’s aim is to use these methods in order to confirm already existing relationships in literature and to optimize them, or to identify new relationships. Sometimes, equations can be created to describe relationships. They will have to be compared to the existing equation or to be interpreted and discussed in order to accept or not the model.

This project is based on data gathered from 53 passenger aircraft (see Appendix A). Data have to be selected with great care before exploitation.

All new trends and equations have one purpose: improve knowledge in aircraft design and improve the design process.

1.2 Definitions

Statistics

Statistic is an activity which consists of gathering, dealing with and interpreting data set. Data processing relates to descriptive statistic. Data interpretation is called inferential statistic and leans on poll theory and mathematical statistic.

Sometimes, statistics also mean reaped data set. The statistic’s aim is to give prominence to proprieties of known variables set only through few of its realizations.

Aircraft design

A good explanation is given in Raymer 1992:

Aircraft design is a separate discipline of aeronautical engineering, different from the analytical disciplines such as aerodynamics, structures, controls, and propulsion. An aircraft designer needs to be well versed in these and many other specialties, but will actually spent little time performing
such analysis in all but the smallest companies. Instead, the designer’s time is spent doing something called “design”, creating the geometric description of a thing to be built. Design is not just the actual layout, but also the analytical processes used to determine what should be designed and how the design should be modified to better meet the requirement.

Aircraft design comprises three major phases:

- Conceptual design, which answers to the basic question of configuration arrangement, size and weight, and performance.
- Preliminary design, which freezes the configuration, allows serious structural and control system analysis and design, performs wind tunnel tests and mathematically models the precise shape of the outside skin of the airplane.
- Detail design, which narrows the design of each aircraft’s piece and determines production design.

1.3 Objectives

The purpose of this project is to determine relationships or trends between different parameters sampled beforehand. This project’s task has many subtasks:

- Theoretical approach of relationships between parameters
- Research and acquisition of data
- Exploitation of data, creating regressions
- Discussion of results compared to theoretical relationships

1.4 Literature review

Most of the relationships can be found or deducted from literature. These equations and/or forecasting of parameters’ evolution are detailed in Chapter 2.

A lot of books deal with aircraft design. Howe 2000, Torenbeek 1982, Jenkinson 1999, Schaufele 2000 and Raymer 1992 are perfectly convenient to process the issues of the project. Dealing only with civil jet aircraft design, Jenkinson 1999 covers almost all issues to study in a synthetic way. Moreover, it contains the closest approach to the project task since it is based on statistical considerations and data regression. The most complete aircraft design book is Toreenbeek 1982. But its approach is founded on aerodynamic theory.
The text of Scholz 1999 has been useful to find several equations and figures. All aircraft design topics are developed in this document and equations can be found in.

About data roots, they come from Jane's All the World’s Aircraft Collection which contains a huge amount of all-time aircraft data.

1.5 Structure arranged of work

Chapter 2 describes equations and relationships between parameters found in literature and give a preliminary trend idea in order to base statistics’ exploitation on. This Chapter only deals with the effects of some parameters on the others.

Chapter 3 analyses selected data thanks to correlations and statistic calculations in Excel. This Chapter foregrounds relationships created by these analyses and compares to the content of the Chapter 2. Furthermore, optimized methods are used and explain so as to obtain best results.
2 Relationship of parameters as found in the literature

2.1 Number of passengers and number of seats abreast

Fuselage can be sized thanks to many methods. According to Raymer 1992, number of passengers and number of seats across determined the fuselage size:

For example, a large passenger aircraft devotes most of its length to the passenger compartment. Once the number of passengers is known and the number of seats across is selected, the fuselage length and diameter is essentially determined.

More precisely, the number of seats across fixes the number of rows and aisles in the cabin which affects respectively the length and the diameter of the cabin. Figure 2.1 shows the evolution of the total number of passengers as function of the number of seats across.

![Figure 2.1 Total number of seats related to the number of seats across the economy cabin section Jenkinson 1999](Image)

A relationship between the two parameters, as illustrated in figure 2.2, has been found in Scholz 1999, p 87:

\[ N_{SA} = 0.45 \sqrt{N_{Pax}} \]  

(2.1)
2.2 Length of the cockpit, fuselage tail section and length of cabin in the fuselage tail section

- Length of the cockpit

The length of the cockpit is determined by many requirements. First, cockpit design depends on crew accommodation and range of pilot size. Then other requirements such as instrumentation panel or flight controls have to be considered (see figure 2.3).
So the cockpit length is not dependent on fuselage length. According to Torenbeek 1982:

*The flight deck accommodation in general aviation is generally limited, so that the length of the flight deck is not more than 3 ft (1.5m) for small touring aircraft, and up to about 6 ft (1.8m) for business aircraft.*

Moreover, adapted from Schmitt 98, the cockpit length should be an average of 4 m, what is shown in figure 2.4.

![Figure 2.4](image)

**Figure 2.4** Length of cockpit, nose section, cabin in the tail section and tail section functions of fuselage diameter (Schmitt 98)

- **Fuselage tail section and cabin in tail section**

In figure 2.4, the fuselage tail section is composed of two parts: the pressurized part which is the cabin in the tail section and the unpressurized part which contains additional equipment like APU.

An equation can be created from the figure 2.4:

\[
L_{Heck} = L_{Kab} + L_{Equipment}
\]  

(2.2)

If this equation is generalized to other tails, it becomes:

\[
L_{Heck} = \alpha L_{Kab} + L_{Equipment}
\]  

(2.3)
2.3  Fuselage length and length of the cargo compartments

According to Torenbeek 1982, the underfloor cargo compartment both ahead and behind the wing, shown in figure 2.5, might be an advantage to control the center of gravity travel. Other configurations are possible for small twin-engined aircraft: in the fuselage nose ahead of the cockpit or in the engine nacelles.

Space dedicated to cargo compartments is limited by the location of electric and electronic bays, air conditioning, hydraulic bay and landing gear.

![Figure 2.5 A320, lower deck compartments (Airbus SAS 2003)](image)

2.4  Geometric relationships for engine integration

The engine integration has a significant impact on the aircraft, affecting safety, structural weight, flutter, drag, control, maximum lift, propulsive efficiency, maintainability, and aircraft growth potential.

Engines can be mounted on different wing positions, as shown in figure 2.6. Schaufele 2000 describes these positions functions of aircraft classes:

> Engine location for single engine personal/utility aircraft is usually at the forward end of the fuselage, while light twin engine aircraft engines are usually located on the inboard portion of the wing. Commuters and regional turboprops also favor inboard wing location. Business jets universally locate engines on the aft fuselage, along with the “T-tail” empennage arrangement. Jet transports mostly utilize engines placed under the wing with the inlet located forward of the wing leading edge. Some transport designs have used aft fuselage mounted engines in nacelles or, on some 3-engine designs, located in or above the extreme aft end of the fuselage.
In the instance of jet aircraft with wing-mounted podded engines as shown in figure 2.7, engine location is influenced by many considerations including the interference between the nacelle and the wing which increases drag. Consequently, nacelles must be sufficiently forward and low to avoid drag increases.

However, to minimize the weight of the landing gear and engine pylon, a general rule is drawn:

*The nacelles are usually located as close to the wing lower surface as possible, without causing undue heating of the wing by the engine exhaust.* (Schaulefe 2000, p. 193)

Two different tools exist for engine integration design analysis: CFD method and wind tunnel. Nevertheless, CFD does not replace the wind tunnel but can only improve results.

Figure 2.8 provides data of engine locations optimized by CFD method or not. Obviously, CFD method allows installing nacelles closer to the wing. The white area corresponds to the theoretical non-acceptable resistance area.
Figure 2.8  Engine locations with CFD Method (Airbus 1991)

Figure 2.9 delineates difficulty areas of engine location. The closer to the wing the nacelle is, the harder the engine integration becomes.

Figure 2.9  Difficulty areas of engine location (Kroo 2005)

Ground clearance is another parameter restricting space between nacelle and wing, as illustrated in Figure 2.10.

Figure 2.10  Ground clearance considerations for under-wing installation (Jenkinson 1999)
2.5 Sweep of the wing, cruise Mach number and entry into service

Raymer 1992, p. 52, gives an explanation of sweep effects on Mach number:

\[ \text{Wing sweep is used primarily to reduce the adverse effects of transonic and supersonic flow. Theoretically, shock formation on a swept wing is determined not by the actual velocity of the air passing over the wing, but rather by the air velocity in a direction perpendicular to the leading edge of the wing. This result, first applied by the Germans during World War II, allows an increase in Critical Mach Number by the use of sweep.} \]

Trends variation of wing sweep functions of Mach Number are described in Torenbeek 1982, p. 251:

For cruising speeds of up to about \( M = 0.65 \) to 0.70 compressibility effects can be catered for straight wing of acceptable thickness ratio. Increasing the Mach number makes it desirable to use sweepback (or sweep forward) in order to avoid severe compressibility problems in a dive. At cruising speeds of \( M = 0.75 \) to 0.80 a straight wing may only be acceptable if it is very thin requires a low aspect ratio (e.g. Learjet 24). The angle of sweepback increases progressively for cruising speeds in excess of \( M = 0.80 \).

The choice of the sweep angle is related to the section thickness/chord ratio. For the same thickness/chord ratio, sweep varies functions of drag divergence Mach number as follows:

\[ M_{\text{eff}} = M.(\cos \Lambda_{25})^x \]

\( x = 0.5 \) according to Torenbeek 1988
\( x = 0.75 \) according to Staufenbil 1992
\( x = 1 \) according to Jenkinson 1999

Nevertheless, a mishandled sweep can have drawbacks:

As a general rule sweep angles should be as low as possible for a given design flight condition and aerofoil configuration, since sweep implies both structural and possible handling penalties. (Howe 2000)
2.6 Sweep of the wing, dihedral and position of the wing

Three positions of the wing are possible:

- Low-wing
- Mid-wing
- High-wing

However, aircraft are most often designed with high or low wings. Reasons to choose the wing position are explained by Raymer 1992, p 96:

The choice of a vertical wing location relative to the fuselage is a compromise between aerodynamics, structural and operational considerations. In some cases there may be overriding issues such as propeller ground clearance on a multi-engined type or powerplant removal on a V/STOL combat aircraft, both of which may well determine the use of a high wing.

Blatantly, dihedral angle of high-wing aircraft absolutely differs from the one of a low-wing aircraft. First and foremost, these two positions and the sweep angle create different inherent dihedral effects:

High wing airplanes have inherent dihedral effect due to the wing position while low wing airplanes tend to be deficient in inherent dihedral effect. For this reason low wing airplanes tend to have considerably greater geometric dihedral than high wing airplanes.
Swept wing airplanes tend to have too much dihedral effect due to sweep. This can be offset in high wing airplanes by giving the wing negative dihedral (anhedral). (Roskam III, p194)

In order to have numerical references, Howe 2000, p; 130-131, gives intervals of dihedral or anhedral angle separated by wing category:

From the point of view of stability the following may be used as an initial guide to the desirable dihedral/anhedral:

<table>
<thead>
<tr>
<th>Wing Type</th>
<th>Dihedral/Anhedral</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low wing, unswept</td>
<td>dihedral 3° to 5°</td>
</tr>
<tr>
<td>Low wing, swept back</td>
<td>dihedral about 3° at 30° aft sweep</td>
</tr>
<tr>
<td>High wing, unswept</td>
<td>no dihedral</td>
</tr>
<tr>
<td>High wing, swept back</td>
<td>anhedral of about minus 3° at 30° aft sweep (increasing somewhat at higher aft sweep)</td>
</tr>
</tbody>
</table>
2.7 Length of the winglet, wing span and entry into service

A good description of winglets effects and usefulness is given by Howe 2000, p. 39:

Wing tip end plates or fins may be used to reduce drag during cruise by effectively increasing the wing span. There may also be some contribution to directional stability if the wing is swept back. It is usual to keep tip fins relatively small to avoid severe structural penalties, and their primary use is when an improvement is required to an existing design or when wing span is limited by operational considerations.

Figure 2.11 shows all wing tips usable.

![Wing tips](image)

Figure 2.11 Wing tips (Raymer 1992, p. 64)

According to wing type, winglets are more and less efficient:

A properly design winglet can potentially provide an effective span increase up to double that bought by adding the winglets’ height to the wing span. Winglets provide the greatest benefit when the wing tip vortex is strong, so a low-aspect-ratio wing will see more advantage from the use of winglets than an already-efficient high-aspect-ratio wing. (Raymer 1992, p. 65)

The use of winglets implies two drawbacks: the possibility of aggravating flutter tendencies and the decrease of performances besides the design speed.
2.8 Sweep of the wing, sweep of the horizontal tail and sweep of the vertical tail

The tail surfaces should have higher sweep than the wing to prevent strong shocks on the tail in normal cruise:

Leading-edge sweep of the horizontal tail is usually set to about 5 deg more than the wing sweep. This tends to make the tail stall after the wing, and also provides the tail with a higher Critical Mach Number than the wing, which avoids loss of elevator effectiveness due to shock formation. For low-speed aircraft, the horizontal tail sweep is frequently set to provide a straight hinge line for the elevator, which usually has the left and right sides connected to reduce flutter tendencies. Vertical-tail sweep varies between about 35 and 55 deg. For a low-speed aircraft, there is little reason for vertical-tail sweep beyond about 20 deg other than aesthetics. For a high-speed aircraft, vertical-tail sweep is used primarily to insure that the tail’s Critical Mach Number is higher than the wing’s. (Raymer 1992, p. 76)

2.9 Aspect ratio of the wing, aspect ratio of the horizontal tail and of the vertical tail

- Aspect ratio of the wing:
  According to Howe 2000, typical values of wing aspect ratio for subsonic design are generally in the range 5 to 10. Concerning commercial airliners, Jenkinson 1999 gives the range 7 to 11.

- Aspect ratio of the vertical and horizontal tail
  As for tail aspect ratio, tail size depends on the wing. Hence, the tail aspect ratio is functions of the wing’s one, as shown in Table 2.1.

Table 2.1 Tails geometry (Howe 2000, p. 255)

<table>
<thead>
<tr>
<th>Surface</th>
<th>A</th>
<th>(N/A_{\text{wing}})</th>
<th>(2N/A_{\text{wing}})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Horizontal</td>
<td>(0.5 to 0.6) (A_{\text{wing}})</td>
<td>1.0</td>
<td>1.2</td>
</tr>
<tr>
<td>Canard</td>
<td>(1.0 to 1.3) (A_{\text{wing}})</td>
<td>0.8 to 1.15</td>
<td>1.3</td>
</tr>
<tr>
<td>Vertical</td>
<td>0.9 to 3.0**</td>
<td>1.0^x</td>
<td>0.5</td>
</tr>
</tbody>
</table>

** 0.9 for single engine otherwise 1.2 or more, upwards of 3 for transport types.
^x Usually not less than 20° on quarter chord.

Table 2.2 shows tail aspect ratio designed for different aircraft. Passenger aircraft forms a part of the others category. It appears clearly that horizontal tail aspect ratio is higher than vertical tail’s one.
Table 2.2  Tail Aspect Ratio and Taper Ratio (Raymer 1992, p. 76)

<table>
<thead>
<tr>
<th></th>
<th>Horizontal tail</th>
<th>Vertical tail</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$A$</td>
<td>$\lambda$</td>
</tr>
<tr>
<td>Fighter</td>
<td>3-4</td>
<td>0.2-0.4</td>
</tr>
<tr>
<td>Sail plane</td>
<td>6-10</td>
<td>0.3-0.5</td>
</tr>
<tr>
<td>Others</td>
<td>3-5</td>
<td>0.3-0.6</td>
</tr>
<tr>
<td>T-Tail</td>
<td>—</td>
<td>—</td>
</tr>
</tbody>
</table>

2.10 Taper ratio of the wing, taper ratio of the horizontal tail and taper ratio of the vertical tail

- **Taper ratio of the wing**
  According to *Howe 2000, p. 101*, commercial aircraft aspect ratio of the wing is in the range 0.2 to 1.0.

- **Taper ratio of the vertical and horizontal tail**
  As for tail taper ratio, tail size depends on the wing. Hence, the tail taper ratio is functions of the wing’s one, as also shown in Table 2.1.
3 Relationships between parameters from own statistics

3.1 Number of passengers and number of seats abreast

Figure 3.1 underlines the possible proportionality between the number of seats abreast and the total number of passengers.

![Figure 3.1](image)

**Figure 3.1** Number of seats abreast versus Total number of passengers

The good correlation coefficient allows creating a straight line equation:

\[ y = 65.494x - 180.15 \]

\[ R^2 = 0.8051 \]

In spite of the good correlation coefficient, this equation can not be validated because a two-across seats correspond to a negative value of the total number of seats which is physically wrong.

The equation (3.1) seems to be a better model representing the relationship, as shown in figure 3.2.
Optimization of the coefficient 0.45:

Two solutions are investigated:

- **Dealing all aircraft**

  The optimized coefficient becomes 0.442 and the coefficient is 0.89. The red series in figure 3.3 is the result of the optimization.

- **Dealing all aircraft except double deck aircraft (A380-800 and B747-400)**

  Double deck airplanes have been removed owing to specific cross section.

  The optimized coefficient is a bit closer of the initial coefficient with 0.457. Nevertheless, the correlation is stronger with a coefficient of 0.92. The light-blue series also in figure 3.3 is the second result of the optimization.
At last, the two optimizations are very close to the initial equation, the second appearing to be the best.
3.2 Length of the cockpit, fuselage tail section and length of cabin in the fuselage tail section

- **Length of the cockpit**

  The average cockpit length calculated from gather aircraft data is 3.70 m. So this average value is lower than Schmitt 98 value of 4 m.

- **Fuselage tail section and cabin in tail section**

  As literature describes, the relationship between length of cabin in the tail section and length of the tail section can be correlated by a straight line with a good correlation coefficient, as shown in figure 3.4.

  ![Figure 3.4](image)

  **Figure 3.4** Length of cabin in the tail section versus length of the tail section

  The equation deduced from the correlation is:

  \[
  L_{\text{Tail}} = 1.1L_{\text{TailCabin}} + 6.3
  \]  

  (3.2)

  Therefore, the total length of equipment compartments in the fuselage is 6.3 m and varies little for different airplanes.
3.3 Fuselage length and length of the cargo compartments

Two interpretations can be discussed.

- **First interpretation**

In figure 3.5 which represents measured data of aircraft, the red straight line establishes the model of the relationship between these two lengths.

The equation corresponding to this strong correlation can be written:

\[
L_{Comp} = 0.6L_{Fus} + 7.5
\]  

(3.3)

The constant of 7.5 m fits the location of electric and electronic bays, air conditioning, hydraulic bay and landing gear. Nevertheless, this equation does not respect physical laws since \(L_{Comp}\) would not be null for \(L_{Fus} = 0\).

- **Second interpretation**

The black straight line establishes the model of the second possible relationship between the two parameters.

The correlation is stronger than the first and the equation is:

\[
L_{Comp} = 0.5L_{Fus}
\]  

(3.4)

Physically better, this equation gives a null \(L_{Comp}\) when \(L_{Fus}\) is null. However, with this equation, length of an equipment compartment such as landing gear can’t be considered and calculated.
Figure 3.5  Length of the fuselage versus length of the cargo compartments
3.4 Geometric relationships for engine integration

Measures of parameters $H$, $Xf$ and $C$ on under-wing engines allow obtaining figure 3.6.

Figure 3.6 $Xf/C$ versus $H/C$

Figure 3.6 can be compared to figure 2.8 using Photoshop (see figure 3.7). Eight engines are situated in the white area. These engines fit A380-800, B767-200, B767-300, B737-700, B737-800, A340-600, A340-500 and B777-200. Obviously, all these engine locations have been devised by CFD Method so as to optimize them.

Figure 3.7 Superposition of Figure 3.6 and Figure 2.8
Figure 3.6 can also be compared with figure 2.9 thanks to Photoshop (see figure 3.8). B767-300 is located in the black area what is certainly due to the combination of CFD Method, ground clearance and short landing gear. In the Challenge area, two aircraft are present, mainly due to CFD Method: B767-200 and B737-700. The Difficult area contains seven aircraft and the Easy area comprises the other airplanes.

Figure 3.8 Superposition of Figure 3.6 and Figure 2.9
3.5 Sweep of the wing, cruise Mach number and entry into service

Figure 3.9 shows the wing sweep angle’s increase required for the cruise Mach number’s increase. This trend confirms the law of sweep increase for high Mach numbers.

![Figure 3.9 ¼ Chord sweep of the wing versus Cruise Mach number](image)

For different airfoils, \( M_{\text{eff}} \) seems to be constant functions of years of entry into service, as shown in figure 3.10, figure 3.11 and figure 3.12.

Obviously, these three kinds of airfoils do not have the same \( M_{\text{eff}} \) average:

- Supercritical airfoil: \( M_{\text{average}}^{\text{eff}} = 0.75 \)
- Peaky airfoil: \( M_{\text{average}}^{\text{eff}} = 0.79 \)
- Conventional airfoil: \( M_{\text{average}}^{\text{eff}} = 0.72 \)
Figure 3.10  Entry into service versus $M_{eff}$ for Supercritical Airfoils

Figure 3.11  Entry into service versus $M_{eff}$ for Peaky Airfoils

Figure 3.12  Entry into service versus $M_{eff}$ for Conventional Airfoils
3.6 Sweep of the wing, dihedral and position of the wing

Figure 3.13 does not confirm intervals given by Howe 2000. A fairly good correlation can be established. The equation of the straight line is:

$$\Gamma = -0.32 \Lambda_{25} + 3.70$$  

(3.5)

So, for unswept low wings and until $\lambda_{25} = 11.6$, positive dihedral is used. $\Gamma$ is null at $\Lambda_{25} = 11.6$. After this value, anhedral is employed to override effects of high sweep. The maximum anhedral is $-6^\circ$ for a maximum sweep of $27^\circ$.

![Figure 3.13](image)  

**Figure 3.13** ¼ Chord sweep of the wing versus Dihedral for High wings

About low wings, Figure 3.14 does not allow creating a regression due to the point cloud which illustrates consequences of ground clearance. Dihedral angle for low wings of Figure 3.14 is included between $1.5^\circ$ and $7^\circ$ and sweep angle between $14^\circ$ and $37.5^\circ$. For low wings, wing dihedral can not oppose to dihedral due to sweep.

![Figure 3.14](image)  

**Figure 3.14** ¼ Chord sweep of the wing versus Dihedral for Low wings
3.7 Length of the winglet, wing span and entry into service

The aircraft data base contains two types of winglets:

- Double winglets, constituted of a lower part and an upper part
- Single winglets, constituted only of an upper part

According to figure 3.15, double winglets length, mainly used by Airbus aircraft, does not appear to evolve functions of years of entry into service. Winglets have a length between 2.9% and 4.9% of the wing span. The average ratio is 3.5%.

![Figure 3.15](image)

**Figure 3.15** Entry into service versus Ratio length of winglets/wing span for double winglets

Figure 3.16 shows also evolution of the same parameters but for single winglets. In this case, the length of the winglets can reach 7.6% and decrease until 1.6% of the wing span. The average ratio of 4.5% is more than the average ratio for double winglets.

![Figure 3.16](image)

**Figure 3.16** Entry into service versus Ratio length of winglets/wing span for single winglets
Finally, the ratio does not seem to evolve historically. This may be explained because of the drawbacks of winglets which limit the length and the ratio.

### 3.8 Sweep of the wing, sweep of the horizontal tail and sweep of the vertical tail

Figure 3.17 shows a regression is possible between sweep of the wing and sweep of the horizontal tail. A quite strong correlation equation can be established thanks to this trend:

\[
\Lambda_{25}^{\text{Horiz. Tail}} = 0.68\Lambda_{25}^{\text{Wing}} + 11.85
\]  

This equation does not confirm the trend of Raymer 1992. However, the physical reality of this equation could lead to a reconsideration since the sweep of the horizontal tail is 1.85° for a wing sweep null.

![Figure 3.17](image)

**Figure 3.17** ¼ Chord sweep of the wing versus ¼ Chord sweep of the horizontal tail

Vertical tail sweep varies between 29° and 56°.
3.9 Aspect ratio of the wing, aspect ratio of the horizontal tail and of the vertical tail

- Aspect ratio of the wing:

Observing the measured aircraft data in Figure 3.19, aspect ratio of the wing is included between 6.6 and 9.4. This range confirms values of \textit{Howe 2000} and \textit{Jenkinson 1999} and is included in.

- Aspect ratio of the vertical and horizontal tail:

In Figure 3.19, aspect ratio of the horizontal tail sprawls from 3.35 to 5.54, which confirms values in literature of \textit{Raymer 1992}.

Moreover, a proportional relationship between aspect ratio of the wing and aspect ratio of the horizontal tail can be found. Its equation with a quite good correlation coefficient is:

\[ A_{HorizTail} = 0.55A_{Wing} \]  \hspace{1cm} (3.7)

The slope of the equation, comprised between 0.5 and 0.6, is in agreement with \textit{Howe 2000}.
Figure 3.19 Aspect ratio of the wing versus aspect ratio of the horizontal tail

In Figure 3.20, aspect ratio of the vertical tail sprawls from 0.64 to 1.87. The minimum value is less than the minimum one announced by Howe 2000 which is 0.9. This value is the aspect ratio of the B727-200Adv’s vertical tail.

If this value is excluded, the minimum value becomes 0.97 and belongs to the range of Howe 2000.

Figure 3.20 Aspect ratio of the wing versus aspect ratio of the vertical tail
3.10 Taper ratio of the wing, taper ratio of the horizontal tail and taper ratio of the vertical tail

- Taper ratio of the wing

As shown in Figure 3.21, wing taper ratio values are include in 0.15 to 0.54 which is a bit inferior than the range 0.2 to 1 (Howe 2000).

- Taper ratio of the horizontal and vertical tail

Concerning the horizontal tail, the taper ratio evolves in the range 0.19 to 0.69, as shown in Figure 3.21. The value 0.19 is less than the minimum value of Raymer 1992 and the value 0.69 is a bit more than the maximum value of Raymer 1992.

In order to verify the relationship of Howe 2000 between the horizontal tail and the wing, a straight line is built in Figure 3.21. The slope value is 1.32, which is very near the coefficient 1.2 given by Howe 2000. Nonetheless, the correlation coefficient is not really good. Thus, Howe’s hypothesis can not be validated for certain.

In Figure 3.22, taper ratio of the vertical tail is included between 0.24 and 0.98. The maximum 0.98 is higher than the maximum value announced by Raymer 1992. About the slope of Howe 2000, any correlation seems possible between taper ratio of the wing and taper ratio of the vertical tail. So the two taper ratios appear independent.
Figure 3.22   Taper ratio of the wing versus Taper ratio of the vertical tail
4 Conclusions

After looking for and studying the different trends and relationships between parameters given in the literature, selected data has been used to confirm literature results or to prove them wrong.

Most of the statistical trends and equations given in the task sheet have been verified. Two equations in this project have confirmed those in the literature, one of them has been optimized. Five relationships were cast in new equations. In some other cases, no equation or relationship could be found. Hence, related equations given in the literature could not be confirmed or improved. Many reasons are possible for this situation: First, an intrinsic parameter was not considered. Second, the relationship is distorted by imprecision of underlying data or measurements. Third, lack of data precluded finding a statistical equation. And finally, there may not be a relationship at all.

To conclude, this project reinforces or casts doubts on existing equations. The project found new equations which could be improved further in the future.
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<table>
<thead>
<tr>
<th>Author</th>
<th>Title</th>
</tr>
</thead>
</table>
Appendix A

Three-view drawings
Figure A.1 Three-view drawing: A300-600

Figure A.2 Three-view drawing: A319-100

Figure A.3 Three-view drawing: A310-300
Figure A.4 Three-view drawing: A320-200

Figure A.5 Three-view drawing: A330-200

Figure A.6 Three-view drawing: A321-100
Figure A.7 Three-view drawing: A330-300

Figure A.8 Three-view drawing: A340-200

Figure A.9 Three-view drawing: A340-500

Figure A.10 Three-view drawing: A340-600
Figure A.15 Three-view drawing: B737-200

Figure A.16 Three-view drawing: B737-400

Figure A.17 Three-view drawing: B737-500

Figure A.18 Three-view drawing: B737-700
Figure A.19 Three-view drawing: B737-800

Figure A.20 Three-view drawing: B747-400

Figure A.21 Three-view drawing: B757-200

Figure A.22 Three-view drawing: B757-300
Figure A.23 Three-view drawing: B767-200

Figure A.24 Three-view drawing: B767-300

Figure A.25 Three-view drawing: B777-200

Figure A.26 Three-view drawing: DC9-30
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Figure A.29 Three-view drawing: Tu134

Figure A.30 Three-view drawing: Tu204
Figure A.31 Three-view drawing: EMB-145

Figure A.32 Three-view drawing: Westwind II

Figure A.33 Three-view drawing: Caravelle

Figure A.34 Three-view drawing: VFW-614
Figure A.35 Three-view drawing: HFB 320

Figure A.36 Three-view drawing: Jetstar II

Figure A.37 Three-view drawing: LearJet 23
Figure A.38 Three-view drawing: Falcon 20

Figure A.39 Three-view drawing: BAC1-11

Figure A.40 Three-view drawing: Super VC10

Figure A.41 Three-view drawing: Citation VI
Figure A.42 Three-view drawing: Hawker 800XP

Figure A.43 Three-view drawing: Beechjet400A

Figure A.44 Three-view drawing: Be-40

Figure A.45 Three-view drawing: Be-200
Figure A.46 Three-view drawing: Global Express

Figure A.47 Three-view drawing: CRJ 200

Figure A.48 Three-view drawing: RJ 85

Figure A.49 Three-view drawing: An-124
Figure A.50 Three-view drawing: An-140

Figure A.51 Three-view drawing: An-72

Figure A.52 Three-view drawing: ATR 72

Figure A.53 Three-view drawing: Dash 8