

Master Thesis

Analytical Balanced Field Length Calculation

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Title of the report

Analytical Balanced Field Length Calculation

Keywords

Aeronautics, Aerodynamics, Airplanes, Airplanes--Take-off, Aviation safety, Aircraft safety measures, Jet transports, Jet engines, Jet planes, Electronic spreadsheets, [Balanced Field Length, decision speed, engine failure, analytical calculation, ground roll distance, safety speed, take-off field length, Excel]

Abstract

Purpose – The Balanced Field Length (BFL) is the take-off distance at which the accelerate-go and accelerate-stop distances are equal after an engine failure. It is normally found by numerical step-wise integration with an iterative search for the decision speed V_1 . This work calculates the BFL analytically with a closed-form solution that removes both the numerical integration and the iteration. The study is focused on jet aircraft with two and four engines.

Methodology – The take-off is described by the equation of motion of an accelerating aircraft, with aerodynamic drag proportional to the square of the speed and thrust modeled as a second-order polynomial in speed. This equation is integrated analytically instead of numerically. The BFL and the decision speed V_1 are then obtained algebraically, from the intersection of the accelerate-go line and the accelerate-stop line, instead of by iteration. The method is implemented in an Excel spreadsheet with a graphical user interface.

Findings – The tool is validated against the numerical MATLAB results of a previous thesis for the Airbus A320 and A340. The analytical BFL is very close to these numerical values, which shows that solving the integral gives the same result as the step-wise integration. The tool is then applied to the Boeing 737-800, Boeing 777-300ER, Embraer E190, and Bombardier CRJ-900, and the results are compared with published take-off field length values for these aircraft.

Research Limitations – If no other values are given, the analysis assumes ISA sea-level conditions, a dry level runway, and no wind. Simplifying assumptions are made for the rotation time and for the asymmetric drag after engine failure.

Practical Implications – The tool is intended for educational use. It provides fast take-off performance estimates for aircraft design and aircraft performance calculations. It helps students to better understand the concept of BFL.

Social Implications – Municipalities could face pressure due to a looming runway extension. The Excel spreadsheet helps them to better understand the need for the runway extension. They can do independent calculations and engage in a discussion with the airport on equal footing.

Originality – Goudreault proposed the principles of an analytical integration in an SAE paper in 2013. At HAW Hamburg, Lucht showed how to apply analytical integration to the drag and thrust models used at the university. To my knowledge, this Master Thesis is the first open

work to calculate BFL with analytical integration (and without iteration of V_I) to a practical level including both ground and air segments of the take-off path. In addition, an Excel spreadsheet is offered.

Name des Studierenden

Alejandro Marín Portugal

Thema der Masterarbeit

Analytical Balanced Field Length Calculation

Stichworte

Luftfahrt, Flugmechanik, Flugzeugaerodynamik, Flugleistung, Auftrieb, Luftwiderstand, Flugzeuge, Tragflügel, Flugtriebwerk, Startbahn, Strahlflugzeug, Tabellenkalkulation, EXCEL, [Balanced Field Length, Startrollstrecke, Entscheidungsgeschwindigkeit, Triebwerksausfall, Sicherheitsstartgeschwindigkeit, Startsicherheitsstartstrecke].

Kurzreferat

Zweck – Die Balanced Field Length (BFL) ist die Startrollstrecke, bei der die Accelerate-Go-Strecke und die Accelerate-Stop-Strecke nach einem Triebwerksausfall gleich sind. Sie wird normalerweise durch numerische schrittweise Integration mit einer iterativen Suche nach der Entscheidungsgeschwindigkeit V_1 berechnet. Diese Arbeit berechnet die BFL analytisch mit einer geschlossenen Lösung, die sowohl die numerische Integration als auch die Iteration entfällt. Die Studie konzentriert sich auf Strahlflugzeuge mit zwei und vier Triebwerken.

Methodik – Der Start wird durch die Bewegungsgleichung eines beschleunigenden Flugzeugs beschrieben, wobei der aerodynamische Widerstand proportional zum Quadrat der Geschwindigkeit ist und der Schub als Polynom zweiter Ordnung in der Geschwindigkeit modelliert wird. Diese Gleichung wird analytisch statt numerisch integriert. Die BFL und die Entscheidungsgeschwindigkeit V_1 werden dann algebraisch aus dem Schnittpunkt der Accelerate-Go-Line und der Accelerate-Stop-Line ermittelt, anstatt durch Iteration. Die Methode ist in einer Excel-Tabelle mit einer grafischen Benutzeroberfläche implementiert.

Ergebnisse – Das Werkzeug wird anhand der numerischen MATLAB-Ergebnisse einer vorherigen Abschlussarbeit für den Airbus A320 und A340 validiert. Die analytische BFL stimmt sehr gut mit diesen numerischen Werten überein, was zeigt, dass die Lösung des Integrals das gleiche Ergebnis liefert wie die schrittweise Integration. Das Werkzeug wird anschließend auf den Boeing 737-800, Boeing 777-300ER, Embraer E190 und Bombardier CRJ-900 angewendet, und die Ergebnisse werden mit veröffentlichten Startstrecken für diese Flugzeuge verglichen.

Grenzen der Anwendbarkeit – Sofern keine anderen Werte angegeben sind, geht die Analyse von ISA-Bedingungen auf Meereshöhe, einer trockenen, ebenen Startbahn und Windstille aus. Vereinfachende Annahmen werden für die Rotationszeit und den asymmetrischen Widerstand nach Triebwerksausfall getroffen.

Bedeutung in der Praxis – Das Werkzeug ist für den Bildungseinsatz gedacht. Es liefert schnelle Startleistungsabschätzungen für den Flugzeugentwurf und Flugleistungsberechnungen. Es hilft Studierenden, das Konzept der BFL besser zu verstehen.

Soziale Bedeutung – Gemeinden könnten unter Druck geraten aufgrund einer bevorstehenden Startbahnverlängerung. Die Excel-Tabelle hilft ihnen, den Bedarf an der

Startbahnverlängerung besser zu verstehen. Sie können unabhängige Berechnungen durchführen und auf Augenhöhe mit dem Flughafen diskutieren.

Originalität – Goudreault schlug die Prinzipien einer analytischen Integration in einem SAE-Papier im Jahr 2013 vor. An der HAW Hamburg zeigte Lucht, wie die analytische Integration auf die an der Universität verwendeten Widerstands- und Schubmodelle angewendet werden kann. Nach meinem Wissen ist diese Masterarbeit die erste öffentlich verfügbare Arbeit, die BFL mit analytischer Integration (und ohne Iteration von V_1) auf einem praktischen Niveau berechnet, einschließlich sowohl der Boden- als auch der Luftsegmente des Startflughafens. Zusätzlich wird eine Excel-Tabelle angeboten.

Analytical Balanced Field Length Calculation

Task for a *Master Thesis*

Background

Calculation of the Balanced Field Length (BFL) traditionally involves a numerical calculation, where the various forces are evaluated as a function of speed, using a step wise integration and an estimate for V_1 . The process is iterated with different values for the engine failure speed until the accelerate-stop and accelerate-go distances are equal. Dennis Lucht wrote a thesis: [Numerical and Analytical Take-off Field Length Calculations for Jet Aircraft](#). He performs a numeric integration with MATLAB (Chapter 5.2.4) to obtain the BFL. The equations are also listed in the [MATLAB m-files](#). Instead of using MATLAB with the function "ode45", now BFL is calculated with an equation obtained from solving the integral of the equation of motion. This is explained in Chapter 5.2.3. As such, all equations from Lucht can be inserted into Excel.

Task

A spread sheet with a friendly Graphical User Interface (GUI) should be produced to allow easy and accurate BFL calculation. These points should be observed in the work:

- Instead of using the Oswald factor (span efficiency factor) from Howe (Chapter 2.4, Equation 2.38), a new equation should be included into Excel. The equation is presented [here](#) in Chapter 3.2.2, 3.2.3, Eq. (6.4), Table 7.1.
- Instead of iterating for V_1 , a direct algebraic solution should be found.
- Apply your Excel-Table to the Airbus A320 and A340 and compare your results with those from Lucht.
- A comparison with Take-Off Filed Length (TOFL) values from selected passenger aircraft (Airbus, Boeing, ...) should also be included.
- A recent paper (2025) "[Development of Improved Empirical Take-Off Equation](#)" should be considered.
- Goudreault wrote an important [SAE paper](#) in 2013. Now, the author shared his industry experience in several e-mails. This input should be used to write about the practical industrial implications of BFL calculations.

The report has to be written in English based on German or international standards on report writing.

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List of Symbols

Latin Letters

a	Acceleration; polynomial coefficient
a_0	Speed of sound at sea level
a_e	Compressibility exponent (Oswald factor)
A	Wing aspect ratio; thrust coefficient
b	Wingspan; polynomial coefficient
b_e	Compressibility exponent (Oswald factor)
c	Polynomial coefficient
$c_{f,eqv}$	Equivalent skin-friction coefficient
C_D	Drag coefficient
C_{D0}	Zero-lift drag coefficient
$C_{D0,clean}$	Zero-lift drag coefficient, clean configuration
$C_{D0,f}$	Flap drag increment
$C_{D0,gear}$	Landing gear drag increment
$C_{D0,OEI}$	OEI zero-lift drag coefficient
C_L	Lift coefficient
$C_{L,g}$	Ground lift coefficient
$C_{L,max}$	Maximum lift coefficient
D	Aerodynamic drag force
d_F	Fuselage diameter
e	Oswald span efficiency factor
e_{geo}	Geometric Oswald factor
e_{theo}	Theoretical Oswald factor
E_{max}	Maximum lift-to-drag ratio
g	Gravitational acceleration
G	Gas generator factor (Bartel model)
h	Geometric altitude
h_{apt}	Airport altitude
h_{arc}	Height gained in circular arc
h_{SC}	Screen height
h_W	Wing height above ground
H	Geopotential height
k_1	Speed-dependent thrust coefficient
k_2	Speed-dependent thrust coefficient
$k_{e,D0}$	Oswald viscous correction factor
$k_{e,F}$	Oswald fuselage interference factor

$k_{e,M}$	Oswald compressibility correction factor
k_E	Aerodynamic efficiency factor
K	Viscous drag slope factor (Oswald Method 2)
L	Aerodynamic lift force; temperature lapse rate
m	Aircraft mass
M	Mach number
M_{crit}	Reference Mach number (Oswald factor)
N	Number of engines; normal force on landing gear
p	Air pressure
p_0	Sea-level pressure
P	Viscous drag term (Oswald Method 2)
q	Dynamic pressure
Q	Inviscid term (Oswald Method 2)
r_{earth}	Mean Earth radius
R	Turn radius (air distance model)
R_L	Specific gas constant for air
s	Distance
s_{ASD}	Accelerate-stop distance
s_{AIR}	Air distance
s_{arc}	Horizontal distance covered in circular arc
s_{climb}	Horizontal distance covered in straight climb
s_g	Ground roll distance
s_{Rec}	Recognition distance
s_{Stop}	Stop distance
s_{BFL}	Balanced Field Length
s_{TOD}	Take-off distance
s_{TOFL}	Take-off Field Length
S_W	Wing reference area
S_{wet}	Total wetted area
$S_{w,f}$	Flapped wing area
T	Thrust force
T_0	Static thrust per engine at sea level
T_{idle}	Idle thrust per engine
t_{buf}	AFM buffer time
t_R	Rotation time
t_{recog}	Recognition time
v	Airspeed
V_1	Decision speed
$V_{1,bal}$	Balanced decision speed
V_2	Take-off safety speed
V_{EF}	Engine failure speed

V_{LOF}	Lift-off speed
V_{MCA}	Minimum airborne control speed
V_{MCG}	Minimum control speed on the ground
V_{MU}	Minimum unstick speed
V_R	Rotation speed
V_S	Stall speed
V_{S1g}	1g stall speed
W	Aircraft weight
x	Empirical TOFL index variable
X	Auxiliary thrust coefficient (Bartel model)
Z	Auxiliary thrust coefficient (Bartel model)

Greek Letters

γ	Runway slope angle
δ	Pressure ratio
δ_f	Flap deflection angle
Δ	Polynomial discriminant; percentage deviation
Δ_T	Temperature deviation from ISA
Δ_λ	Taper ratio sweep correction
θ	Climb angle; temperature ratio
λ	Taper ratio
λ_{BPR}	Engine bypass ratio
μ	Rolling friction coefficient
μ_B	Braking coefficient
ρ	Air density
ρ_0	Sea-level air density
σ	Density ratio
φ	Ground effect correction factor
φ_{25}	Wing sweep angle at 25% chord
φ_{LG}	Gear load factor

List of Abbreviations

<i>AEO</i>	All engines operative
<i>ASD</i>	Accelerate-stop distance
<i>BFL</i>	Balanced Field Length
<i>GUI</i>	Graphical User Interface
<i>ISA</i>	International Standard Atmosphere
<i>MSL</i>	Mean sea level
<i>MTOW</i>	Maximum take-off weight
<i>OEI</i>	One engine inoperative
<i>SL</i>	Sea level
<i>TOD</i>	Take-off distance
<i>TOFL</i>	Take-off Field Length

List of Definitions

Analytical:

"Of or relating to analysis; separating something into component parts or constituent elements." (Merriam-Webster 2026)

In the context of this thesis, the term *analytical* refers to the derivation of a closed-form mathematical expression that yields a result directly, without requiring numerical iteration or step-wise integration.

Balanced Field Length (BFL):

"The runway length at which, for a given aircraft weight, engine thrust, aircraft configuration, and runway condition, the one-engine-inoperative take-off distance required and the accelerate-stop distance required are equal." (Crocker 2005)

The BFL corresponds to the take-off decision speed V_1 at which aborting or continuing the take-off both require the same runway distance. It is the shortest runway on which an aircraft can operate safely following an engine failure at the most critical moment (Lucht 2022).

Calculation:

"The process or act of performing mathematical operations to find a value; the result obtained by such a process." (Merriam-Webster 2026)

Engine Failure Speed V_{EF} :

" V_{EF} is the calibrated airspeed at which the critical engine is assumed to fail. V_{EF} must be selected by the applicant but may not be less than V_{MCG} ." (EASA 2022, CS-25.107)

Jet Aircraft:

"An aircraft powered by a jet engine." (Crocker 2005)

More specifically, the jet aircraft considered in this thesis are turbofan-powered transport category airplanes with two or four engines, certified under EASA CS-25 or FAA FAR Part 25.

Take-off Decision Speed V_1 :

"The take-off decision speed, V_1 , is the calibrated airspeed on the ground at which, as a result of engine failure or other reasons, the pilot is assumed to have made a decision to continue or discontinue the take-off. The take-off decision speed, V_1 , must be selected by the applicant but must not be less than V_{EF} plus the speed gained with the critical engine inoperative during the time interval between the instant at which the critical engine is failed and the instant at which the pilot recognizes and reacts to the engine failure." (Gudmundsson 2014)

Take-off Field Length (TOFL):

"The greater of the Balanced Field Length or 115% of the all-engines-operative take-off distance." (Lucht 2022, based on EASA 2022, CS-25.113)

1 Introduction

1.1 Motivation

The Take-off Field Length (TOFL) is one of the most critical performance parameters in aircraft design. It determines whether an aircraft can operate safely from a given runway, and it directly influences key design decisions such as wing loading, thrust-to-weight ratio, and the complexity of the high-lift system. Because aircraft design is an iterative process, a reliable estimate of the TOFL is needed from the earliest stages, when the aircraft dimensions and the maximum take-off weight are still being fixed (Takahashi 2025).

The TOFL is governed by the Balanced Field Length (BFL) for most two-engine jet aircraft. In current practice, the BFL is obtained by numerical integration of the equation of motion over a range of engine failure speeds. The forces acting on the aircraft are evaluated as a function of speed, and the process is repeated with different decision speeds until the accelerate-stop distance and the one-engine-inoperative take-off distance are equal. This is typically done with dedicated software such as MATLAB (Lucht 2022). The method is accurate, but it requires access to numerical tools and a certain level of programming knowledge, so the calculation is difficult to reproduce or verify outside specialized engineering environments.

A simpler and more transparent approach is possible. Lucht (2022) shows that the equation of motion can be solved analytically in closed form, giving explicit equations for every component distance of the take-off. If these equations are implemented in a spreadsheet, the full BFL calculation becomes available to anyone with access to Microsoft Excel, without MATLAB or any other numerical tool. This is the central contribution of the present thesis.

The developed tool may also be useful for preliminary airport planning purposes. It provides an independent estimation of runway length requirements and helps users understand how aircraft parameters and operating conditions influence take-off performance. The aim is not to replace certified performance software, but to provide a transparent calculation method that can be easily accessed and reproduced.

1.2 Title Terminology

The title of this thesis, *Analytical Balanced Field Length Calculation*, is built from three terms. Each one defines a part of the work and is explained below.

Balanced Field Length

The Take-off Field Length (TOFL) is the take-off distance of an aircraft including a margin of safety. It is the greater of two values, the Balanced Field Length (BFL) and 115% of the all-engines-operative take-off distance (Lucht 2022). For most aircraft with two engines, the TOFL is governed by the BFL.

The Balanced Field Length is the runway distance for the condition in which the distance to continue the take-off after an engine failure is equal to the distance required to abort it (Lucht 2022). If the engine fails at a low speed, the distance needed to stop is short, but the distance needed to continue is long. If the engine fails at a high speed, the opposite is true. There is one speed at which both distances are equal, and the runway length at that point is the BFL.

This behavior is shown in Figure 1.1. The accelerate-stop distance (ASD) grows with the decision speed V_1 , because a higher speed needs more distance to stop. The accelerate-go distance (AGD), which is the one-engine-inoperative take-off distance, decreases with V_1 , because less distance is left to reach the screen height. The two curves cross at one point. The speed at that point is the take-off decision speed V_1 , and the corresponding distance is the Balanced Field Length. Below V_1 the take-off must be aborted, and above V_1 it must be continued.

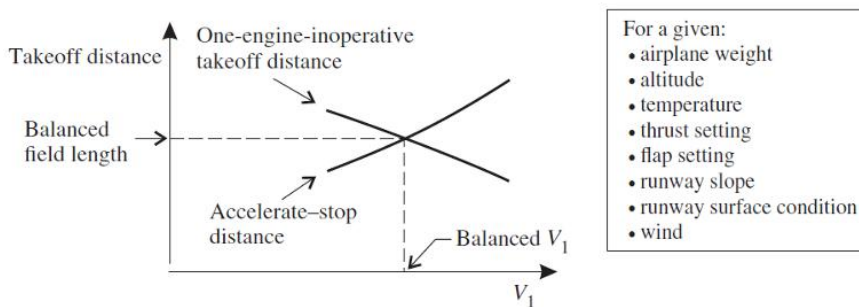


Figure 1.1 BFL as the intersection of ASD and OEI-TOD (Young 2017)

Analytical

The term analytical refers to how the equation of motion is solved. In the traditional method, the forces acting on the aircraft are evaluated as a function of speed, and the equation of motion is integrated numerically, step by step. Lucht (2022) follows this method in MATLAB, using the function ode45, and repeats the process for different engine failure speeds until the balanced condition is found. In this thesis the same integral is solved in closed form. The result is a set of explicit equations that give each component distance of the take-off directly, without step wise integration and without iteration for V_1 .

Calculation

The term calculation defines the scope of the work. The objective is not a flight test or an experimental campaign, but the development of a computational tool. This tool is a Microsoft

Excel spreadsheet with a graphical user interface, and it applies the analytical equations to determine the BFL and the TOFL of a given aircraft.

1.3 Objectives

The main objective is to implement an analytical BFL calculation in a Microsoft Excel spreadsheet with a GUI. Rather than integrating the equation of motion step by step as in Lucht (2022), the integral is solved in closed form. This gives BFL and V_1 directly, without iteration.

The first objective is to implement the analytical equations in the spreadsheet. The closed-form solution derived by Lucht (2022) is used for every component distance of the take-off, from the ground roll to the air distance. These equations are entered into Excel and combined into a single calculation that returns the BFL and the Take-off Field Length (TOFL). A graphical user interface is built around this calculation, so that the tool can be used without programming knowledge and without any numerical software.

The second objective is to improve the estimation of the induced drag. The Oswald span efficiency factor used by Lucht (2022) is replaced by the method of Niță and Scholz (2012). This method estimates the Oswald factor from the basic geometry of the aircraft, separating it into identifiable physical contributions such as the non-elliptical lift distribution of the wing and the effect of the fuselage. It is a traceable and verified procedure, better suited to a transparent calculation tool than the statistical correlation it replaces.

The third objective is to validate the tool. The results of the spreadsheet are compared with the numerical results of Lucht (2022) for the Airbus A320-200 and the Airbus A340-300. The deviations are reported and discussed for both a two-engine and a four-engine aircraft.

The fourth objective is to extend the application beyond the two reference aircraft. The tool is applied to four additional transport aircraft, and the calculated Take-off Field Length is compared with published empirical predictions for each type. This shows whether the analytical method remains reliable when applied to aircraft that were not part of the original numerical study.

1.4 Main Literature

The equations for the essential flight-mechanical relationships, assumptions, and parameters are mainly taken from Lucht (2022), supplemented by Young (2017) and Nicolai (2010). The analytical solution of the equation of motion, which is the core of this thesis, is derived from the closed-form integration presented by Lucht (2022), who applied it to the Airbus A320-200 and A340-300. The algebraic approach to the take-off integration was first proposed by Goudreault (2013) in an SAE paper. The basic mathematical relationships needed to solve the integrals are taken from Papula (2015).

The speed-dependent thrust calculation follows Bartel and Young (2008). The zero-lift drag coefficient is estimated using the method of Scholz (2015). The Oswald span efficiency factor is calculated with the method of Niță and Scholz (2012), which replaces the formulation used by Lucht (2022). The atmosphere model follows the ISA equations as presented by McClamroch (2011).

The aircraft parameters for the Airbus A320-200 and A340-300 are taken from the same sources used by Lucht (2022). For the four additional aircraft, the parameters are taken from the Aircraft Database of Hirsch (2022) and the EASA Type Certificate Data Sheets (EASA 2024). The V speeds are derived from performance data published by SKYbrary (2024).

The empirical methods used as secondary reference in Chapter 6, specifically the Torenbeek (1982), Loftin (1980), and Kroo (2001) methods, are implemented in the BFL Calculator following the formulations presented by Lucht (2022). The recent empirical equations of Takahashi (2025) are used as an additional external benchmark. The paper by Krull (2025) on wave drag and the generic drag polar equation provided useful context for the aerodynamic modeling in Chapter 3.

1.5 Structure of the Work

This thesis is organized into the following chapters.

Chapter 2 presents the regulatory framework. It summarizes the certification standards of EASA CS-25 and FAA FAR Part 25 that define the take-off distances, the decision speed, and the safety margins on which the Take-off Field Length is based.

Chapter 3 describes the theoretical background. It covers the atmosphere model, the aerodynamic forces acting on the aircraft, the speed-dependent thrust model, and the equation of motion that governs the take-off ground roll. The calculation of the Oswald span efficiency

factor following Niță and Scholz (2012) is also presented here, together with the empirical methods used as secondary reference in Chapter 6.

Chapter 4 is where the analytical calculation method is explained. It shows how the equation of motion is solved in closed form for every component distance of the take-off, and how the balanced decision speed $V_{1,bal}$ and the Balanced Field Length are obtained directly without numerical iteration.

Chapter 5 describes the Excel tool. It explains the structure of the spreadsheet, the graphical user interface, the input parameters, and the implementation of the analytical equations of Chapter 4.

Chapter 6 contains the application and validation. The tool is first applied to the Airbus A320-200 and A340-300, and the results are compared with the numerical reference of Lucht (2022). The tool is then applied to four additional transport aircraft. All six aircraft are compared with the five empirical methods presented in Chapter 3, including the recent equations of Takahashi (2025).

Chapter 7 summarizes the main findings and draws conclusions on the accuracy and applicability of the analytical method.

Chapter 8 gives recommendations for future work, covering extensions to the tool and improvements to the calculation method.

2 Regulatory Framework

2.1 Certification Standards: EASA CS-25 and FAA FAR Part 25

The take-off performance of large commercial jet aircraft is governed by two closely harmonized sets of certification standards: the Certification Specifications for Large Aeroplanes CS-25, issued by the European Union Aviation Safety Agency (EASA), and the Federal Aviation Regulations Part 25 (FAR Part 25), codified as 14 CFR § 25, issued by the Federal Aviation Administration (FAA). Both define the airworthiness requirements that an aircraft must demonstrate during certification flight tests before entering commercial service (EASA 2022, FAA 2025). This thesis follows the EASA CS-25 convention throughout; where no amendment difference is explicitly stated, the corresponding FAR Part 25 article is numerically identical.

The certification standards defined in CS-25 are based on a worst-case approach: all take-off requirements are built around the scenario where the most critical engine fails at the most unfavorable moment during the take-off roll. This ensures that the aircraft is always able to either continue the take-off safely or stop within the available runway length, regardless of when the failure takes place. The CS-25 articles most relevant to this thesis are summarized below, based on Lucht (2022) and Scholz (2015).

$$V_2 \geq 1.13 \cdot V_{S1g} \quad (2.1)$$

where V_{S1g} is the reference stall speed at a load factor of 1g in the take-off configuration. The 1g stall speed is the speed at which the maximum lift coefficient C_{Lmax} is reached under level flight conditions, and it represents the lowest speed at which the aircraft can sustain its own weight in straight and level flight. The factor 1.13 provides a safety margin above this limit to ensure adequate climb performance and controllability after engine failure. For aircraft with fly-by-wire systems such as the Airbus A320 and A340, the airworthiness authorities have agreed that $V_S = 0.94 \cdot V_{S1g}$, which means the requirement becomes effectively $V_2 \geq 1.2 \cdot V_S$ (Lucht 2022). The speed hierarchy and the regulatory constraints between all take-off speeds are illustrated in Figure 2.7.

CS-25.111 - Take-off path

CS-25.111 defines the conditions under which the one-engine-inoperative (OEI) take-off path must be demonstrated. The key requirement is that:

(a)....

- (2) The aeroplane must be accelerated on the ground to V_{EF} , at which point the critical engine must be made inoperative and remain so for the remainder of the take-off.
- (3) After reaching V_{EF} , the aeroplane must be accelerated to V_2 .

- (c) (2) The aeroplane must reach V_2 before it is 35 ft above the take-off surface.

The engine failure speed V_{EF} is the speed at which the critical engine is assumed to fail during the certification test. It is always slightly below V_1 , the decision speed, to account for the one second of recognition time that elapses before the pilot identifies the failure. This one-second interval is not negligible: at typical take-off speeds of around 140 kt, the aircraft covers approximately 70 m during that second. The regulatory scenario is therefore conservative by design: the pilot recognizes the failure only after the aircraft has already traveled beyond V_{EF} (Lucht 2022, Young 2017).

CS-25.109 - Accelerate-Stop Distance

CS-25.109 defines the Accelerate-Stop Distance (ASD) as the sum of:

(a)...

(2)...

- (i) The distance to accelerate from a standing start to V_1 , plus the distance covered during 2.0 additional seconds after V_1 is reached, with all engines operating.
- (ii) The distance to come to a full stop from the point reached at the end of the 2-second period, assuming no retarding means are applied until that point.

The 2-second period referred to in (a)(2)(i) is known as the AFM buffer. It represents the time between the pilot's recognition of the failure at V_1 and the first physical braking action.

During this interval, the aircraft continues moving, either at constant speed or still accelerating, before any retardation begins. Physically, this buffer accounts for the reaction time and the sequential actuation of braking systems: brakes first, then thrust to idle, then spoilers as illustrated in Figure 2.1 (Lucht 2022). The distinction between the two relevant amendments is summarized in Table 2.1.

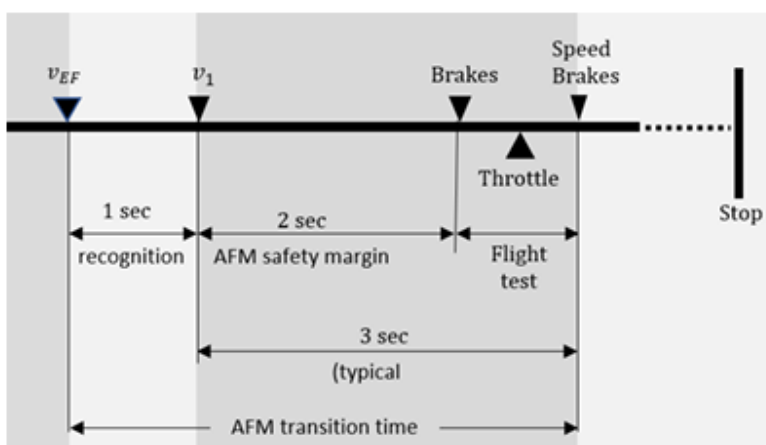


Figure 2.1 AFM transition time sequence, Amendment 25-42 and 25-92

The treatment of this 2-second buffer differs between two amendments of particular relevance to this thesis:

Amendment 25-92: The aircraft is assumed to maintain a constant speed V_1 during the 2 second AFM buffer before braking begins. This is the more conservative assumption, as it does not credit any deceleration during the delay. It is applicable to the Airbus A340, among other types.

Amendment 25-42: The aircraft is assumed to continue accelerating during the 2 second buffer with the operative engines still at full thrust. It is applicable to the Airbus A320, among other types (Lucht 2022, Young 2017).

The two assumptions do not differ in a simple way. Under Amendment 25-42 the aircraft reaches a higher speed before braking begins, which tends to increase the stop distance. At the same time, the higher speed reduces the aerodynamic lift relative to the weight, so more of the aircraft weight rests on the braked wheels and the deceleration is more vigorous. The two effects act in opposite directions, and in practice both amendments yield similar ASD values for the aircraft types considered here. The distinction is still important for a rigorous BFL calculation, and it means that the A320 and A340 must be treated with different time-interval assumptions in the ASD calculation, even though both are certificated under CS-25.

Table 2.1 AFM transition time sequence

Event	Time elapsed [s]	A320 (Amdt. 25-42)	A340 (Amdt. 25-92)
Engine failure at V_{EF}	0	Accelerating	Accelerating
Pilot recognizes failure at V_1	+1	Accelerating	Accelerating
AFM buffer ends	+3	Accelerating	Constant V_1
Brakes applied	+3.5	Decelerating	Decelerating
Thrust to idle	+4.0	Decelerating	Decelerating

CS-25.113 - Take-off distance and take-off run

CS-25.113 defines the Take-off Distance (TOD) as the greater of:

(a)...

- (1) The horizontal distance from brake release to the point at which the aeroplane reaches 35 ft above the take-off surface, with the critical engine failed and the aircraft accelerated to V_2 in accordance with CS-25.111.
- (2) 115% of the horizontal distance from brake release to 35 ft, with all engines operative.

The 15% factor in (a)(2) is a safety margin applied to the all-engines-operative case to account for variability in pilot technique, atmospheric conditions, and aircraft-to-aircraft scatter in performance. The greater of these two distances constitutes the Take-off Field Length (TOFL).

2.2 Take-off Field Length

The Take-off Field Length (TOFL) is the runway length that an aircraft requires to take off safely under a given set of conditions: weight, flap configuration, airport pressure altitude, outside air temperature, wind component, and runway slope. It is the governing parameter for determining whether an aircraft can operate from a given airport at a given dispatch weight, and it therefore plays a central role both in aircraft certification and in day-to-day airline operations planning.

As established by CS-25.113 in Section 2.1, the TOFL is defined as the greatest of three distances: the Accelerate-Stop Distance, the one-engine-inoperative Take-off Distance, and the factored all-engines-operative Take-off Distance, expressed as the following equation.

$$s_{\text{TOFL}} = \max(s_{\text{ASD}}, s_{\text{TOD,OEI}}, 1.15 \cdot s_{\text{TOD,AEO}}) \quad (2.2)$$

Each of these three distances represents a distinct physical scenario and is discussed in detail in the following subsections. Before going into each distance, it is worth defining the screen height h_{SC} . This is the height of an imaginary obstacle at the end of the take-off path. For aircraft certified under CS-25, the screen height is 35 ft (10.7 m). All three distances end at this point (CS-25.111).

2.2.1 All-Engines-Operative Take-off Distance

The all-engines-operative take-off distance $s_{\text{TOD,AEO}}$ is the total horizontal distance covered from brake release to the point at which the aircraft reaches the screen height of 35 ft, with all engines functioning normally throughout the entire maneuver. It represents the nominal take-off scenario, with no failure of any kind, and it is therefore the shortest of the two distances that contribute to the TOFL under typical conditions. CS-25.113(a)(2) requires that this distance, multiplied by a safety factor of 1.15, be one of the candidates for the TOFL. The 15% factor accounts for variability in pilot technique, atmospheric turbulence, and aircraft-to-aircraft performance scatter observed during certification flight testing.

The AEO take-off distance is composed of two sequential phases, which together describe the complete trajectory from the point of brake release to the screen height.

The first phase is the ground roll $s_{\text{g,AEO}}$. This phase begins when the brakes are released and full take-off thrust is applied, and it ends when the main landing gear wheels leave the runway surface at the lift-off speed V_{LOF} . The ground roll has two sub-phases. In the first, $s_{\text{g1,AEO}}$, the aircraft accelerates from standstill to V_2 with all wheels on the ground. As speed increases, lift

grows and rolling friction decreases. Thrust also drops slightly with speed for turbofan engines, so the net accelerating force reduces as the aircraft approaches V_2 . The second sub-phase, $s_{R,AEO}$, is the rotation phase, during which the pilot applies a nose-up input at V_2 , pitching the aircraft to its lift-off angle of attack. This phase ends when the lift force equals the aircraft weight and the main gear leaves the ground at V_{LOF} .

The rotation distance is typically approximated as the product of the average of V_2 and V_{LOF} and the rotation time t_R , which is a function of the aircraft type and pilot technique, with representative values of 2 to 3 seconds for transport category aircraft (Nicolai 2010, Young 2017). The total ground roll is the sum of the two sub-phases.

$$s_{g,AEO} = s_{g1,AEO} + s_{R,AEO} \quad (2.3)$$

The second phase is the air distance $s_{AIR,AEO}$. This phase begins at lift-off, when the main gear wheels leave the runway, and ends when the aircraft reaches the screen height of 35 ft. After lift-off the aircraft follows a curved transition trajectory, during which the flight path angle increases progressively from zero to a steady climb angle, before continuing in a linear climb up to the screen height. The air distance depends primarily on the lift-off speed, the available thrust excess over drag, and the aircraft weight, since these together determine the climb gradient. The total AEO take-off distance is therefore the following equation.

$$s_{TOD,AEO} = s_{g,AEO} + s_{AIR,AEO} \quad (2.4)$$

Figure 2.2 illustrates the AEO take-off trajectory with all component distances labeled.

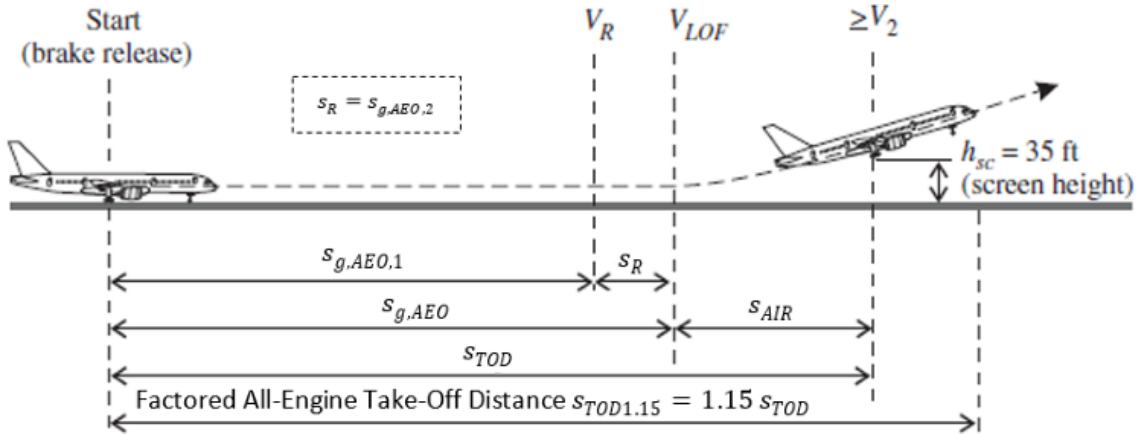


Figure 2.2 AEO take-off distance components

A safety factor of 1.15 is applied to give the factored AEO take-off distance.

$$s_{TOD,AEO} = 1.15 \cdot s_{TOD,AEO} \quad (2.5)$$

This distance does not depend on V_1 , since there is no engine failure in the AEO scenario

2.2.2 One-Engine-Inoperative Take-off Distance

The one-engine-inoperative take-off distance $s_{\text{TOD,OEI}}$, also referred to in this thesis as the Accelerate-Go Distance (AGD), is the total distance from brake release to the screen height of 35 ft in the event that the critical engine fails during the take-off roll and the pilot decides to continue the take-off. This distance represents the worst-case continued take-off scenario and is the one that most directly governs the BFL for two-engine aircraft, as will be discussed in future sections.

The regulatory scenario set by CS-25.111 defines the engine failure conditions precisely. The critical engine is assumed to fail at the engine failure speed V_{EF} , which is always slightly below V_1 . CS-25.107 requires $V_1 \geq V_{EF}$ plus the speed gained during the one-second recognition interval. By the time the pilot reacts, the aircraft has already passed V_{EF} and reached V_1 . In practice, for the purposes of BFL calculation it is commonly assumed that $V_{EF} \approx V_1$, since the speed increment during one second at typical take-off accelerations is small (Lucht 2022).

The OEI ground roll is divided into three distinct segments, reflecting the changing force balance as the failure scenario unfolds. The first segment is the all-engines-operative acceleration from standstill to V_{EF} , which is identical to the first sub-phase of the AEO ground roll described in Section 2.2.1. The second segment, $s_{g,\text{OEI},1}$, covers the continued acceleration from V_{EF} to the rotation speed V_2 with one engine inoperative. During this segment the available thrust is reduced by approximately half for a two-engine aircraft, but the retarding forces remain unchanged, so the net accelerating force is substantially reduced, and the acceleration rate drops significantly. In addition, the failed engine and its associated windmilling compressor generate a significant asymmetric drag increment, which further reduces the net thrust available for acceleration. The pilot must also apply rudder to counteract the yawing moment produced by the asymmetric thrust, and the resulting rudder deflection generates an additional drag force that penalises the acceleration further (Lucht 2022). The third segment, $s_{R,\text{OEI}}$, is the rotation distance with one engine inoperative, from V_2 to the OEI lift-off speed $V_{\text{LOF,OEI}}$.

The total OEI ground roll follows from the sum of all three segments as can be seen in the next equation.

$$s_{g,\text{OEI}} = s_{g,\text{AEO}} + s_{g,\text{OEI},1} + s_{R,\text{OEI}} \quad (2.6)$$

After lift-off, the aircraft must still climb to the screen height of 35 ft with asymmetric thrust and elevated drag. The OEI air distance $s_{\text{AIR,OEI}}$ is longer than the AEO air distance because the

climb gradient is significantly lower with one engine inoperative. CS-25.111(c)(2) requires the aircraft to reach the safety speed V_2 before it is 35 ft above the take-off surface, ensuring adequate controllability and a minimum climb gradient at screen height. The total OEI take-off distance is the sum of the ground roll and the air distance.

$$S_{TOD,OEI} = S_{g,OEI} + S_{g,AIR,OEI} \quad (2.7)$$

Figure 2.3 illustrates the OEI take-off trajectory with all component distances labeled. The effect of the engine failure on the force balance is clearly visible in the significant increase in drag immediately after V_{EF} , which produces a pronounced reduction in the net accelerating force even though only a fraction of the total thrust is lost.

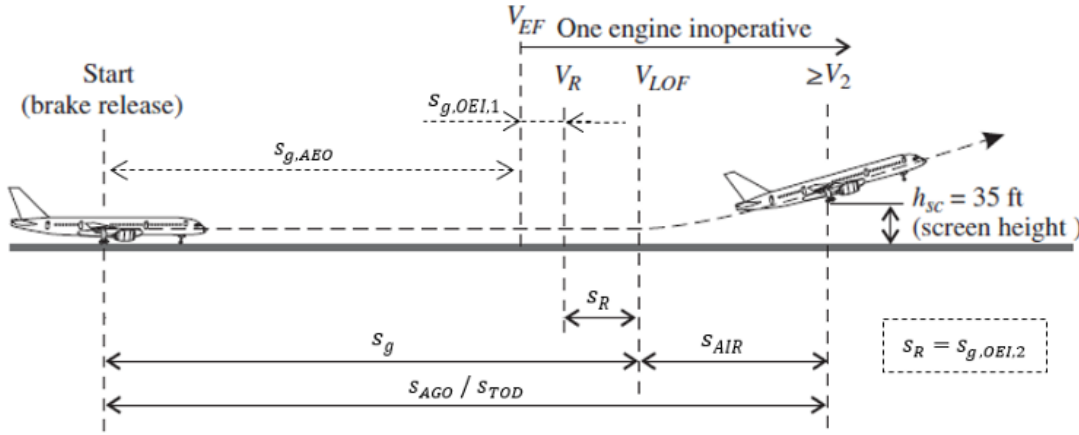


Figure 2.3 OEI take-off distance components

Unlike the factored AEO distance, $S_{TOD,OEI}$ depends directly on V_1 , because a higher V_1 means the engine failure is recognized later and closer to V_2 , leaving a shorter remaining OEI acceleration distance.

2.2.3 Accelerate-Stop Distance

The Accelerate-Stop Distance (ASD) is the total distance covered from brake release to a complete standstill if the pilot decides to abort the take-off at the decision speed V_1 . It represents the rejected take-off scenario and is the distance that limits how high V_1 can be set: the higher V_1 is, the more runway is consumed during the acceleration, and the longer the subsequent stop distance becomes. CS-25.109 mandates that the ASD be included as one of the two candidates for the TOFL, and it specifies the precise sequence of events and timing assumptions that must be used in its calculation.

The ASD is the sum of three components as it can be seen in the following Equation. Each component corresponds to a distinct phase of the rejected take-off maneuver, as described below.

$$s_{ASD} = s_{g,AEO} + s_{Rec} + s_{Stop} \quad (2.8)$$

Acceleration distance

$s_{g,AEO}$ is the distance covered from brake release to the engine failure speed V_{EF} , with all engines operative. This phase is identical in both the continue and the abort scenarios: the aircraft accelerates normally until the critical engine fails or the pilot decides to abort. As in the OEI take-off case, the simplification $V_{EF} \approx V_1$ is commonly applied for BFL calculations, meaning that the acceleration distance is treated as the ground roll from standstill to V_1 with all engines operative (Lucht 2022).

Recognition distance

s_{Rec} is the distance covered during the one-second interval between the engine failure at V_{EF} and the pilot's formal recognition of the failure at V_1 . CS-25.109 mandates a minimum recognition time of one second, representing the average pilot response time measured across a statistically significant number of flight tests. During this one second the aircraft continues to accelerate, since no retarding action has yet been applied. At typical take-off speeds of 140 kt, one second of continued acceleration at normal take-off thrust corresponds to approximately 70 m, which is not negligible.

Stop distance

s_{Stop} is the distance from the first retarding action to a complete standstill. Once the pilot has recognized the failure and made the abort decision at V_1 , the retarding means are applied sequentially in the following order, as established by certification flight testing: first the wheel brakes are applied, then the thrust levers are moved to idle, and finally the ground spoilers are deployed. The brakes come first because they give the greatest deceleration. The spoilers then collapse the wing lift, pushing more weight onto the wheels and increasing the braking force (Lucht 2022).

For dry runway calculations, reverse thrust is not credited in the ASD, as required by CS-25.109. This is a conservative regulatory choice: reverse thrust is available on most jet aircraft but is considered an unreliable retarding means for certification purposes, since its availability and effectiveness depend on maintenance condition, ambient temperature, and pilot technique (Scholz 2015). The braking coefficient μ_B for dry asphalt or concrete is typically in the range of 0.3 to 0.6 (Gudmundsson 2014, Nicolai 2010), with a value of $\mu_B = 0.35$ commonly used for certification calculations (Lucht 2022).

The stop distance is further influenced by the residual aerodynamic lift during the deceleration roll. As long as the wing is still generating lift, part of the aircraft weight is carried

aerodynamically and is not transferred to the landing gear, reducing the normal force and therefore the braking force available. This is where the ground spoilers play a key role: when deployed, they disrupt the airflow over the wing surface, collapsing the lift almost immediately and transferring the full aircraft weight back onto the wheels, which directly increases the braking force. This is why spoiler deployment is one of the first actions in the rejected take-off sequence, despite being listed after the brakes in the actuation order. Furthermore, it is worth noting that not all of the aircraft weight acts on braked wheels, since the nose gear on most transport aircraft is not equipped with brakes. The proportion of the total weight that rests on the braked main landing gear is accounted for through the load factor f_L , which takes typical values between 0.80 and 0.95 for jet transport aircraft, depending on the center of gravity position (Lucht 2022).

A critical regulatory distinction affects the stop distance calculation depending on the aircraft type. As introduced in Section 2.1, two amendments govern the treatment of the two-second AFM buffer that follows the one-second recognition interval. Under **Amendment 25-92**, applicable to the Airbus A340 and most newer aircraft types, the aircraft is assumed to maintain a constant speed equal to V_1 during the two-second buffer before braking commences. This is the more conservative assumption, since it does not allow the aircraft to decelerate during the buffer period, resulting in a longer overall stop distance. Under **Amendment 25-42**, applicable to the Airbus A320, the aircraft is assumed to continue accelerating during the two-second buffer with the operative engines still at full thrust. In practice both amendments give similar ASD values, but the distinction matters for a rigorous BFL calculation (Lucht 2022).

Figure 2.4 illustrates the ASD with its component distances, and Table 2.1 in Section 2.1 summarises the complete time sequence for both amendments.

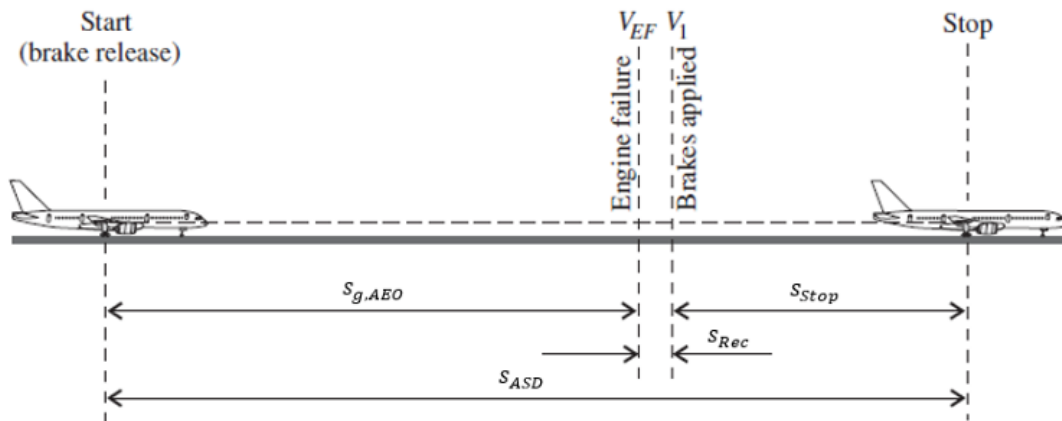


Figure 2.4 Accelerate-stop distance components

2.2.4 Influence of V_1 on the Three Distances

The decision speed V_1 is not a fixed parameter. It is selected by the aircraft operator for each take-off, within the regulatory limits established in CS-25.107. Its value directly determines the magnitudes of two of the two TOFL candidate distances and understanding how it affects each distance is essential for understanding the concept of the Balanced Field Length.

As V_1 increases, the ASD grows monotonically. A higher V_1 means the pilot accelerates to a higher speed before the abort decision is made, which increases the acceleration distance and, since braking begins from a higher speed, also increases the stop distance. The relationship between ASD and V_1 is approximately quadratic, since the kinetic energy that must be dissipated by the brakes grows with the square of the speed.

At the same time, as V_1 increases, the OEI take-off distance $s_{\text{TOF,OEI}}$ decreases. A higher V_1 means the engine failure is recognized later, and the aircraft is already at a higher speed when the OEI acceleration phase begins. The remaining OEI acceleration distance from V_1 to V_2 is therefore shorter, and since the aircraft is already moving faster, the time spent at low speed with reduced thrust and high drag is reduced. The air distance is also slightly shorter because the aircraft is closer to V_{LOF} when it leaves the ground.

The TOFL at any given V_1 is the greatest of the two distances at that speed. At low V_1 values, the OEI take-off distance dominates: the aircraft fails at low speed and needs a long distance to accelerate to V_2 with reduced thrust. At high V_1 values, the ASD dominates: the aircraft has committed to a high speed and cannot stop before the end of the runway. Between these two extremes there is a speed at which ASD equals $s_{\text{TOF,OEI}}$. This intersection defines the Balanced Field Length condition and gives the lowest possible TOFL in the OEI case. Figure 2.5 illustrates this behavior graphically.

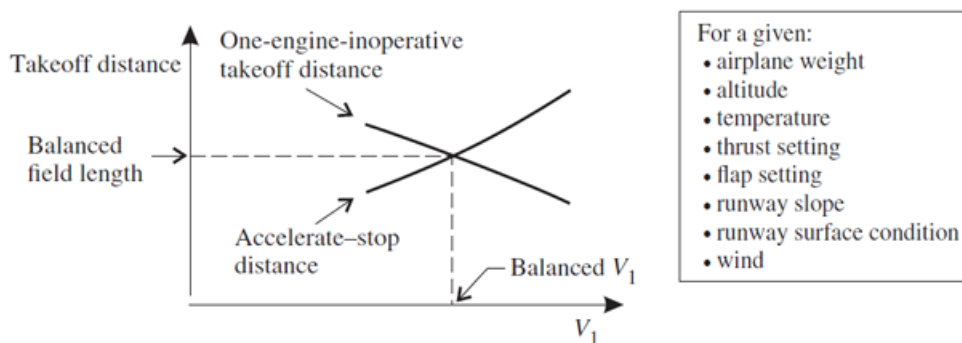


Figure 2.5 TOFL candidate distances as a function of V_1

2.3 Balanced Field Length

The Balanced Field Length (BFL) is the central concept of this thesis and the performance parameter around which all the calculations presented in the following chapters are built. It is defined as the runway distance at which the Accelerate-Stop Distance equals the one-engine-inoperative Take-off Distance, both evaluated at the same decision speed. The speed at which this equality holds is called the balanced decision speed $V_{1,BFL}$, and the corresponding distance is the BFL, defined as follows.

$$s_{BFL}(V_{1,BFL}) = s_{ASD}(V_{1,BFL}) = s_{TOD,OEI}(V_{1,BFL}) \quad (2.9)$$

To understand why this condition is significant, it is useful to think about what happens when an engine fails during the take-off roll. At that moment, the pilot faces a critical choice: continue the take-off with one engine inoperative and try to reach 35 ft safely or abort the take-off and stop before the end of the runway. Both options require runway distance, and the amount needed depends directly on the speed at which the failure occurs. If the failure happens at a low speed, stopping is easy but completing the take-off requires a long distance because the aircraft must still accelerate to V_R and V_2 with reduced thrust. If the failure happens at a high speed, close to V_2 , completing the take-off is relatively straightforward because the aircraft only needs a short additional distance to reach V_{LOF} , but stopping safely may require more runway than is available. The BFL condition captures the exact speed at which both outcomes require the same runway length, which is the most efficient use of the available field.

The balanced condition can also be seen directly on the runway. Figure 2.6 shows the take-off as a speed versus distance diagram. The aircraft accelerates with all engines until the critical engine fails near V_1 . From that point, two paths share the same starting point. In the first path the take-off is continued, and the aircraft keeps accelerating with one engine inoperative until it lifts off and reaches the screen height. In the second path the take-off is rejected, and the aircraft is brought to a full stop. At the balanced decision speed, both paths end at the same distance from brake release. That distance is the Balanced Field Length.

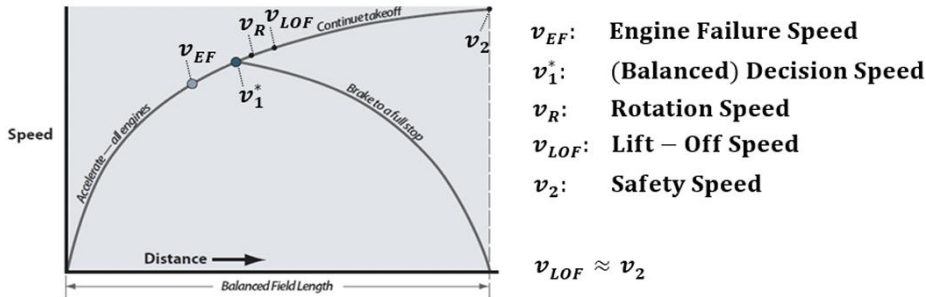


Figure 2.6 BFL condition: continued and rejected take-off paths

From a regulatory standpoint, the BFL is important because it represents the minimum required runway length in the OEI case. For any V_1 below $V_{1,\text{bal}}$, the OEI take-off distance exceeds the ASD, and the required field length is governed by the continued take-off. For any V_1 above $V_{1,\text{bal}}$, the ASD exceeds the OEI take-off distance, and the required field length is governed by the rejected take-off. In both cases, the required field length is longer than the BFL. The BFL is therefore the shortest runway on which the aircraft can operate safely in the event of an engine failure, for a given set of conditions.

In real operations V_1 is not always set at the balanced speed. On a long runway V_1 may be lower, making the OEI distance the critical constraint. On a short runway V_1 may be higher, making the ASD critical. The BFL is the reference for these trade-offs (Takahashi 2025, Young 2017).

For the aircraft types considered in this thesis, specifically two-engine jet aircraft such as the Airbus A320, the BFL typically governs the TOFL. This is because the loss of one engine on a two-engine aircraft represents a 50% reduction in available thrust, which makes the OEI take-off distance significantly longer than for a four-engine aircraft such as the A340, where the thrust reduction is only 25%. For the A340, the TOFL may instead be governed by the factored AEO distance $s_{\text{TOD},1.15}$ rather than by the BFL, depending on the conditions. This difference between the two aircraft types is one of the reasons why both are used as validation cases in this thesis.

Finally, the BFL in this thesis is calculated for dry runway, sea-level, and ISA conditions. In real operations it also depends on altitude, temperature, wind, and slope, but these cases are outside the scope of this work.

2.4 V-Speeds and Their Regulatory Basis

During take-off, the crew uses a set of reference speeds defined in CS-25.107. Each speed has a regulatory minimum and a specific order. Figure 2.7 shows the complete hierarchy of take-off speeds and their mutual constraints.

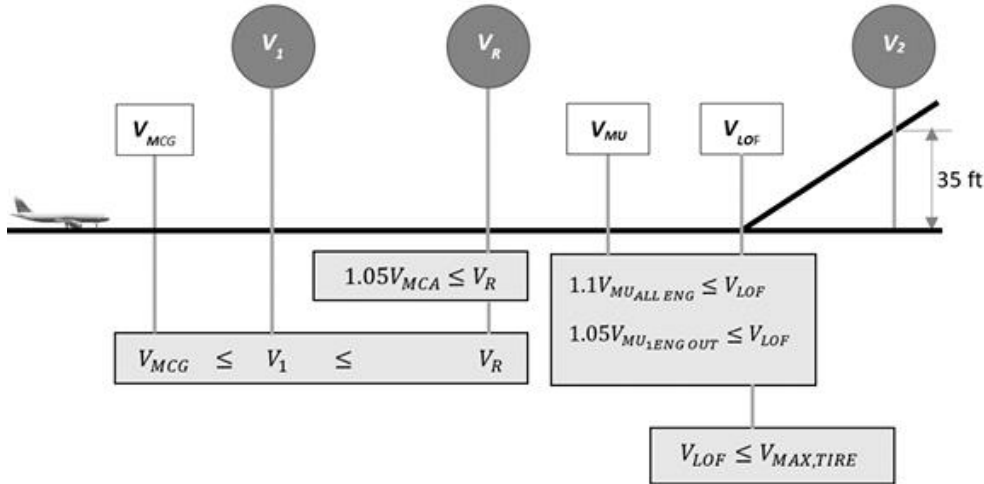


Figure 2.7 Take-off speed hierarchy (CS-25.107)

V_{MCG}	Minimum Control speed
V_{EF}	Engine Failure
V_1	Decision speed
V_{MU}	Minimum Unstick speed
V_R	Rotation speed
V_{MCA}	Minimum Control Speed, Airborne
V_{LOF}	Lift-Off speed
V_2	Safety speed

The speeds are defined below in the order in which they appear during the take-off roll.

Stall Speed V_{S1g} is the reference speed at which the maximum lift coefficient is reached under a load factor of 1g. It is not used operationally but serves as the baseline from which all other minimum speed requirements are derived. For fly-by-wire aircraft such as the A320 and A340, the operational stall speed is defined as $V_S = 0.94 \cdot V_{S1g}$, and all speed margins are computed relative to V_{S1g} (Lucht 2022, EASA 2022).

Minimum Control Speed on the Ground V_{MCG} is the lowest speed at which directional control can be maintained on the runway after a sudden engine failure, using only rudder without nose wheel steering. It is determined during certification flight tests and sets the absolute lower limit for V_1 . Below V_{MCG} , a failed engine cannot be controlled and the take-off must always be rejected (CS-25.149).

Engine Failure Speed V_{EF} is the speed at which the critical engine is assumed to fail. CS-25.107 requires $V_{EF} \geq V_{MCG}$. For BFL calculations, the approximation $V_{EF} \approx V_1$ is commonly used since the speed gained during the one-second recognition interval is small (Lucht 2022).

Decision Speed V_1 is the speed at which the pilot is assumed to have decided to continue or reject the take-off. Above V_1 the pilot must fly; below it, stopping is still possible. Its value is selected for each departure within the limits.

$$V_{MCG} \leq V_1 \leq V_R \quad (2.10)$$

Rotation Speed V_R is the speed at which the pilot applies nose-up elevator to begin the rotation to the lift-off attitude. CS-25.107 establishes the following minimum constraints.

$$V_R \geq V_1, \quad V_R \geq 1.05 \cdot V_{MCG}, \quad V_R \geq 1.05 \cdot V_{MCA} \quad (2.11)$$

Minimum Unstick Speed V_{MU} is the minimum speed at which the aircraft can safely leave the ground. It is established by flight test and sets a lower limit on V_{LOF} : the lift-off speed must be at least $1.08 \cdot V_{MCA}$ with all engines and $1.04 \cdot V_{MCA}$ with one engine inoperative (CS-25.107).

Minimum Airborne Control Speed V_{MCA} is the minimum flight speed at which directional control can be maintained after an engine failure with maximum thrust on the remaining engines. It is generally lower than V_{MCG} because the vertical tail is more effective in flight than on the ground. CS-25.149 requires $V_{MCA} \leq 1.13 \cdot V_{S1g}$.

Lift-Off Speed V_{LOF} is the speed at which the main gear leaves the runway. It is not set directly by the pilot but results from the rotation initiated at V_2 . Typical values are 3 to 5 kt above V_2 for transport jet aircraft (Nicolai 2010).

Safety Speed V_2 is the minimum speed at which the aircraft must reach the screen height of 35 ft with one engine inoperative. It must meet the following regulatory requirements.

$$V_2 \geq 1.13 \cdot V_{S1g}, \quad V_2 \geq 1.10 \cdot V_{MCA} \quad (2.12)$$

For the A320 and A340, applying $V_S = 0.94 \cdot V_{S1g}$ gives the operational requirement $V_2 \geq 1.2 \cdot V_S$ (Lucht 2022). V_2 is shown on the Primary Flight Display throughout the take-off as the target speed in case of engine failure.

Two features of this speed set matter for the rest of this thesis. Most of the speeds are fixed once the aircraft configuration and weight are known. V_{S1g} , V_{MCG} , V_{MCA} and V_2 all follow directly from CS-25 margins and do not depend on the decision speed. V_1 is different. It is the only speed that is not set by a single margin. It must satisfy Equation (2.10), and it must also make the accelerate-stop distance equal to the accelerate-go distance, as introduced in Section 2.3. This second condition is solved analytically in Chapter 4.

The speeds in Table 2.2 also correspond to the input structure of the BFL Calculator described in Chapter 5. V_{MCG} , V_2 and $C_{L,max}$ are read directly from the aircraft data table, and every other speed in the hierarchy is derived from them inside the spreadsheet.

Table 2.2 Take-off V-speeds

Speed	Name	Regulatory minimum
V_{S1g}	1g stall speed	Reference (CS-25.103)
V_{MCG}	Min. control speed, ground	By flight test (CS-25.149)
V_{EF}	Engine failure speed	$\geq V_{MCG}$ (CS-25.107)
V_1	Decision speed	$V_{MCG} \leq V_1 \leq V_2$
V_{MU}	Min. unstick speed	By flight test (CS-25.107)
V_R	Rotation speed	$\geq V_1, \geq 1.05 \cdot V_{MCG},$ $\geq 1.05 \cdot V_{MCA}$
V_{MCA}	Min. control speed, airborne	$\leq 1.13 \cdot V_{S1g}$ (CS-25.149)
V_{LOF}	Lift-off speed	$\geq 1.08 \cdot V_{MCA}$ (AEO), $\geq 1.04 \cdot V_{MCA}$ (OEI)
V_2	Safety speed	$\geq 1.13 \cdot V_{S1g}, \geq 1.10 \cdot V_{MCA}$

3 Theoretical Models for Take-off Performance

3.1 Atmosphere Model

All aerodynamic forces acting on an aircraft during take-off depend on the local air density, pressure, and temperature. These properties change with altitude, and their values at any given point are defined by the International Standard Atmosphere (ISA), an idealized atmospheric model established by the International Civil Aviation Organization (ICAO) and used universally in aircraft performance analysis (Young 2017).

The ISA defines sea-level reference conditions as a temperature of $T_0 = 288.15 \text{ K}$ (15°C), a pressure of $p_0 = 101325 \text{ Pa}$, and a density of $\rho_0 = 1.225 \text{ kg/m}^3$. In the troposphere, which extends up to approximately 11 km and covers all airport elevations relevant to this thesis, the temperature decreases linearly with altitude at a rate of $L = 6.5 \cdot 10^{-3} \text{ K/m}$. The temperature at a given geopotential height H follows the linear lapse rate.

$$T(H) = T_0 - L \cdot H \quad (3.1)$$

The temperature ratio θ , pressure ratio δ , and density ratio σ are defined as the following equations.

$$\theta = \frac{T}{T_0} \quad (3.2)$$

$$\delta = \frac{p}{p_0} = \left(\frac{T}{T_0}\right)^{\frac{g}{R_L \cdot L}} \quad (3.3)$$

$$\sigma = \frac{\rho}{\rho_0} = \left(\frac{T}{T_0}\right)^{\frac{g}{R_L \cdot L} - 1} \quad (3.4)$$

where $g = 9.80665 \text{ m/s}^2$ is the gravitational acceleration and $R_L = 287.053 \text{ J/(kg}\cdot\text{K)}$ is the specific gas constant for air.

Departures from standard conditions are accounted for through a temperature deviation ΔT from the ISA value at sea level. On an ISA + ΔT day, the temperature at any altitude is given by.

$$T = T_0 + \Delta T - L \cdot H \quad (3.5)$$

and the pressure and density ratios are recalculated accordingly. Higher temperatures mean

lower air density, which increases ground roll and air distance. The tool accounts for this through the altitude and temperature deviation inputs.

Table 3.1 summarises the standard constants used throughout this thesis.

Table 3.1 ISA reference constants

Parameter	Symbol	Value	Unit
Sea-level temperature	T_0	288.15	K
Sea-level pressure	p_0	101325	Pa
Sea-level density	ρ_0	1.225	kg/m ³
Temperature gradient	L	$6.5 \cdot 10^{-3}$	K/m
Gravitational acceleration	g	9.80665	m/s ²
Gas constant, air	R_L	287.053	J/(kg·K)
Speed of sound, MSL	a_0	340.294	m/s

3.2 Aerodynamics

During take-off, the two relevant aerodynamic forces are lift and drag. Both depend on dynamic pressure, wing geometry, and dimensionless coefficients. This section presents the models used to compute these forces throughout the BFL calculation.

3.2.1 Lift and Drag Polar

The aerodynamic lift force L and drag force D are expressed through dimensionless coefficients C_L and C_D , which depend on the aircraft configuration, angle of attack, and Mach number. Both forces share the same expression for dynamic pressure q .

$$q = \frac{1}{2} \rho v^2 \quad (3.6)$$

The lift and drag forces are expressed as.

$$L = C_L \cdot q \cdot S_w \quad (3.7)$$

$$D = C_D \cdot q \cdot S_w \quad (3.8)$$

where S_W is the reference wing area. During the ground roll, the lift coefficient at a given speed is derived from the aircraft weight and the maximum lift coefficient C_{Lmax} , which corresponds to the stall speed V_{S1g} .

$$C_{L,max} = \frac{2W}{S_W \rho v_{S1g}^2} \quad (3.9)$$

The total drag coefficient C_D is composed of several contributions. The general drag polar used in this thesis follows the formulation of Krull (2025).

$$C_D = C_{D0} + \frac{C_L^2}{\pi A e} \quad (3.10)$$

where C_{D0} is the zero-lift drag coefficient, A is the wing aspect ratio, and e is the Oswald span efficiency factor. The first term accounts for all drag that does not depend on lift, and the second term is the induced drag, which grows with the square of the lift coefficient.

During take-off, the aircraft operates with extended flaps and the landing gear down, so C_{D0} includes several increments beyond the clean configuration value.

$$C_{D0} = C_{D0, clean} + \Delta C_{D0, f} + \Delta C_{D0, gear} \quad (3.11)$$

where $\Delta C_{D0, f}$ is the drag increment due to flap deflection and $\Delta C_{D0, gear}$ is the drag increment due to the extended landing gear. In the OEI case, three additional drag terms appear: the windmill drag of the failed engine $\Delta C_{D0, wm}$, the spillage drag of the inlet $\Delta C_{D0, sp}$, and the asymmetric drag $\Delta C_{D0, asym}$ due to the rudder deflection needed to counteract the yawing moment from asymmetric thrust (Young 2017, Lucht 2022).

$$C_{D0, OEI} = C_{D0} + \Delta C_{D0, wm} + \Delta C_{D0, sp} + \Delta C_{D0, asym} \quad (3.12)$$

The asymmetric drag term $\Delta C_{D0, asym}$ is the dominant OEI drag contribution and increases significantly at low speeds, since the rudder must deflect further to balance the yaw moment when dynamic pressure is low (Lucht 2022).

Ground effect reduces the induced drag when the aircraft is flying close to the ground. As the wing approaches the runway, the vortex sheet is distorted and vortex drag decreases. This is accounted through a correction factor ϕ .

$$\phi = \frac{(16 \frac{h_w}{b})^2}{1 + (16 \frac{h_w}{b})^2} \quad (3.13)$$

where h_w is the average height of the wing above the ground and b is the wingspan (Lucht 2022). The induced drag term in the ground roll phase is multiplied by ϕ , which takes values between 0 and 1, with $\phi \rightarrow 1$ as the aircraft climbs away from the ground.

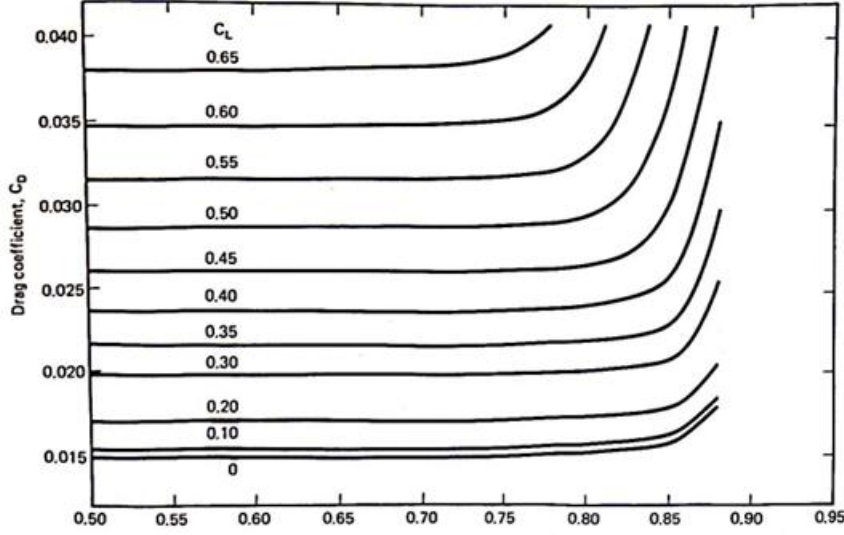


Figure 3.1 Drag coefficient vs. Mach number (Shevell 1989)

3.2.2 Induced Drag and Oswald Efficiency Factor

The Oswald span efficiency factor e corrects the aspect ratio for the deviation of the actual induced drag from the ideal elliptical lift distribution. A wing with a perfectly elliptical distribution has $e = 1$; real aircraft have values typically between 0.7 and 0.85 for clean configurations (Lucht 2022).

In this thesis, e is computed following the method of Niță (2012), which is an improvement over the simpler formulation of Lucht (2022) used by Lucht (2022). The Niță method distinguishes between two cases depending on the available input data.

Method 1 is used when C_{D0} is not yet known, which is the case during preliminary sizing. The Oswald factor is expressed as the product of a theoretical value and three corrections factors.

$$e = e_{theo} \cdot k_{e,F} \cdot k_{e,D0} \cdot k_{e,M} \quad (3.14)$$

The theoretical Oswald factor e_{theo} accounts for the inviscid induced drag of a tapered wing and is given by.

$$e_{\text{theo}} = \frac{1}{1 + f(\lambda) \cdot A} \quad (3.15)$$

where $f(\lambda)$ is a function of the taper ratio $\lambda = c_{\text{tip}}/c_{\text{root}}$, approximated by (Niță 2012) as.

$$f(\lambda) = 0.0524\lambda^4 - 0.15\lambda^3 + 0.1659\lambda^2 - 0.0706\lambda + 0.0119 \quad (3.16)$$

For a swept wing, the taper ratio λ is replaced by a corrected value $(\lambda - \Delta\lambda)$, where computed as following.

$$\Delta\lambda = -0.357 + 0.45e^{-0.0375\varphi_{25}} \quad (3.17)$$

and φ_{25} is the wing sweep angle at 25% chord, inserted in degrees (Niță 2012, Krull 2025). The three correction factors account for the following effects. The fuselage interference factor $k_{e,F}$ accounts for the drag increase caused by the fuselage interrupting the span-wise lift distribution. It depends on the ratio of fuselage diameter d_F to wingspan b .

$$k_{e,F} = 1 - 2\left(\frac{d_F}{b}\right)^2 \quad (3.18)$$

For all-jet transport aircraft, a representative value of $k_{e,F} = 0.974$ can be taken from Table 3.2 (Niță 2012). The viscous correction factor $k_{e,D0}$ accounts for the increase in induced drag due to the skin friction contribution and takes values around 0.873 for jet transport aircraft. Representative values for both factors are given in Table 3.2.

The compressibility correction factor $k_{e,M}$ captures the Mach number dependency of induced drag.

$$k_{e,M} = 1 + a_e \left(\frac{M}{M_{\text{crit}}} - 1 \right)^{b_e} \quad (3.19)$$

with generic parameter values $a_e = -0.00152$ and $b_e = 10.82$, and a reference Mach number $M_{\text{crit}} = 0.3$ (Niță 2012, Krull 2025). Since $a_e < 0$, this factor is less than 1 at Mach numbers above 0.3, reducing the effective Oswald factor with increasing speed. For the take-off phase, where Mach numbers remain well below 0.3, $k_{e,M} \approx 1$ and its effect is negligible.

The three geometry-dependent terms can be grouped into a single geometric Oswald factor.

$$e_{geo} = e_{theo} \cdot k_{e,F} \cdot k_{e,D0} \quad (3.20)$$

Since Method 1 does not require C_{D0} as input, it does not take into account the effect of flap deflection on induced drag. To correct for this, the geometric Oswald factor is scaled proportionally using the reference values given by Scholz (2015) for the clean configuration ($e = 0.85$) and with flaps extended ($e = 0.70$).

$$e = e_{geo} \cdot k_{e,M} \cdot \frac{0.70}{0.85} \quad (3.21)$$

Table 3.2 Oswald correction factors (Niță 2012)

Parameter	All Jet	Business Jet	Turboprop	General Aviation
dF/b	0.114	0.116	0.120	0.102
$k_{e,F}$	0.974	0.973	0.971	0.979
$k_{e,D0}$	0.873	0.873	0.864	0.804

Method 2 — C_{D0} known

When C_{D0} is already available from handbook calculations, the Oswald factor can be estimated with greater accuracy using a method that incorporates C_{D0} as a direct input. In this case, e is defined as (Niță 2012).

$$e = \frac{k_{e,M}}{Q + P\pi A} \quad (3.22)$$

The auxiliary terms Q and P are defined as.

$$Q = \frac{1}{e_{theo} \cdot k_{e,F}} \quad (3.23)$$

$$P = K \cdot C_{D0} \quad (3.24)$$

The definitions of e_{theo} , $k_{e,F}$, and $k_{e,M}$ are identical to those of Method 1. An additional factor K with a generic value of $K = 0.38$ is used in the denominator, which represents the slope of the viscous drag contribution (Niță 2012). This method can be extended to include twist effects, as described in Niță (2012b).

Both methods are available in the Excel tool, and the user can select either one for the calculation.

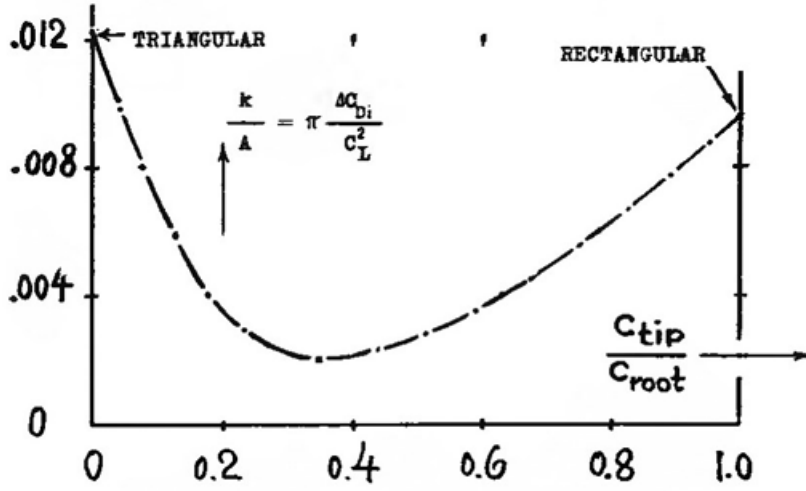


Figure 3.2 Oswald factor vs. taper ratio (Hoerner 1965)

3.2.3 Zero-Lift Drag Coefficient

The zero-lift drag coefficient C_{D0} represents all drag that does not depend on lift: skin friction, pressure drag, and interference drag from the various components of the airframe. In the clean configuration, with flaps retracted and gear up, $C_{D0, \text{clean}}$ is estimated using the method of Scholz (2015), which relates it to the maximum lift-to-drag ratio E_{max} of the aircraft.

$$C_{D0, \text{clean}} = \frac{\pi A e}{4 E_{\text{max}}^2} \quad (3.25)$$

The maximum lift to drag ratio is estimated as the following equation.

$$E_{\text{max}} = k_E \sqrt{\frac{A}{S_{\text{wet}}/S_W}} \quad (3.26)$$

where S_{wet}/S_W is the ratio of total wetted area to wing reference area and k_E is a factor that captures the aerodynamic efficiency level of the aircraft, defined as.

$$k_E = \frac{1}{2} \sqrt{\frac{\pi e}{C_{f, \text{eqv}}}} \quad (3.27)$$

where $c_{f,eqv}$ is the equivalent skin-friction drag coefficient, which represents the average friction drag per unit of wetted surface area. For jet transport aircraft, $c_{f,eqv} = 0.003$ is a representative value (Scholz 2015, Krull 2025). When k_E is not computed explicitly, a statistical value of $k_E = 15.8$ can be used for jet transport aircraft (Lucht 2022). Typical values of $C_{D0, clean}$ for transport jets lie in the range 0.015 to 0.04.

The wetted area ratio S_{wet}/S_w depends on the aircraft configuration and can be read from statistical data for different aircraft types. For the Airbus A320 and A340, the values used in this thesis are taken from Lucht (2022).

For the take-off configuration, two drag increments are added to $C_{D0, clean}$. The flap drag increment $\Delta C_{D0, f}$ is estimated for single-slotted Fowler flaps using the method of Nicolai (2010).

$$\Delta C_{D0, f} = k_1 k_2 \frac{S_{w, f}}{S_w} \quad (3.28)$$

where $S_{w, f}$ is the flapped wing area and k_1, k_2 are factors depending on flap angle, taken from statistical charts (Nicolai 2010, Lucht 2022). The landing gear drag increment $\Delta C_{D0, gear}$ is estimated from statistical mean values as a polynomial function of the flap angle δ_f . For large transports such as the A340, the expression is the following.

$$\Delta C_{D0, gear} = 4 \cdot 10^{-8} \delta_f^3 - 7.485 \cdot 10^{-6} \delta_f^2 + 3.542 \cdot 10^{-5} \delta_f + 2.551 \cdot 10^{-2} \quad (3.29)$$

and for small to medium transports such as the A320, the corresponding expression is.

$$\Delta C_{D0, gear} = -1.8 \cdot 10^{-6} \delta_f^2 - 3.48 \cdot 10^{-5} \delta_f + 1.57 \cdot 10^{-2} \quad (3.30)$$

based on the statistical data of Nicolai (2010) as applied by Lucht (2022). Numerical values of all drag coefficients for the A320 and A340 in each take-off configuration are given in Chapter 6.

3.3 Thrust Model

The net thrust produced by a turbofan engine decreases with increasing airspeed because the ram pressure at the inlet reduces the pressure ratio across the fan and reduces the net momentum change of the airflow. This speed dependency must be accounted for in the BFL calculation, since the net accelerating force changes continuously throughout the ground roll.

A simple and widely used approximation for the speed-dependent thrust of a jet engine is given by (Lucht 2022).

$$T(v) = N \cdot T_0[A - K_1 v + K_2 v^2] \quad (3.31)$$

where N is the number of engines, T_0 is the static thrust per engine at sea level, and A , K_1 , K_2 are coefficients that depend on the bypass ratio λ_{BPR} . With $A = 1$, the coefficients are the following form.

$$K_1 = [2.44 \times 10^{-4} \cdot \lambda_{BPR} + 1.66 \times 10^{-3}] \frac{1}{m/s} \quad (3.32)$$

$$K_2 = [6.16 \times 10^{-7} \cdot \lambda_{BPR} + 4.08 \times 10^{-6}] \frac{1}{(m/s)^2} \quad (3.33)$$

This formulation is valid for sea-level standard conditions and gives a reasonable approximation for the take-off speed range. However, it does not account for the effect of altitude on thrust. For airports at non-zero elevation, the reduced air pressure decreases the mass flow through the engine and therefore reduces the available thrust. To include this effect, the model of Bartel (2008) extends the thrust equation to include altitude through the pressure ratio p/p_0 .

$$T(M) = N \cdot T_0[A - K_1 M + K_2 M^2] \quad (3.34)$$

where M is the Mach number and the altitude-dependent coefficients are defined as.

$$k1 = \frac{0.377(1 + \lambda_{BPR})}{\sqrt{G(1 + 0.82\lambda_{BPR})}} \cdot Z \cdot \frac{p}{p_0} \quad (3.35)$$

$$k2 = (0.23 + 0.19\sqrt{\lambda_{BPR}}) \cdot X \cdot \frac{p}{p_0} \quad (3.36)$$

$$A = -0.4327 \left(\frac{p}{p_0}\right)^2 + 1.3855 \left(\frac{p}{p_0}\right) + 0.0472 \quad (3.37)$$

The auxiliary factors X and Z are defined as.

$$X = 0.1377 \left(\frac{p}{p_0} \right)^2 - 0.4374 \left(\frac{p}{p_0} \right) + 1.3003 \quad (3.38)$$

$$Z = 0.9106 \left(\frac{p}{p_0} \right)^2 - 1.7736 \left(\frac{p}{p_0} \right) + 1.8697 \quad (3.39)$$

The gas generator factor G in Equation (3.40) accounts for the thermodynamic cycle of the engine core and is estimated from the bypass ratio as (Bartel 2008).

$$G = 0.061\lambda_{BPR} + 0.633 \quad (3.40)$$

The Bartel (2008) model is used in this thesis for all calculations involving altitude variations, since it provides a physically consistent way to scale the thrust from sea-level static conditions to any pressure altitude. For sea-level standard-day conditions, both models give equivalent results.

During the ASD calculation, once the pilot moves the thrust levers to idle after the abort decision, the engines produce a residual idle thrust rather than zero thrust. Since idle thrust data for the A320 and A340 are not publicly available, it is estimated following Lucht (2022) as approximately 80% of the rolling friction force at standstill.

$$T_{idle} \approx 0.8 \cdot \frac{\mu \cdot m \cdot g}{N} \quad (3.41)$$

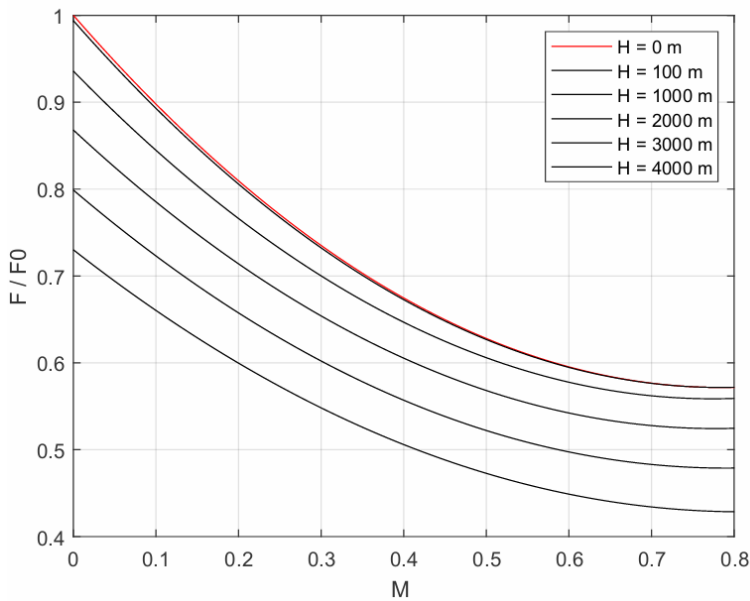


Figure 3.3 Speed-dependent thrust, Bartel (2008)

3.4 Equation of Motion on the Ground

During the ground roll, the aircraft accelerates along the runway under the combined action of thrust, aerodynamic lift and drag, rolling friction, and the weight component due to runway slope. Applying Newton's second law in the direction of motion gives the equation of motion (Lucht 2022).

$$m \cdot a = T - D - F_f - W \sin \gamma \quad (3.42)$$

where m is the aircraft mass, a is the acceleration, T is the net thrust, D is the aerodynamic drag, F_f is the friction force, $W = mg$ is the aircraft weight, and γ is the runway slope angle. The normal force N on the landing gear is obtained from the equilibrium of forces perpendicular to the runway surface.

$$N = W \cos \gamma - L \quad (3.43)$$

where L is the aerodynamic lift. Since the runway slopes considered in this thesis are small, $\cos \gamma \approx 1$ is a valid approximation. The friction force follows as.

$$F_f = \mu \cdot N = \mu \cdot (mg - L) \quad (3.44)$$

where μ is the rolling friction coefficient. For dry concrete or asphalt, $\mu = 0.02$ is the standard value used throughout this thesis, consistent with Lucht (2022) and Scholz (2015). Substituting Equations (3.43) and (3.44) into (3.42) gives the complete equation of motion for the ground roll.

$$m \cdot a = T - D - \mu(mg - L) - mg \sin \gamma \quad (3.45)$$

Figure 3.4 illustrates the force diagram for the ground roll with all contributing forces labeled.

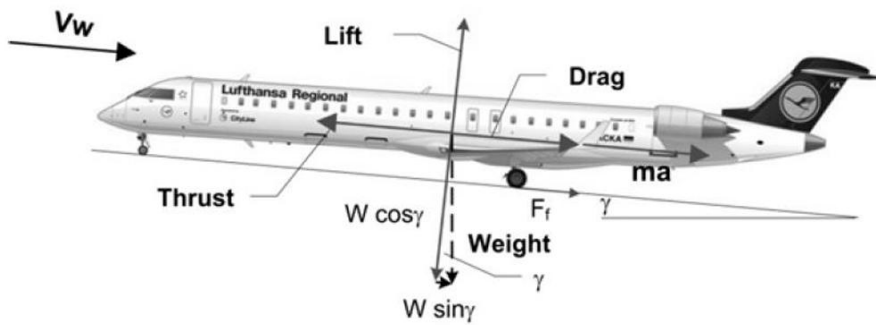


Figure 3.4 Ground roll force diagram

To obtain the ground roll distance, the equation of motion must be integrated over speed rather than time. Using the kinematic identity.

$$a = \frac{dv}{dt} = v \cdot \frac{dv}{dx} \quad (3.46)$$

The equation of motion becomes in terms of distance is.

$$m \cdot v \cdot \frac{dv}{dx} = T - D - \mu(mg - L) - mg \sin \gamma \quad (3.47)$$

Rearranging and integrating over the speed range from v_A to v_B gives the general integral for the ground roll distance.

$$S_g = \int_{v_A}^{v_B} \frac{m \cdot v}{T(v) - D(v) - \mu[mg - L(v)] - mg \sin \gamma} \quad (3.48)$$

This is the fundamental equation from which all ground roll distance calculations in this thesis are derived. The integration limits depend on the scenario: for the AEO ground roll, $v_A = 0$ and $v_B = V_2$. For the first OEI segment, $v_A = 0$ and $v_B = V_{EF}$; and for the second OEI segment, $v_A = V_{EF}$ and $v_B = V_2$.

The key step that distinguishes this thesis from the numerical approach of Lucht (2022) is the analytical solution of Equation (3.46). Since thrust, drag, and lift are all polynomial functions of speed, the integrand can be expressed in a standard algebraic form that is solvable in closed form.

3.5 Empirical Methods for TOFL Estimation

The methods described in Sections 3.1 to 3.4 form the physical basis of the BFL Calculator. However, several simpler empirical methods exist that estimate the TOFL directly from a small number of global aircraft parameters, without solving the equation of motion. These methods are used in Chapter 6 as secondary reference values. This section presents the five methods implemented in the BFL Calculator: the Torenbeek analytical approximation, the Loftin method, the modified Loftin method, the Kroo method, and the Takahashi (2025) method.

All four methods share the same underlying parameter grouping. The TOFL is primarily governed by the wing loading W/SW , the thrust-to-weight ratio T/W , and the maximum lift coefficient $C_{L,max}$. These are combined into an index variable x (Lucht 2022, Loftin 1980).

$$x = \frac{1}{\sigma \cdot C_{L,max,TO}} \cdot \frac{m_{MTO}/S_W}{T_{TO}/(m_{MTO} \cdot g)} \quad (3.49)$$

where σ is the density ratio at the airport altitude. This parameter groups the dominant influences on field length into a single value, making it a useful quantity for rapid estimates in the conceptual design phase.

Torenbeek Method

Torenbeek (1982) derives an analytical approximation for the BFL by solving a simplified equation of motion with average thrust. The result is a closed-form expression (Lucht 2022).

$$BFL = \frac{0.8863}{1 + 2.3G} \left(\frac{W/S}{\rho g C_{L,2}} + h_{SC} \right) \left(\frac{1}{T_{av}/W - u} + 2.7 \right) + \frac{655}{\sqrt{\rho/\rho_{SL}}} \quad (3.50)$$

where the parameter u with flaps in take-off position is defined as.

$$u = 0.01 \cdot C_{L,max,TO} + \mu \quad (3.51)$$

The average thrust for jet engines is given by the following equation.

$$T_{av} = 0.75 \cdot T_0 \left[\frac{5 + \lambda_{BPR}}{4 + \lambda_{BPR}} \right] \quad (3.52)$$

The factor G is the difference between the actual OEI climb angle γ_2 and the minimum climb gradient γ_{min} required by CS-25.121.

$$G = \gamma_2 - \gamma_{min} \quad (3.53)$$

The minimum climb gradient γ_{min} is 0.024 for two-engine aircraft and 0.030 for four-engine aircraft. The Torenbeek method requires knowledge of the drag polar, which makes it more demanding than the statistical methods below.

Loftin Method

Loftin (1980) proposed a simpler approach based on the observation that TOFL is approximately proportional to x .

$$s_{TOFL} = k_{TO} \cdot x \quad (3.54)$$

A statistical fit across jet transport aircraft gives $k_{TO} = 2.34$ (Lucht 2022). This method requires only three global parameters and gives a result in a single step, useful at very early design stages.

Modified Loftin Method

Lucht (2022) improves the Loftin method by fitting a linear regression to the same index variable x but allowing a non-zero intercept, unlike the original Loftin fit which forces the line through the origin. Using aircraft data from Lucht (2022), the modified equation is defined as the following expression.

$$s_{TOFL} = 1.876 \cdot x + 543.28 \quad (3.55)$$

This fit achieves a coefficient of determination of $R^2 = 0.8553$, with maximum deviations of -293 m to $+393$ m across the statistical sample. Lucht (2022) identifies this as the most accurate of the empirical methods evaluated, with deviations below 5.4% for the A320 and A340 reference cases.

Kroo Method

Kroo (2001) follows a similar approach but distinguishes between aircraft with two and four engines, and uses a quadratic fit rather than a linear one. The index variable is defined but evaluated with the thrust at $0.7 \cdot V_{LOF}$ rather than the static thrust, to account for the speed-dependent thrust loss during the ground roll.

The equation for two engines.

$$s_{TOFL,2eng} = 857.4 + 28.43 \cdot x + 0.0185 \cdot x^2 \quad (3.56)$$

For four engines expression is defined as.

$$s_{TOFL,2eng} = 486.7 + 26.20 \cdot x + 0.0083 \cdot x^2 \quad (3.57)$$

where x is in $\text{lbs} \cdot \text{ft}^{-2}$. The Kroo method generally overestimates the TOFL, with deviations of 1.9% to 13.9% reported by Lucht (2022) for the A320 and A340, but it captures the engine count effect that the Loftin method ignores.

Takahashi Method

Takahashi (2025) proposes a simulation-based empirical relationship for the Critical Field Length, calibrated against a dataset of certified flight manual values for contemporary transport aircraft. The method uses the Take-off Parameter TOP, defined as the following equation.

$$TOP = \frac{W/S}{T/W \cdot C_{L,max}} \quad (3.58)$$

where W/S is the wing loading in lb/ft^2 , computed as $(m/S_w) \cdot 0.20482$, T/W is the thrust-to-weight ratio, and $C_{L,max}$ is the maximum lift coefficient in the take-off configuration. The conversion factor 0.20482 converts kg/m^2 directly to lb/ft^2 and does not include the gravitational acceleration. The predicted Critical Field Length in feet follows as the following equation.

$$CFL = \max(750 + 31 \cdot TOP, 3300)\text{ft} \quad (3.59)$$

The 3300 ft floor reflects the V_{MCG} -limited minimum accelerate-stop distance and is not reached by any of the six aircraft studied here. The result is converted to meters by multiplying by 0.3048. This method is substantially more accurate than the legacy Roskam and Raymer correlations for large transport aircraft, with errors around 5% or less for widebody types. However, it underestimates significantly for regional jets, with an error of 17% reported by Takahashi (2025) for the E190-E2. In this thesis, the method shows a comparable pattern for the CRJ-900, with a deviation of -11.5% from the analytical BFL, consistent with Takahashi's own finding that regional jets are less well represented in the underlying regression. All five methods are implemented in the comparison block of the BFL Calculator and evaluated automatically for the active aircraft configuration. Their results are discussed alongside the analytical BFL in Chapter 6.

4 Analytical Calculation Method

This Chapter presents the equations used to calculate each distance component of the BFL. The starting point is the integral of the equation of motion from Section 3.4, solved following Lucht (2022). Speed-dependent thrust is included throughout, which removes the need for numerical integration. The result is a closed-form calculation covering the ground roll, rotation, air distance, and stop distance.

4.1 Ground Roll Distance – All Engines Operative (AEO)

The AEO ground roll distance covers the acceleration from standstill to rotation speed V_2 with all engines delivering full take-off thrust. Substituting the speed-dependent expressions for thrust $T(v)$, drag $D(v)$, and lift $L(v)$ from Chapter 3 into the integral of Equation (3.48) results in the following expression.

$$S_{g,AEO} = \int_0^{v_R} \frac{m \cdot v}{NT_0[A - K_1v + K_2v^2] - \frac{\rho}{2}C_{D,g}S_wv^2 - mg(\mu + \sin \gamma) + \mu\frac{\rho}{2}C_{L,g}S_wv^2} dv \quad (4.1)$$

Grouping the terms by power of V , the denominator takes the form $aV^2 + BV + c$, giving an integral of the standard type (Lucht 2022).

$$\int \frac{x dx}{ax^2 + bx + c} \quad (4.2)$$

with the coefficients of the denominator defined as.

$$a = NT_0K_2 - \frac{\rho}{2}C_{D,g}S_wv^2 + \mu\frac{\rho}{2}C_{L,g}S_wv^2 \quad (4.3)$$

$$b = -NT_0K_1 \quad (4.4)$$

$$c = NT_0A - mg(\mu + \sin \gamma) \quad (4.5)$$

The solution depends on the sign of the discriminant $\Delta = 4ac - b^2$ (Papula 2015, Goudreault 2013).

For the case where $\Delta > 0$, the analytical solution of the integral takes the following form.

$$\int \frac{x dx}{ax^2 + bx + c} = \frac{1}{2a} \ln|ax^2 + bx + c| - \frac{b}{2a} \cdot \frac{2}{\sqrt{\Delta}} \arctan\left(\frac{2x + b}{\sqrt{\Delta}}\right) \quad (4.6)$$

For the case where $\Delta < 0$, the analytical solution of the integral takes the following form.

$$\int \frac{x dx}{ax^2 + bx + c} = \frac{1}{2a} \ln|ax^2 + bx + c| - \frac{b}{2a} \cdot \frac{1}{\sqrt{\Delta}} \ln \left| \frac{2ax + b - \sqrt{|\Delta|}}{2ax + b + \sqrt{|\Delta|}} \right| \quad (4.7)$$

For the case where $\Delta = 0$, the analytical solution of the integral takes the following form.

$$\int \frac{x dx}{ax^2 + bx + c} = \frac{1}{2a} \ln|ax^2 + bx + c| - \frac{b}{2a} \cdot \frac{2}{2ax + b} \quad (4.8)$$

In practice, for typical jet transport aircraft at take-off, $\Delta < 0$ and Equation (4.7) applies. The ground roll distance is then evaluated between the limits $v_A = 0$ and $v_B = V_2$. The ground effect correction factor ϕ from Equation (3.15) is applied to the induced drag term in $C_{D,g}$ throughout the integration.

4.2 Rotation Distance

The rotation distance s_R covers the phase from V_2 to lift-off speed V_{LOF} , during which the pilot applies nose-up elevator and the aircraft pitches to the lift-off angle of attack. The rotation is modeled as a phase of approximately constant speed, equal to the average of V_2 and V_{LOF} , lasting for a fixed rotation time t_R (Lucht 2022, Nicolai 2010).

$$s_R = \frac{v_R + v_{LOF}}{2} \cdot t_R \quad (4.9)$$

The rotation time t_R is not directly prescribed by CS-25 but is determined from the angular velocity of the pitch-up maneuver. Following Lucht (2022), t_R is computed from the rotational dynamics of the aircraft about the main gear contact point, accounting for the nose up pitching moment applied by the elevator, the aerodynamic pitching moment, and the moment of inertia. Typical values for large transport aircraft lie between 2 and 3 seconds. Table 4.1 gives the values used for the A320 and A340.

Table 4.1 Rotation times, A320-200 and A340-300

Aircraft	Config	$t_{R,AEO}$ [s]	$t_{R,OEI}$ [s]
A320-200	CONF 1+F	3.83	4.50
A340-300	CONF 1+F	4.50	5.50

The lift-off speed V_{LOF} is the speed at which the lift force equals the aircraft weight at the lift-off angle of attack α_{LOF} . It must satisfy the regulatory requirement $V_{LOF} \geq 1.08 \cdot V_{MCA}$ for the AEO case (CS-25.107). In practice V_{LOF} is typically 3 to 5 kt above V_2 (Lucht 2022).

For the OEI case, the rotation distance is calculated using the same formula 4.9 but with a slightly higher $V_{LOF,OEI}$, since the lower available thrust and higher drag result in a different pitch response.

4.3 Air Distance

The air distance s_{AIR} covers the trajectory from lift-off to the screen height of 35 ft. It must be reached at the safety speed V_2 before the screen height is achieved, as required by CS-25.111. The trajectory is modeled geometrically following Lucht (2022): after lift-off the aircraft follows a circular arc, and then a straight climb segment until the screen height is reached.

The arc is flown at a load factor increment of 0.15, which is the value conventionally assumed for this maneuver. The turn radius follows from the balance between centripetal acceleration, and this load factor increment.

$$R = \frac{V_{LOF}^2}{0.15 \cdot g} \quad (4.10)$$

The climb angle θ is obtained from the thrust and drag balance at the lift-off speed.

$$\theta = \arcsin\left(\frac{T - D}{W}\right) \quad (4.11)$$

For the OEI case, the thrust of the remaining engines and the drag increment of the failed engine are used. The height gained inside the arc follows from the geometry of the circle.

$$h_{arc} = R \cdot (1 - \cos\theta) \quad (4.12)$$

Two cases arise depending on whether the screen height is reached inside or outside the arc. If $h_{arc} \geq h_{SC}$, the aircraft reaches the screen height while still inside the arc and the straight climb segment vanishes. The horizontal air distance is given by.

$$s_{AIR} = \sqrt{R^2 - (R - h_{SC})^2} \quad (4.13)$$

If $h_{arc} < h_{SC}$, the arc is completed and the remaining height is covered by the straight climb segment at the constant angle θ . The two contributions are the following equations.

$$s_{arc} = R \cdot \sin \theta \quad (4.14)$$

$$s_{climb} = \frac{h_{SC} - h_{arc}}{\tan \theta} \quad (4.15)$$

and the total air distance follows as.

$$s_{AIR} = s_{arc} + s_{climb} \quad (4.16)$$

The same geometric construction is applied to both the OEI and AEO cases, with the corresponding values of thrust, drag, and lift-off speed.

4.4 One-Engine-Inoperative Take-off Distance

The OEI take-off distance is the sum of the two segments described in Section 2.2.2. The ground roll is divided into an AEO phase from standstill to V_{EF} , followed by an OEI phase from V_{EF} to V_2 , each solved using Equation (4.6) with the appropriate drag polar and thrust (AEO or OEI). The rotation distance and air distance are then added to the final expression.

$$s_{TOD,OEI} = s_{g,AEO}(0 \rightarrow v_{EF}) + s_{g,OEI}(v_{EF} \rightarrow v_R) + s_{R,OEI} + s_{AIR,OEI} \quad (4.17)$$

4.5 Accelerate-Stop Distance

The ASD is calculated following the time sequence defined in CS-25.109 and summarized in Table 2.1. The acceleration phase from standstill to V_1 is solved analytically using equation 4.6 with AEO drag and thrust, as for the first segment of the OEI ground roll. After V_1 is reached,

the AFM buffer applies for the A340 (Amendment 25-92) the aircraft maintains constant speed V_1 for 2 seconds, while for the A320 (Amendment 25-42) it continues to accelerate.

The stop distance from the first braking action to a standstill is also solved analytically. During the stop phase, the thrust is reduced to idle, and the braking force replaces rolling friction as the retarding force. Substituting the braking force $F_B = \mu_B \cdot N$ and the idle thrust T_{idle} into the equation of motion and applying the same integral technique, the stop distance takes the same algebraic form as Equation (4.6), with the denominator coefficients redefined accordingly (Lucht 2022).

$$a_{STOP} = -\frac{\rho}{2}(C_{D,g} + \mu_B C_{L,g})S_W \quad (4.18)$$

$$b_{STOP} = -NT_0K_1 \quad (4.19)$$

$$c_{STOP} = T_{idle} - mg(\mu_B + \sin \gamma) \quad (4.20)$$

The total ASD follows as.

$$S_{ASD} = s_{g,AEO}(0 \rightarrow v_1) + s_{buffer} + s_{stop}(v_1 \rightarrow 0) \quad (4.21)$$

where $s_{buffer} = V_1 \cdot \Delta t_{buffer}$ for Amendment 25-92 and is calculated by continued AEO acceleration over Δt_{buffer} for Amendment 25-42.

4.6 Analytical Solution for Balanced Field Length

The BFL is found by evaluating $s_{TOD,OEI}$ and s_{ASD} as functions of V_I over a range of speeds between V_{MCG} and V_2 , and identifying the speed $V_{1,bal}$ at which the two distances are equal. Both functions are computed analytically using the equations derived in Sections 4.4 and 4.5, so no numerical iteration is required: both curves are evaluated at a set of discrete V_1 values, and the crossing point is found by linear interpolation (Lucht 2022).

$$s_{BFL} = s_{ASD}(v_{1,bal}) = s_{TOD,OEI}(v_{1,bal}) \quad (4.22)$$

This is the core analytical result of this thesis. The full calculation chain, from aircraft parameters and atmospheric conditions to the final BFL and $V_{1,bal}$, is implemented in the Excel tool described in Chapter 5.

Figure 4.1 shows the BFL calculation result for the Airbus A320 at MTOW and sea-level standard conditions, with both distance curves plotted as a function of V_1 .

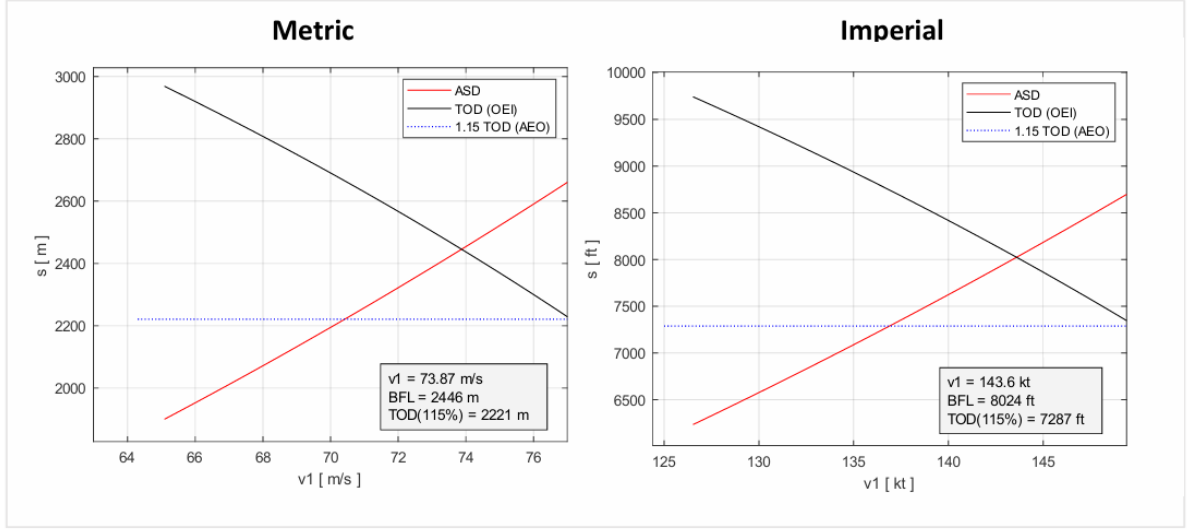


Figure 4.1 ASD and OEI-TOD vs. V_1 , A320-200, MTOW (Lucht 2022)

4.7 Take-off Field Length

The TOFL is the final output of the calculation and is determined following CS-25.113 as defined in Equation (2.2). Once the BFL and the factored AEO take-off distance are known, the TOFL is simply the following expression.

$$s_{TOFL} = \max(s_{BFL}, 1.15 \cdot s_{TOD,AEO}) \quad (4.23)$$

For two-engine aircraft such as the A320, the BFL is typically the governing distance. For four engine aircraft such as the A340, the factored AEO distance often governs, since the OEI thrust loss is proportionally smaller.

5 Excel Tool – Description and Implementation

The analytical equations derived in Chapter 4 have been implemented in a Microsoft Excel spreadsheet titled *BFL_Calculator*. The tool computes the Balanced Field Length and the Take-off Field Length for jet transport aircraft in a single worksheet, without requiring any numerical solver or iterative procedure. All calculations are performed through closed-form Excel formulas, making every intermediate result visible and traceable. The workbook contains three sheets: *BFL_Calculator*, which holds the full calculation; *Documentation*, which explains each formula block with its physical meaning and source reference; and *(c)*, which contains the copyright notice.

5.1 Structure of the Spreadsheet

The calculation sheet is organized into four column groups. The left group (columns B–F) holds the user inputs and their labels, symbols, units, and source references. The center-left group (columns G–I) holds all derived quantities, updated automatically when any input changes. The center-right group (columns J–L) holds the formula explanations and physical meaning of each block. The right group (columns M–W) holds the reference data for the six sample aircraft and a Custom column for user-defined aircraft. Figure 5.1 shows the complete view of the tool. Because the complete sheet spans many columns, the figure is provided for orientation only; each functional block is shown in detail in the figures that follow.

BALANCED FIELD LENGTH CALCULATOR — analytical (closed-form) solution									
Select aircraft →		A340-300		This is a dropdown — click and pick an aircraft or 'Custom' (last aircraft sample data column) to enter your own values. Full instructions: click the title of Section 1.					
Parameter	Symbol	Value	Unit	Derived Quantity	Value	Unit	Formula and physical meaning		
AIRCRAFT & GEOMETRY							1. HOW TO USE THIS GUI		
Mass (MTOW)	m	271000	kg	Oswald clean [e_clean]	0.645097632	—	Pick an aircraft in D2 (6 presets + Custom). Full instructions → click the title above.		
Weight W = m·g	W	2657602.15	N	Oswald take-off [e_TO]	0.645097632	—	2. ATMOSPHERE (ISA)		
Wing area	S _w	363.1	m²	Induced drag factor [k_TO]	0.032593237	—	T[m] = T0 - L·H σ = p/p0 = (T/T0)^(g0/(R(L·U - 1))		
Wingspan	b	58	m	CD0 ground = clean + gear + flap	0.046055495	—	3. AERODYNAMICS		
Aspect ratio	A	9.26	—	CDp (ground polar)	0.046540154	—	q = ½ρV² L = CL·q·S _w CD = CD0 + CL²/(π·A·e)		
Wing sweep A/c/4	φ ₂₁	29.7	deg	CD0a (wing profile drag)	0.01484305	—	CD0, TO = CD0, clean + ΔCD0, gear + ΔCD0, flap CD0, OEI = CD0, OEI (single term, 0 unless mo		
Taper ratio	λ	0.251	—	CD0 airclean + flap (gear up)	0.02191	—	4. OSWALD FACTOR - Method 1 [geometric]		
Fuselage diameter	d _f	5.64	m	REFERENCE SPEEDS			e _{theo} = 1/(1+(h·(A-Δ))·A) (h·(A-Δ)) = 0.52(A-Δ)·(A-Δ)²/(1+(A-Δ)²) · 0.1859/(h·(A-Δ)) · 0.0708/(h·(A-Δ)) · 0.119		
AERODYNAMICS							ΔL = 1/(1+(h·(A-Δ))·A) · 0.1859/(h·(A-Δ)) · 0.0708/(h·(A-Δ)) · 0.119		
CL _{max} (take-off config)	C _{L,max}	2.24	—	Safety speed v ₂ = 1.13·v _{s1g}	82.38798907	m/s	e = e _{theo} · ke · F ₁ · ke · D ₀ · ke · M ₀ · (0.70/0.85)		
CD0 clean	C _{D0, clean}	0.019	—	Rotation speed [V _R]	80.84468907	m/s	5. OSWALD FACTOR - Method 2 [CDO-based, <u>used for all validated results</u>]		
Oswald method (1 = geom - 2 = CDO)	method	2	1 2	Lift-off speed [V _{LOF, OEI}]	82.38798907	m/s	Q = 1/(1+(h·(A-Δ))·A) · 0.1859/(h·(A-Δ)) · 0.0708/(h·(A-Δ)) · 0.119		
Oswald factor e (selected)	e	0.645097632	—	U _{LOF} speed [V _{LOF, AEO}]	84.96018907	m/s	Q = 1/(1+(h·(A-Δ))·A) · 0.1859/(h·(A-Δ)) · 0.0708/(h·(A-Δ)) · 0.119		
ΔCD0 gear	ΔC _{D0, gear}	0.024145495	—	V _{MCG}	56.58901967	m/s	e = e _{theo} · ke · F ₁ · ke · D ₀ · ke · M ₀ · (0.70/0.85)		
ΔCD0 flap	ΔC _{D0, flap}	0.003	—	AEO GROUND ROLL COEFFICIENTS			6. OSWALD FACTOR - Method 2 [CDO-based, <u>used for all validated results</u>]		
ΔCD0 additional (asym. + windmill, OEI)	ΔC _{D0, OEI}	0.004	—	a _g [AEO]	-6.577924507	N/(m/s)²	Q = 1/(1+(h·(A-Δ))·A) · 0.1859/(h·(A-Δ)) · 0.0708/(h·(A-Δ)) · 0.119		
CL _{ground}	C _{L,g}	0.1	—	b _g [AEO]	-1781.450208	N·s/m	7. THRUST & ENGINES		
Flap angle (take-off config)	δ _f	17	deg	c _g [AEO]	500659.957	N	Static thrust / engine		
THRUST & ENGINES							T ₀		
Static thrust / engine	T ₀	138800	N	Δ _g (discriminant)	-16346778.45	—	Idle thrust / engine		
Idle thrust / engine	T _{idle}	10630.4086	N	OEI GROUND ROLL COEFFICIENTS			T _{idle}		
N engines (total)	N	4	—	a _{OEI}	-8.299457027	N/(m/s)²	N engines (total)		
N engines (OEI)	N _{OEI}	3	—	b _{OEI}	-1336.087656	N·s/m	N engines (OEI)		
Bypass ratio	BPR	6	—	c _{OEI}	362206.957	N	Bypass ratio		
Throttle ratio	θ	0.9975	—	Δ _{OEI} (discriminant)	-13809614.52	—	Throttle ratio		
Thrust coefficient K ₁	K ₁	1.094625853	—	BRAKING COEFFICIENTS			Thrust coefficient K ₁		
Thrust coefficient K ₂	K ₂	0.695820293	—	a _{sid} (Lucht 5.87)	-3.012206316	N/(m/s)²	Thrust coefficient K ₂		
Thrust coefficient A ₁	A ₁	1	—	b _{sid}	-136.4376341	N·s/m	Thrust coefficient A ₁		
GROUND & BRAKING							c _{sid}		
Rolling friction	μ _{GR}	0.02	—	Δ _{sid} (discriminant)	9669013.248	—	Rolling friction		
Brake coefficient	μ _B	0.35	—	OSWALD CALCULATIONS			Brake coefficient		
Gear load factor	q _{LG}	0.91	—	ΔΔ shift	-0.209253557	—	Gear load factor		
Screen height (35 ft)	h _{sc}	10.7	m	f(Δ) - ΔΔ	0.002276057	—	Screen height (35 ft)		
E _{max} = v(t·A·e)/(4·CD0, clean)	E _{max}	1778324274	—	e _{theo}	0.979348572	—	E _{max} = v(t·A·e)/(4·CD0, clean)		
V _{MCG} (min. control speed, ground)	V _{MCG}	110	kt	k ₁ = F ₁ - 1/2(g ₁ /h²)	0.981088328	—	V _{MCG} (min. control speed, ground)		
TIMING (Amendment 25-42)							k ₂ = M ₀ (M ₀ ≤ 0.873)		
Recognition time	t _{recog}	1	s	k ₃ = M ₀ (M ₀ ≤ 0.3 → 1)	1	—	Recognition time		
AFM buffer time	t _{buf}	2	s	M1: e (geometric)	0.690778349	—	AFM buffer time		
							M2: Q = 1/(e _{theo} ·k ₁ ·e _f)		
							M2: P = 0.38·CD0		
							0.017501088		

Figure 5.1 Complete view of the BFL Calculator spreadsheet

The sheet is organized into the following functional blocks, each identified by a bold header row.

The **Aircraft & Geometry** block collects the geometric parameters: take-off mass m , weight W , wing area S_w , wingspan b , aspect ratio A (computed from b^2/S_w), wing sweep ϕ_{25} , taper ratio λ , and fuselage diameter d_f . Figure 5.2 shows this block.

AIRCRAFT & GEOMETRY			
Mass (MTOW)	m	271000.	kg
Weight $W = m \cdot g_0$	W	2657602.15	N
Wing area	S_w	363.1	m ²
Wingspan	b	58.	m
Aspect ratio	A	9.26	—
Wing sweep $\Lambda(c/4)$	ϕ_{25}	29.7	deg
Taper ratio	λ	0.251	—
Fuselage diameter	d_f	5.64	m

Figure 5.2 Aircraft and Geometry block of the BFL Calculator

The **Aerodynamics** block contains the lift and drag inputs: $C_{L,max}$ for the selected take-off configuration, $C_{D0,clean}$, the Oswald factor method selector (1 = geometric, 2 = CD0-based), the active Oswald factor e , the gear drag increment $\Delta C_{D0,gear}$, the flap drag increment $\Delta C_{D0,f}$, a consolidated additional OEI drag cell $\Delta C_{D,OEI}$ for windmill and asymmetric drag, the ground lift coefficient $C_{L,g}$, and the flap angle δ_f . Figure 5.3 shows this block.

AERODYNAMICS			
CL_max (take-off config)	$C_{L,max}$	2.24	—
CD0 clean	$C_{D0,clean}$	0.019	—
Oswald method (1 = geom · 2 = CD0)	method	2	1 2
Oswald factor e (selected)	e	0.645097632	—
$\Delta CD0$ gear	$\Delta C_{D0,gear}$	0.024145495	—
$\Delta CD0$ flap	$\Delta C_{D0,f}$	0.003	—
$\Delta CD0$ additional (asym. + windmill, OE)	$\Delta C_{D,OEI}$	0.004	—
CL_ground	$C_{L,g}$	0.1	—
Flap angle (take-off config)	δ_f	17.	deg

Figure 5.3 Aerodynamics block of the BFL Calculator

The **Thrust & Engines** block collects the propulsion parameters: static thrust per engine T_0 , idle thrust per engine T_{idle} , total number of engines N , number of operative engines in the OEI case N_{OEI} , bypass ratio BPR, throttle ratio G , and the thrust coefficients K_1 , K_2 , and A_t following Bartel and Young (2008). Figure 5.4 shows this block.

THRUST & ENGINES			
Static thrust / engine	T_0	138800.	N
Idle thrust / engine	T_{idle}	10630.4086	N
N engines (total)	N	4.	—
N engines (OEI)	N_{OEI}	3	—
Bypass ratio	BPR	6.	—
Throttle ratio	G	0.9975	—
Thrust coefficient K_1	K_1	1.094625853	—
Thrust coefficient K_2	K_2	0.695820293	—
Thrust coefficient A_t	A_t	1	—

Figure 5.4 Thrust and Engines block of the BFL Calculator

The **Ground & Braking** block contains the rolling friction coefficient μ_{GR} , the braking coefficient μ_B , the gear load factor ϕ_{LG} , the screen height h_{SC} , the maximum lift-to-drag ratio E_{max} , and the minimum control speed on the ground V_{MCG} . Figure 5.5 shows this block.

GROUND & BRAKING			
Rolling friction	μ_{GR}	0.02	—
Brake coefficient	μ_B	0.35	—
Gear load factor	ϕ_{LG}	0.91	—
Screen height (35 ft)	h_{sc}	10.7	m
$E_{max} = v(\pi \cdot A \cdot e / (4 \cdot CD0, clean))$	E_{max}	17.78324274	—
VMCG (min. control speed, ground)	V_{MCG}	110.	kt

Figure 5.5 Ground and Braking block of the BFL Calculator

The **Timing** block specifies the recognition time t_{recog} , the AFM buffer time t_{buf} , and the rotation times for the AEO and OEI cases, following Amendment 25-42. Figure 5.6 shows this block.

TIMING (Amendment 25-42)			
Recognition time	t_{recog}	1.	s
AFM buffer time	t_{buf}	2.	s
Rotation time AEO	$t_{R,AEO}$	4.5	s
Rotation time OEI	$t_{R,OEI}$	5.5	s

Figure 5.6 Timing block of the BFL Calculator

The **Atmosphere** block consists of two sub-blocks. The **ISA Constants** sub-block stores the fixed reference values: sea-level temperature T_0 , pressure p_0 , density ρ_0 , temperature lapse rate L , specific gas constant R , and ratio of specific heats γ . The **Derived Atmosphere** sub-block computes the temperature, pressure, pressure ratio σ_p , and the thrust scaling coefficients Z_t and X_t at the specified airport altitude, using the equations of Section 3.1. Figure 5.7 shows both sub-blocks.

ISA CONSTANTS			
ISA sea-level temperature	T_0	288.15	K
ISA sea-level pressure	p_0	101325	Pa
ISA sea-level density	ρ_0	1.225	kg/m ³
Temperature lapse rate	L	0.0065	K/m
Specific gas constant	R	287.053	J/(kg·K)
Ratio of specific heats	γ	1.4	—
DERIVED ATMOSPHERE			
Temperature at altitude	T_h	288.15	K
Pressure at altitude	p_h	101325	Pa
Pressure ratio	σ_p	1	—
Thrust coefficient Z_t	Z_t	1.0067	—
Thrust coefficient X_t	X_t	1.0006	—

Figure 5.7 ISA Constants and Derived Atmosphere blocks of the BFL Calculator

The center group begins with the **Derived Aerodynamics** block, which displays the computed Oswald factors for both clean and take-off configurations, the induced drag factor k_{TO} , the total ground drag polar CD_{0t} , and the ground polar CD_p . Below it, the **Reference Speeds** block shows V_2 , V_R , $V_{LOF,OEI}$, $V_{LOF,AEO}$, and V_{MCG} in m/s. Figure 5.8 shows both blocks.

DERIVED AERODYNAMICS		
Oswald clean [e_clean]	0.645097632	—
Oswald take-off [e_TO]	0.645097632	—
Induced drag factor [k_TO]	0.053259237	—
CD0 ground = clean + gear + flap	0.046055495	—
CDp (ground polar)	0.046540154	—
CD0a (wing profile drag)	0.01484305	—
CD0 air = clean + flap (gear up)	0.02191	—
REFERENCE SPEEDS		
Safety speed $v_2 = 1.13 \cdot v_{s1g}$	82.38798907	m/s
Rotation speed [V_R]	80.84468907	m/s
Lift-off speed [V_LOF,OEI]	82.38798907	m/s
Lift-off speed [V_LOF,AEO]	84.96018907	m/s
VMCG	56.58901967	m/s

Figure 5.8 Derived Aerodynamics and Reference Speeds blocks of the BFL Calculator

The **AEO Ground Roll Coefficients**, **OEI Ground Roll Coefficients**, and **Braking Coefficients** blocks display the polynomial coefficients a , b , c and the discriminant Δ for the three distance integrals, as defined in Chapter 4. Figure 5.9 shows these blocks.

AEO GROUND ROLL COEFFICIENTS		
a_d [AEO]	-6.577924507	N/(m/s) ²
b_d [AEO]	-1781.450208	N·s/m
c_d [AEO]	500659.957	N
Δ_d (discriminant)	-16346778.45	—
OEI GROUND ROLL COEFFICIENTS		
a_OEI	-8.299457027	N/(m/s) ²
b_OEI	-1336.087656	N·s/m
c_OEI	362206.957	N
Δ_OEI (discriminant)	-13809614.52	—
BRAKING COEFFICIENTS		
a_sid (Lucht 5.87)	-3.012206316	N/(m/s) ²
b_sid	-136.4376341	N·s/m
c_sid	-804030.9545	N
Δ_sid (discriminant)	9669013.248	—

Figure 5.9 Ground roll and braking coefficient blocks of the BFL Calculator

The **Oswald Calculations** block shows all intermediate values of the Niță and Scholz (2012) method: the taper ratio sweep correction $\Delta\lambda$, the polynomial function $f(\lambda - \Delta\lambda)$, the theoretical Oswald factor e_{theo} , the correction factors $k_{e,F}$, $k_{e,D0}$, and $k_{e,M}$, and the results of both Method 1 and Method 2 with the active selection highlighted. Figure 5.10 shows this block.

OSWALD CALCULATIONS		
Δλ shift	-0.209253557	—
f(λ-Δλ)	0.002276057	—
e_theo	0.979348572	—
$k_{e,F} = 1 - 2(d_f/b)^2$	0.981088228	—
$k_{e,D0}$ (jet = 0.873)	0.873	—
$k_{e,M}$ ($M \leq 0.3 \rightarrow 1$)	1	—
M1: e (geometric)	0.690778349	—
M2: $Q = 1/(e_{\text{theo}} \cdot k_{e,F})$	1.040769702	—
M2: $P = 0.38 \cdot CD0$	0.017501088	—
M2: e (CD0-based)	0.645097632	—
→ selected e	0.645097632	—
e_ground (clean + gear + flap)	0.645097632	—
e_air (gear up: clean + flap)	0.779363204	—

Figure 5.10 Oswald Calculations block of the BFL Calculator

The **Fixed Distances** block holds all distance components that do not depend on V_1 : the AEO ground roll $s_{GR,AEO}$, the OEI and AEO rotation distances $s_{ROT,OEI}$ and $s_{ROT,AEO}$, and the complete geometric air distance model for both the OEI and AEO cases, as described in Section 4.3. The factored AEO distance $1.15 \cdot s_{TOD,AEO}$ is also computed here. Figure 5.11 shows this block.

AIR DISTANCE — GEOMETRIC MODEL · OEI		
CL at v_2 [C_L,v2]	1.760474825	—
Thrust OEI at V_LOF,OEI [T_OEI]	322222.4533	N
CD OEI at C_L,v2 [C_D,OEI]	0.162538132	—
Drag OEI at V_LOF,OEI [D_OEI]	245366.5807	N
Climb angle OEI $\theta = \text{asin}((T-D)/W)$	0.028923287	rad
Turn radius OEI $R = V_{LOF}^2/(0.15 \cdot g_0)$	4614.406716	m
Height gained in arc (OEI)	1.929971469	m
s_arc (OEI)	133.4452016	m
s_climb straight segment (OEI)	303.1323133	m
s_AIR_OEI (total)	436.5775149	m
AIR DISTANCE — GEOMETRIC MODEL · AEO		
Thrust AEO at V_LOF,AEO [T_AEO]	426480.1314	N
CD AEO at C_L,v2 [C_D,AEO]	0.158538132	—
Drag AEO at V_LOF,AEO [D_AEO]	254505.4099	N
Climb angle AEO θ_{AEO}	0.064755731	rad
Turn radius AEO $R = V_{LOF,AEO}^2/(0.15 \cdot g_0)$	4907.033307	m
Height gained in arc (AEO)	10.28474833	m
s_arc (AEO)	317.5365006	m
s_climb straight segment (AEO)	6.403619766	m
s_AIR_AEO (total)	323.9401203	m

Figure 5.11 Fixed Distances block of the BFL Calculator

The **Totals and CS-25 Compliance** block computes the one-engine-inoperative climb gradient from the climb angle θ obtained in the air distance model and compares it against the 2.4% minimum required by CS-25.121(b), returning PASS or FAIL. Figure 5.12 illustrates this calculation block, and Table 5.1 summarizes the resulting compliance data for all six aircraft under both Oswald methods.

TOTALS & CS-25 COMPLIANCE		
1.15 × TOD_AEO	2231.619789	m
Climb gradient OEI	2.435292508	%
CS-25.121(b) check ($\geq 2.4\%$ twin)	PASS	—

Figure 5.12 Totals and CS-25 Compliance block of the BFL Calculator

Aircraft	Grad.M1	M1	Grad. M2	M2
A320-200	1.39%	FAIL	2.25%	FAIL
A340-300	1.90%	FAIL	2.89%	PASS
B737-800	2.05%	FAIL	2.84%	PASS
B777-300ER	3.82%	PASS	4.85%	PASS
E190	3.03%	PASS	4.00%	PASS
CRJ-900	4.13%	PASS	4.98%	PASS

Table 5.1 Climb gradient compliance for all sample aircraft

Method 1 fails for three of the six aircraft, including both validation types. Method 2 fails only for the A320-200, and by a small margin (2.25% against the 2.4% threshold). This shortfall is a known limitation of the air distance model: the climb angle θ is evaluated at a single point (V_{LOF}) rather than integrated over the full climb trajectory, which underestimates the effective

gradient. Since all six aircraft are certified types, Method 1 results are physically inadmissible, and Method 2 is used for all results reported in this thesis.

The **BFL 2-Point Interpolation** block evaluates $ASD(V_1)$ and $s_{TOD,OEI}(V_1)$ at two discrete values of V_1 : Point A at $0.88 \cdot V_R$ and Point B at $0.98 \cdot V_R$. The balanced speed $V_{1,bal}$ is found by linear interpolation of the crossing point. The result is passed to the final block. Figure 5.13 shows this block.

Point	V_1 (m/s)	V_1 (kt)	s_{GR_AEO} (0→ V_1)	$s_{reaction}$ ($V_1 \times 3s$)	s_{brake}	ASD	s_{GR_OEI} ($V_1 \rightarrow V_R$)	TOD_OEI	$1.15 \times TOD$ (m)
Point A (0.88· V_R)	68.107443	132.3899726	996.5801	204.3223	772.7182263	1973.620667	844.0956	2683.37	2231.61979
Point B (0.98· V_R)	75.846926	147.4342877	1270.502	227.5408	956.5912024	2454.633674	154.7465	2267.942	2231.61979

Figure 5.13 BFL two-point interpolation block of the BFL Calculator

To the right of the interpolation block, the **Comparison with Analytical/Empirical Methods** block shows the TOFL predicted by the Torenbeek, Loftin, modified Loftin, and Kroo methods alongside the BFL Calculator result, with the absolute and percentage deviation from the GUI value. This allows a direct comparison for any aircraft configuration without leaving the spreadsheet.

▼ RESULT						
V_{1_BFL} (m/s)	V_{1_BFL} (kt)	BFL (m)	BFL (ft)	$1.15 \times TOD$ (m)	TOFL (m)	Governed by
74.23511081	144.30118	2354.458518	7724.601	2231.62	2354.458518	BFL

Figure 5.14 Result block of the BFL Calculator

The Comparison with Analytical/Empirical Methods block, located beside the interpolation table, shows the TOFL predicted by five empirical methods (Torenbeek, standard Loftin, modified Loftin, Kroo, and Takahashi 2025) alongside the BFL Calculator result, with the absolute and percentage deviation for each. This allows an immediate sanity check for any aircraft configuration without leaving the spreadsheet. The theoretical background of each method is presented in Section 3.5. Figure 5.15 shows this block.

Method	BFL / TOFL (m)	Δ vs GUI (m)	Δ vs GUI (%)
Torenbeek	1783.9	-104.0	-5.51%
Loftin	1742.9	-145.0	-7.68%
Loftin (modified)	1940.6	+52.7	+2.79%
Kroo	2127.0	+239.1	+12.66%
Takahashi	1670.1	-217.8	-11.54%
BFL GUI	1887.9	+0.0	+0.00%

Figure 5.15 Comparison with empirical methods block of the BFL Calculator

The Sample Aircraft Data columns (columns P-V) store the reference input parameters for the six sample aircraft covered in this thesis: the Airbus A320-200, Airbus A340-300,

Boeing 737-800, Boeing 777-300ER, Embraer E190, and Bombardier CRJ-900. When one of these aircraft is selected from the dropdown in cell D2, all input parameters in column D are automatically populated from the corresponding reference column. This eliminates manual data entry and ensures that the preset configurations are always applied consistently. The sources for each parameter set are documented in Chapter 6. Figure 5.16 shows the sample aircraft data columns.

Aircraft sample data					
A320-200	A340-300	B737-800	B777-300ER	EMBRAER E190-2	BOMBARDIER CRJ-900
78000.	271000.	79016	351534	47790	36514
122.6	363.1	124.6	436.8	92.53	70.6
34.1	58.	35.8	64.8	28.72	24.85
25.	29.7	26	30	22.7	24.8
0.24	0.251	0.22	0.14	0.252	0.27
3.95	5.64	3.76	6.20	3.01	2.69
2.08	2.24	2.33	2.31	2.19	1.9
0.019	0.019	0.0188	0.0178	0.0204	0.0210
0.003	0.003	0.0031	0.0026	0.0031	0.0031
0.021	0.004	0.0000	0.0000	0.0000	0.0000
0.1	0.1	0.2	0.2	0.2	0.2
10.	17.	10	17	10	10
117900.	138800.	117000.	513950	82300	65432
2.	4.	2	2	2	2
6.	6.	5.3	9	5	5

Figure 5.16 Sample aircraft data columns of the BFL Calculator

The Custom Aircraft option is available as the last entry in the dropdown selector. When selected, no preset data is loaded and all input cells in column D remain at their previous values. The user must fill in each parameter manually. Column W provides a plain-language description of each required input to guide the user. This option allows the tool to be applied to any jet transport aircraft for which the required parameters are available, beyond the six sample types included in the spreadsheet. Figure 5.17 shows the Custom column with its parameter descriptions.

Custom	What to enter (Custom)	
	Mass (MTOW)	kg
	Wing area	m ²
	Wingspan	m
	Wing sweep $\Lambda(c/4)$	deg
	Taper ratio	—
	Fuselage diameter	m
	CL_max (take-off config)	—
	CD0 clean	—

Figure 5.17 Custom aircraft column of the BFL Calculator with parameter descriptions

Every formula in the spreadsheet is linked to the corresponding equation in this thesis through the formula reference column (column K) and the Documentation sheet. The column K entries follow a numbered structure from 1 to 12 that mirrors the section order of Chapters 3 and 4, so any intermediate result can be traced back to its theoretical basis without consulting an external source. Figure 5.18 shows an extract of the formula reference column for the atmosphere and aerodynamics blocks.

Formula and physical meaning
1. HOW TO USE THIS GUI Pick an aircraft in D2 (6 presets + Custom). Full instructions -> click the title above.
2. ATMOSPHERE (ISA) $T(H) = T_0 - L \cdot H$ $\sigma = \rho/\rho_0 = (T/T_0)^{\gamma/(g_0 \cdot L - 1)}$
3. AERODYNAMICS $q = \frac{1}{2} \rho v^2$ $L = CL \cdot q \cdot S_W$ $D = CD \cdot q \cdot S_W$ $CD = CD_0 + CL^2/(\pi \cdot A \cdot e)$ $CD_0, TO = CD_0, clean + \Delta CD_0, gear + \Delta CD_0, flap$ $CD_0, OEI = CD_0, air + \Delta CD_0, OEI$ (single term, 0 unless
4. OSWALD FACTOR - Method 1 (geometric) $e_{theo} = 1/(1+f(\lambda-\Delta\lambda) \cdot A)$ $f(\lambda-\Delta\lambda) = 0.0524(\lambda-\Delta\lambda)^4 - 0.15(\lambda-\Delta\lambda)^3 + 0.1659(\lambda-\Delta\lambda)^2 - 0.0706(\lambda-\Delta\lambda) + 0.0119$ $\Delta\lambda = -0.357 + 0.45 \cdot e^{-(0.0375 \cdot \phi 25)}$ $ke, F = 1 - 2(dF/b)^2$ $ke, D0 = 0.873$ $ke, M \approx 1$ ($M < 0.3$) $e = e_{theo} \cdot ke, F \cdot ke, D0 \cdot ke, M \cdot (0.70/0.85)$
5. OSWALD FACTOR - Method 2 (CD0-based) [used for all validated results] $Q = 1/(e_{theo} \cdot ke, F)$ $P = 0.38 \cdot (CD_0, clean + \Delta CD_0, gear + \Delta CD_0, flap)$ $e = ke, M/(Q + P \cdot \pi \cdot A)$
6. ZERO-LIFT DRAG & GEAR/FLAP INCREMENTS $E_{max} = \sqrt{(\pi \cdot A \cdot e)/(4 \cdot CD_0, clean)}$ $\Delta CD_0, gear(large, \geq 150t) = 4e-8\delta f^3 - 7.485e-6\delta f^2 + 3.542e-5\delta f + 2.551e-2$ $\Delta CD_0, gear(small) = -1.8e-6\delta f^2 - 3$
7. THRUST MODEL (Bartel & Young 2008) $T(M) = N \cdot T_0 \cdot [At - k_1 \cdot M + k_2 \cdot M^2]$ $k_1 = 0.377(1+BPR) / (\sqrt{1+0.82BPR} \cdot G) \cdot Z \cdot p/p_0$ $k_2 = (0.23+0.19\sqrt{BPR}) \cdot X \cdot p/p_0$ $G = 0.061BPR + 0.633$

Figure 5.18 Formula reference column (column K) of the BFL Calculator.

5.2 Graphical User Interface

The GUI is contained within a single worksheet and is designed to produce a BFL result from a small number of clearly defined input cells. The aircraft is selected from a dropdown menu in cell D2, which offers six preset aircraft and a Custom option for user-defined configurations. When a preset is selected, all input parameters in column D are automatically populated from the reference data in columns P-U. For the Custom option, the user fills in each parameter manually, guided by the descriptions in column W. Figure 5.19 shows the dropdown selector and the input area.



Figure 5.19 Aircraft dropdown selector and input area of the BFL Calculator

All calculations update automatically when any input changes. The derived quantities in the center group allow the user to verify that the aerodynamic and propulsion parameters are physically reasonable before reading the final result. The formula explanations in column K and the Documentation sheet provide a complete reference for each calculation block, linking every formula to the corresponding equation in this thesis.

5.3 Calculation Workflow

The calculation proceeds automatically from top to bottom as soon as the input cells are filled. The atmosphere block first computes ρ and a_0 at the given altitude. The aerodynamics block then evaluates the Oswald factor, builds the take-off drag polars for the AEO and OEI configurations, and derives the reference speeds V_R , V_{LOF} , and V_2 . With these quantities available, the tool computes the ground roll coefficients a , b , c and the braking coefficients, and evaluates all fixed distances, including the air distance using the geometric circular arc model described in Section 4.3.

The BFL is found through a two-point linear interpolation scheme. $ASD(V_1)$ and $s_{TOD,OEI}(V_1)$ are evaluated at Point A ($0.88 \cdot V_R$) and Point B ($0.98 \cdot V_R$) using the closed-form integral of Equation (4.6). The balanced speed $V_{1,bal}$ is found by linear interpolation between the two adjacent points where the sign of $ASD - s_{TOD,OEI}$ changes. Finally, the TOFL is computed following Equation (4.23)

5.4 Input Parameters

Table 5.2 lists all input parameters required by the tool, together with their symbols, units, and representative values for the Airbus A320-200 in CONF 1+F at MTOW, sea level, ISA.

Table 5.2 BFL Calculator input parameters

Parameter	Symbol	Value	Unit
Take-off mass	m	78000	kg
Wing area	S_W	122.6	m^2
Wingspan	b	34.1	m
Aspect ratio	A	9.50	-
Taper ratio	λ	25.0	deg
Sweep angle at c/4	ϕ_{25}	0.240	-
Fuselage diameter	d_F	3.95	m
$C_{L,max}$ (CONF 1+F)	$C_{L,max}$	2.08	-
$C_{D0,clean}$	$C_{D0,c}$	0.0194	-
Oswald method	-	2	1 2
ΔC_{D0} gear	$\Delta C_{D0,g}$	0.0152	-
ΔC_{D0} flaps	$\Delta C_{D0,f}$	0.00307	-
Flap angle	δf	10	deg
$C_{L,ground}$	$C_{L,gr}$	0.1	-
Static thrust / engine	T_0	117900	N
Idle thrust / engine	T_{idle}	8253	N
Number of engines	N	2	-
Bypass ratio	BPR	6.0	-
Rolling friction	μ_{GR}	0.02	-
Braking coefficient	μ_B	0.35	-
Gear load factor	ϕ_{LG}	0.91	-
Screen height	h_{SC}	10.7	m
Maximum L/D	E_{max}	17.89	-
V_{MCG}	V_{MCG}	125	kt
Recognition time	t_{recog}	1.0	s
AFM buffer time	t_{buf}	2.0	s
Rotation time AEO	$t_{R,AEO}$	3.83	s
Rotation time OEI	$t_{R,OEI}$	4.5	s
Airport altitude	h_{apt}	0	m

5.5 Outputs and Results Visualization

The result block at the bottom of the sheet displays $V_{1,bal}$ in both m/s and kt, the BFL in both meters and feet, the factored AEO distance, the TOFL, and the governing condition. For the A320-200 at MTOW in CONF 1+F at sea level under ISA conditions, the tool gives $V_{1,bal} = 143.6$ kt, a BFL of 2390 m, and a TOFL of 2390 m. The BFL governs the TOFL for

this twin-engine aircraft, as expected from the discussion in Section 2.3. Figure 5.20 shows the result block.

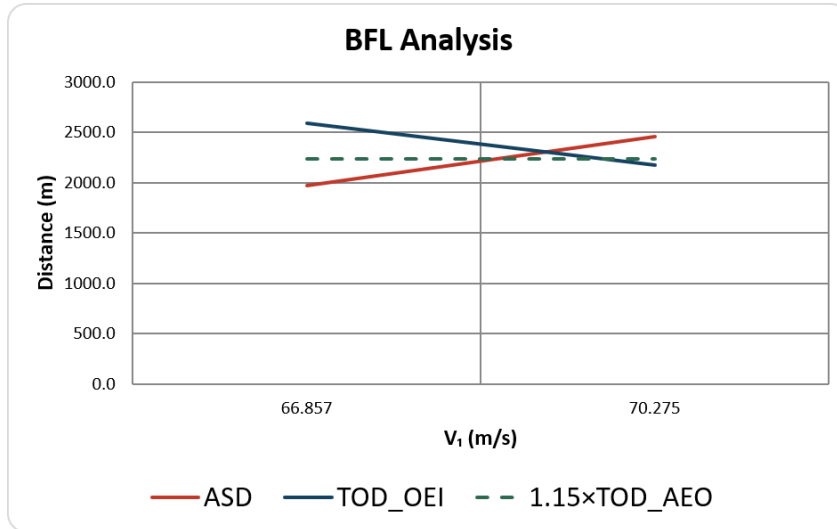


Figure 5.20 Result block of the BFL Calculator, A320-200, CONF 1+F, MTOW, SL, ISA.

The two-point interpolation table shows ASD and $s_{TOD,OEI}$ at the two evaluated V_1 points, letting the user verify that their difference changes sign so the crossing lies within range. The tool also displays the individual fixed distances (AEO ground roll, rotation, air distance) for diagnosing discrepancies against reference values. The comparison block to the right shows the Torenbeek, Loftin, modified Loftin, Kroo, and Takahashi predictions alongside the BFL Calculator result and their deviations.

Figure 5.21 shows this comparison in bar chart format for the active aircraft configuration. The chart updates automatically with the aircraft selection, allowing a direct visual comparison for any of the six preset aircraft or a custom configuration.

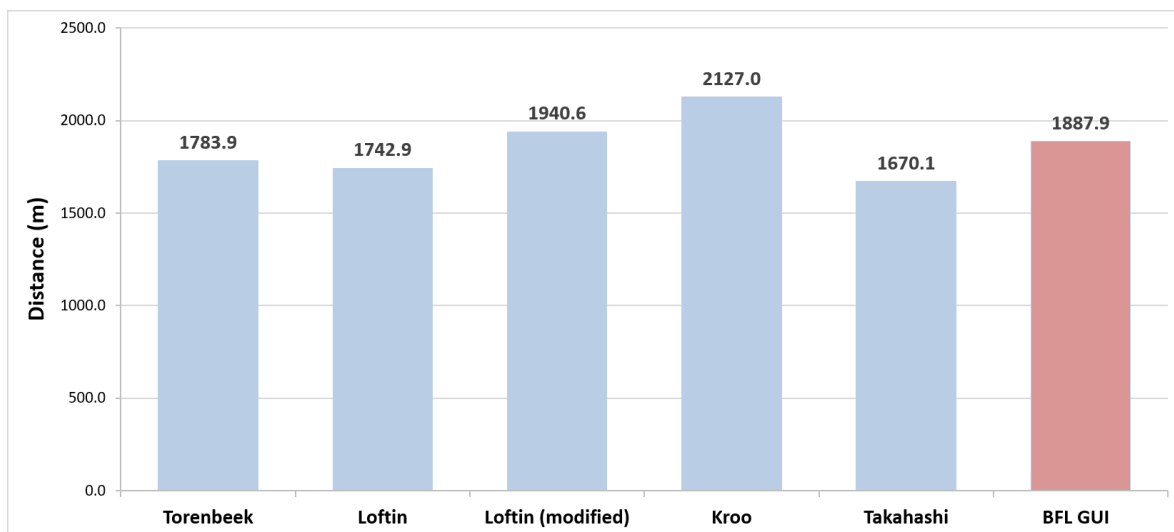


Figure 5.21 Comparison between the BFL Calculator and empirical methods, A320-200

6 Application and Validation

This Chapter presents the results of the BFL Calculator for the Airbus A320-200 and A340-300 and compares them with the numerical results of Lucht (2022). The comparison covers three flap configurations and three airport altitudes: sea level, 1000 ft, and 2000 ft. All calculations are performed at the default take-off mass used by Lucht (2022): 78 t for the A320-200 and 271 t for the A340-300. Wind and runway slope are set to zero in all cases, consistent with the conditions used by Lucht (2022).

6.1 Sample Aircraft Parameters

The aircraft parameters used for the validation are taken directly from Lucht (2022), which in turn sources them from Airbus airport planning documentation Lucht (2022), and the Airbus Flight Crew Operating Manuals. Table 6.1 summarises the main geometric and propulsion parameters for both aircraft.

Table 6.1 Aircraft parameters, A320-200 and A340-300

Parameter	Symbol	A320-200	A340-300	Unit
Take-off mass	m	78000	271000	kg
Wing area	S_W	122.6	363.1	m ²
Wingspan	b	34.1	58.0	m
Aspect ratio	A	9.50	9.26	-
Taper ratio	λ	0.240	0.251	-
Sweep angle at c/4	ϕ_{25}	25.0	29.7	deg
Fuselage diameter	d_F	3.95	5.64	m
Wing height above ground	h_W	3.31	4.73	m
$C_{D0, \text{clean}}$	$C_{D0, c}$	0.0194	0.0193	-
Static thrust per engine	T_o	117900	138800	N
Idle thrust per engine	T_{idle}	8253	14176	N
Number of engines	N	2	4	-
Bypass ratio	BPR	6.0	6.0	-
Rolling friction	μ_{GR}	0.02	0.02	-
Braking coefficient	μ_B	0.35	0.35	-
Gear load factor	ϕ_{LG}	0.91	0.91	-
V_{MCG}	V_{MCG}	125	109.5	kt

6.2 Comparison of TOFL Results

Table 6.2 and Table 6.3 compare the TOFL values from the analytical Excel tool with the numerical results of Lucht (2022) for both aircraft types at sea level. In the first table is going to be the results for the Oswald method 1 and the second table the Oswald method 2. The deviation is expressed as the percentage difference relative to Lucht's numerical result.

$$\Delta = \frac{S_{TOFL,analytical} - S_{TOFL,Lucht}}{S_{TOFL,Lucht}} \times 100\% \quad (6.1)$$

Table 6.2 TOFL comparison, Oswald Method 1

Aircraft	Config	TOFL GUI [m]	TOFL Lucht [m]	$\Delta\%$
A320-200	CONF 1+F	2524	2446	-3.20
A340-300	CONF 1+F	3498	3337	+4.80

Table 6.3 TOFL comparison, Oswald Method 2

Aircraft	Config	TOFL GUI [m]	TOFL Lucht [m]	$\Delta\%$
A320-200	CONF 1+F	2390	2446	-2.30
A340-300	CONF 1+F	3491	3337	+4.60

Figure 6.1 shows the ASD and OEI-TOD curves for the A320-200 using Oswald Method 2. The two evaluation points are visible and the crossing point is clearly identified between Point A and Point B. For a twin-engine aircraft the BFL governs the TOFL, since the OEI distance is significantly longer than the AEO distance. The crossing of the two curves confirms this behavior.

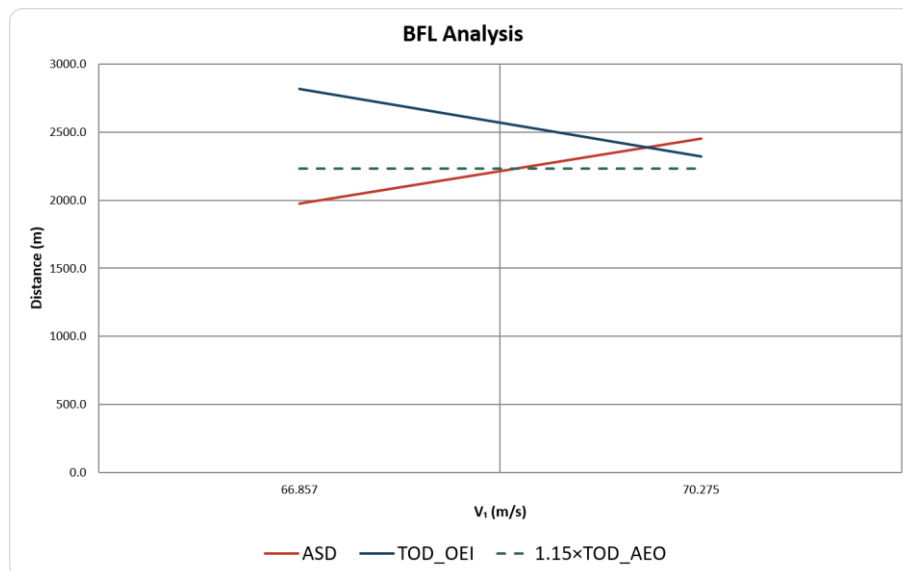


Figure 6.1 ASD and OEI-TOD vs. V_1 , A320-200, MTOW, SL, ISA

Figure 6.2 shows the equivalent curves for the A340-300. In this case the two curves still cross, giving a valid BFL, but the factored AEO distance $1.15 \cdot S_{TOD,AEO}$ lies above both curves across

the entire V_1 range. This confirms that the TOFL for the A340-300 is governed by the AEO case, which is the expected behavior for a four-engine aircraft where the thrust loss from one engine is only 25%.

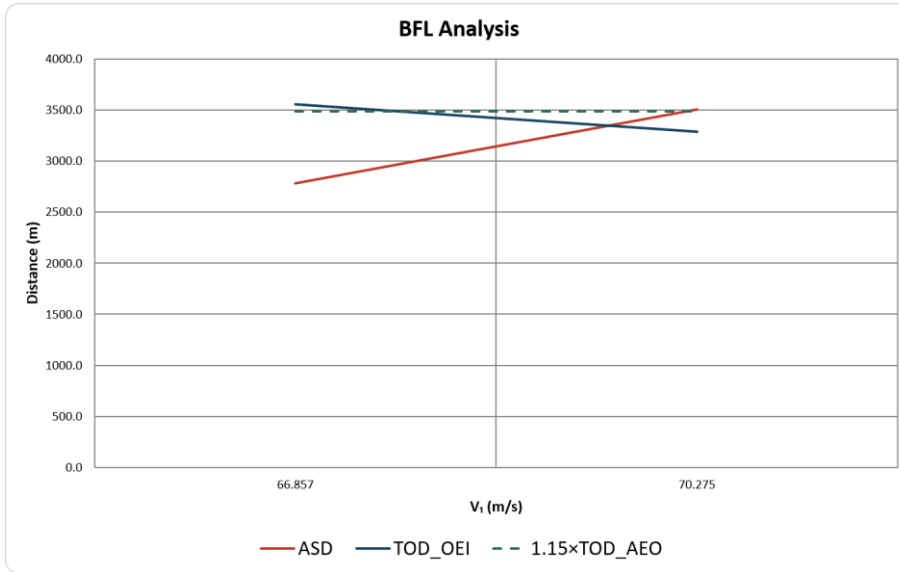


Figure 6.2 ASD and OEI-TOD vs. V_1 , A340-300, MTOW, SL, ISA

For the A320-200, both methods underestimate the TOFL relative to Lucht (2022) by approximately 2.3% to 3.2%. For the A340-300, both methods overestimate by approximately 4.6% to 4.8%. The switch from Method 1 to Method 2 has a small effect on the A320-200, reducing the TOFL by 134 m, while the A340-300 result is nearly identical under both methods.

The Oswald factor method is the most likely source of the deviations: this thesis uses Niță and Scholz (2012), while Lucht (2022) uses a different formulation. Since the ground roll accounts for the majority of both governing distances (63.3% for the A320-200's BFL, 77% for the A340-300's factored AEO distance), a systematic difference in induced drag between the two methods propagates directly into the final result for both aircraft. The opposite sign of the deviation between the two types (−2.3% to −3.2% for the A320-200, +4.6% to +4.8% for the A340-300) is consistent with the two aircraft having markedly different wing geometry (aspect ratio, sweep, taper), for which the Niță and Scholz (2012) and Lucht (2022) Oswald formulations need not diverge in the same direction. Isolating the exact contribution of this and any remaining factors would require access to Lucht's (2022) intermediate MATLAB values, which are not available; this is noted as a limitation of the present comparison rather than resolved further.

These deviations are within acceptable range for a preliminary design tool. For comparison, Lucht (2022) reports deviations of up to 28.2% for the empirical methods he tests, and even the best one deviates up to 5.4%. The analytical tool here achieves similar accuracy while using the same physical model as the numerical reference.

6.3 Comparison with Empirical Methods

To assess whether the BFL Calculator produces consistent results beyond the two validation aircraft, it is applied to four additional transport jet aircraft: the Boeing 737-800, the Boeing 777-300ER, the Embraer E190, and the Bombardier CRJ-900. For these aircraft no numerical reference equivalent to Lucht (2022) is available. The results are therefore compared against the empirical methods described in Section 3.5: the Torenbeek analytical approximation, the standard Loftin method, the modified Loftin method, the Kroo method, and the Takahashi (2025) empirical equation. All results correspond to sea level, ISA, MTOW, and dry runway conditions.

The aircraft parameters for the four additional types are summarized in Table 6.4. Geometric parameters are taken from the Aircraft Database of Hirsch (2022). Engine thrust and bypass ratio are taken from the corresponding EASA Type Certificate Data Sheets (EASA 2024). Performance data including V_2 and $VMCG$ are taken from the individual aircraft pages of SKYbrary, a safety knowledge platform maintained by EUROCONTROL and ICAO (SKYbrary 2024a, SKYbrary 2024b, SKYbrary 2024c, SKYbrary 2024d). The maximum lift coefficient $C_{L,max}$ is derived from the take-off safety speed V_2 using Equation (3.11) and the regulatory relation $V_2 = 1.13 \cdot V_{S1g}$.

Table 6.4 Aircraft parameters, additional aircraft

Parameter	Symbol	B737-800	B777-300ER	E190	CRJ-900	Unit
Take-off mass	m	79016	351534	47790	36514	kg
Wing area	S_w	124.6	436.8	92.53	70.6	m ²
Wingspan	b	35.8	64.8	28.72	24.85	m
Aspect ratio	A	10.29	9.61	8.91	8.75	-
Taper ratio	λ	26.0	30.0	22.7	24.8	deg
Sweep angle at $c/4$	ϕ_{25}	0.220	0.140	0.252	0.270	-
Fuselage diameter	d_F	3.76	6.20	3.01	2.69	m
$C_{L,max}$	$C_{L,max}$	2.33	2.31	2.19	1.90	-
$C_{D0, clean}$	$C_{D0,c}$	0.0188	0.0178	0.0204	0.0210	-
Static thrust	T_0	117000	513950	82300	65432	N
Idle thrust	T_{idle}	6673	35910	5761	4580	N
Number of engines	N	2	2	2	2	-
Bypass ratio	BPR	5.3	9.0	5.0	5.0	
$VMCG$	$VMCG$	110	120	110	105	kt
Rotation time AEO	$t_{R,AEO}$	3.83	4.50	3.83	3.83	s
Rotation time OEI	$t_{R,OEI}$	4.5	5.50	4.5	4.50	s

For all five aircraft the TOFL is governed by the BFL, consistent with the two-engine configuration of each type. This mirrors the behavior observed for the A320-200 and contrasts with the A340-300, where the four-engine arrangement makes the factored AEO distance the critical constraint. Table 6.5 presents the BFL Calculator results alongside the empirical predictions. Table 6.6 presents the percentage of these deviations against the BFL GUI values.

Table 6.5 TOFL results, all aircraft

Aircraft	$V_{1,bal}$ [kt]	BFL GUI [m]	Torenbeek [m]	Loftin [m]	Loftin mod. [m]	Kroo [m]	Takahashi [m]
A320-200	145.4	2390	2258	2322	2405	2693	2149
A340-300	150.6	3491	3525	3732	3535	3719	3315
B737-800	135.6	2175	2092	2109	2234	2524	1973
B777-300ER	150.2	2392	2613	2734	2735	2712	2489
E190	124.6	1708	1704	1571	1803	1905	1528
CRJ-900	132.3	1889	1784	1743	1941	2127	1670

Airbus A320-200

Torenbeek and Loftin give 2258 m and 2322 m, both 3 to 6% below the analytical result. The modified Loftin method gives 2404.7 m, within 1%. The Kroo method gives 269 m, 12.7% above, consistent with its systematic overestimation across the fleet. The Takahashi (2025) method gives 2149 m, 10.1% below the analytical result. Applying Takahashi's own regression directly to his own reference case (a dispatch mass of 77111 kg, close to the 78000 kg used here) gives 2052 m against a certified handbook value of 1981 m, a 3.6% pessimism consistent with his reported accuracy for this aircraft type. Figure 6.3 shows the comparison between the analytical result and the five empirical methods for this aircraft.

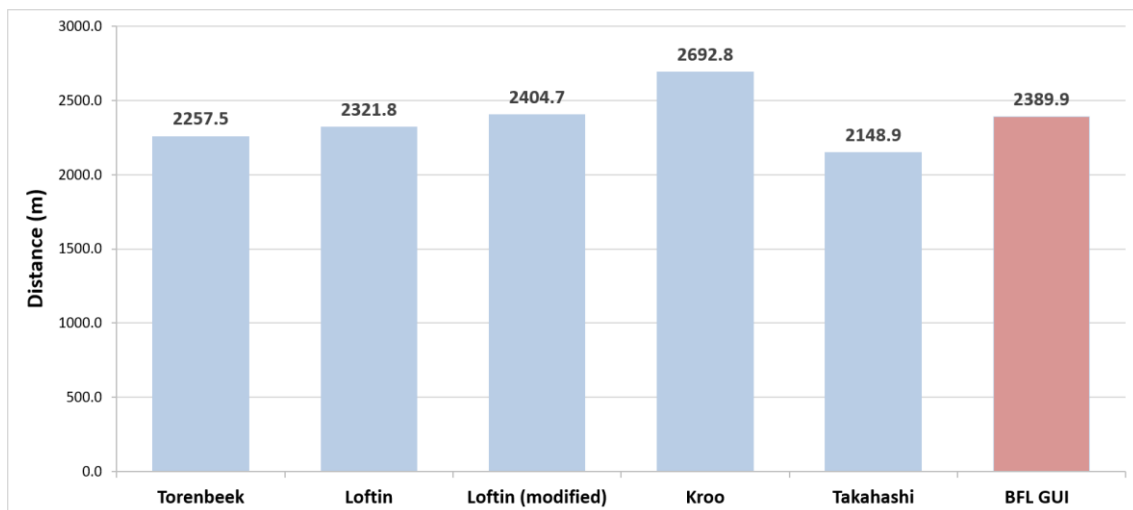


Figure 6.3 TOFL comparison for the Airbus A320-200, MTOW, SL, ISA

Airbus A340-300

Torenbeek and the modified Loftin method give 3525 m and 3535 m, both within 1.3% of the analytical result — the closest agreement of any aircraft in the comparison. The standard Loftin

method gives 3732 m, 6.9% above, and the Kroo method gives 3719 m, 6.5% above. The Takahashi (2025) method gives 3315 m, 5.0% below the analytical result, the smallest deviation of Takahashi's predictions among the four larger aircraft in the comparison. A direct comparison with Takahashi's own reference case is not meaningful here, since his published dispatch weight for this aircraft corresponds to the A340-200 variant at 251230 kg, well below the 271000 kg used in this thesis. Figure 6.4 shows the comparison between the analytical result and the five empirical methods for this aircraft.

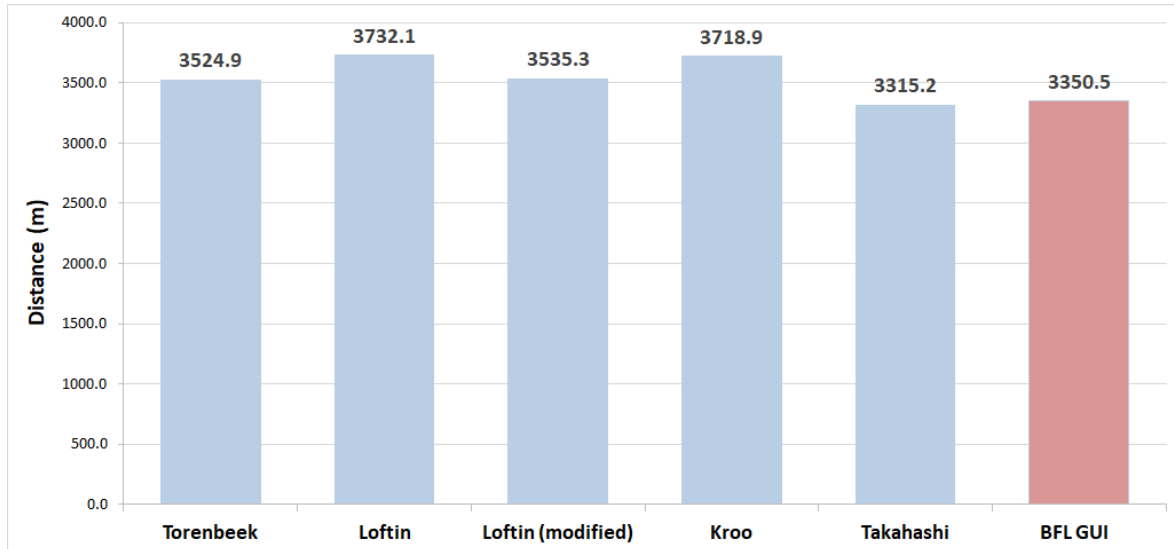


Figure 6.4 TOFL comparison for the Airbus A340-300, MTOW, SL, ISA

Boeing 737-800

The BFL Calculator gives 2176 m for the B737-800 at MTOW. Figure 6.4 shows the ASD and OEI-TOD curves as a function of V_1 .

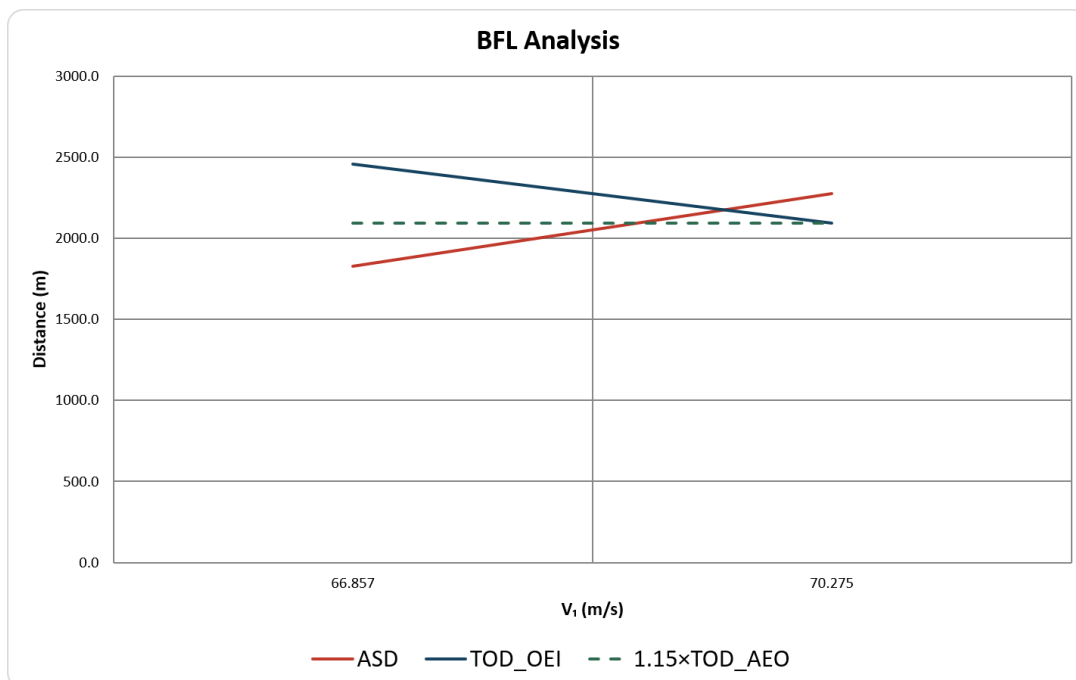


Figure 6.5 ASD and OEI-TOD vs. V_1 , B737-800, MTOW, SL, ISA

The Torenbeek and standard Loftin methods give 2092 m and 2109 m, both approximately 3 to 4% below the analytical result. The modified Loftin method gives 2234 m, 58 m above. The Kroo method gives 2524 m, the highest estimate, consistent with the tendency to overestimate reported by Lucht (2022). The Takahashi (2025) method gives 1973 m, 9.2% below the analytical result. The five methods bracket the analytical result reasonably well. Figure 6.6 shows the comparison between the analytical result and the empirical methods for this aircraft.

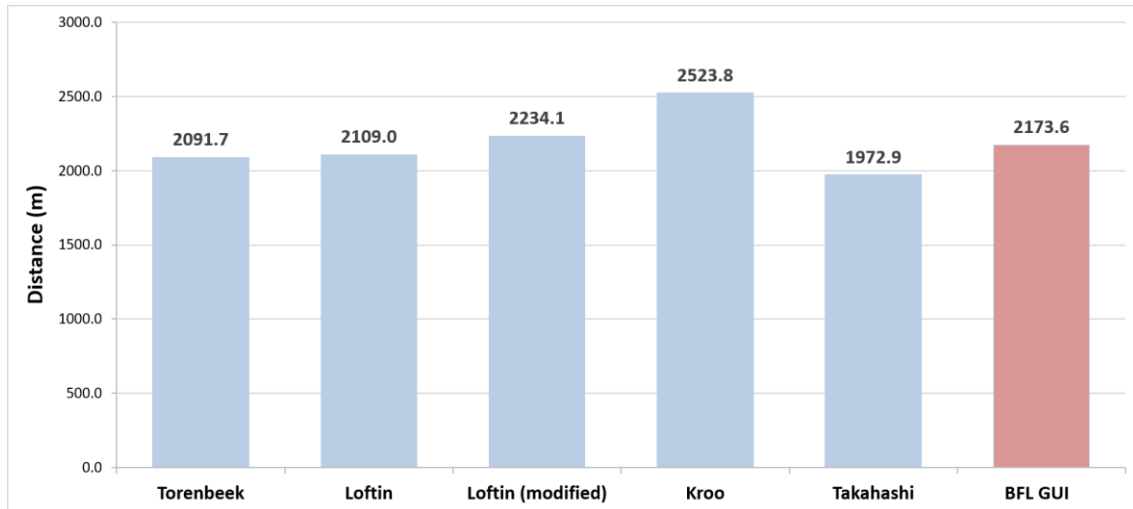


Figure 6.6 TOFL comparison for the Boeing 737-800, MTOW, SL, ISA

Boeing 777-300ER

The BFL Calculator gives 2398 m for the B777-300ER at MTOW. Figure 6.7 shows the ASD and OEI-TOD curves.

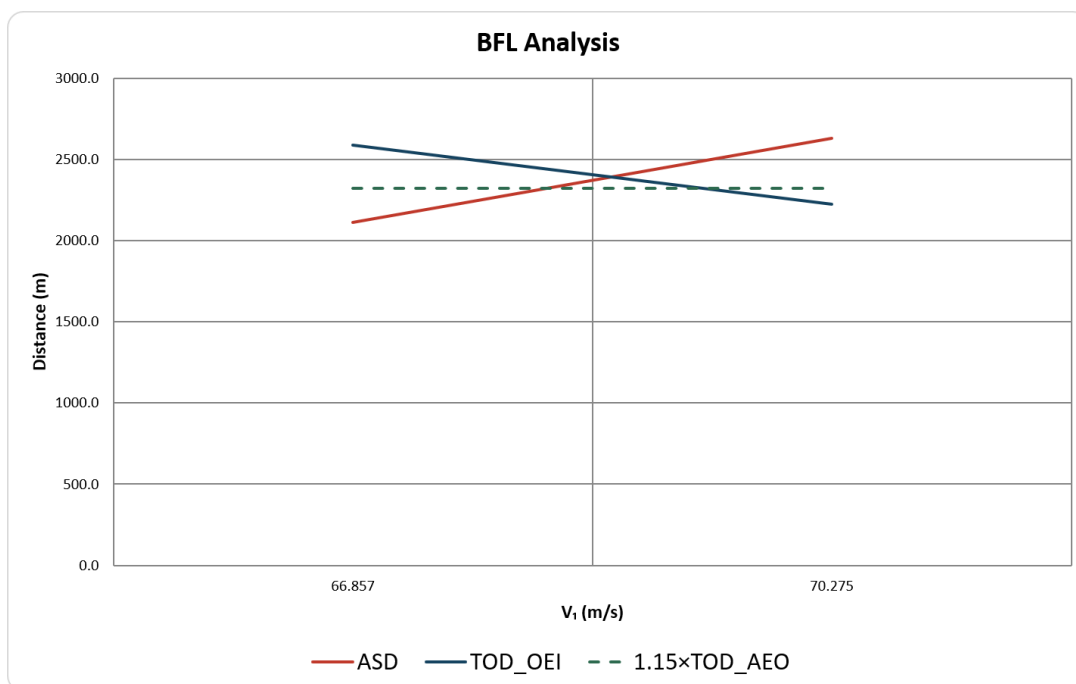


Figure 6.7 ASD and OEI-TOD vs. V_1 , B777-300ER, MTOW, SL, ISA

All five empirical methods overestimate the result, ranging from 2576 m to 2735 m, with deviations between 7% and 14%. The high thrust loading and high $C_{L,max}$ of the 777-300ER, combined with a bypass ratio of 9.0 that reduces the Kroo thrust index, explain why the empirical correlations perform less well for this type. The Takahashi (2025) method gives 2490 m, 4.3% above the analytical result. This is the only aircraft for which this method overestimates, and it is also the smallest absolute deviation of the five methods for this aircraft type. Figure 6.8 shows the comparison.

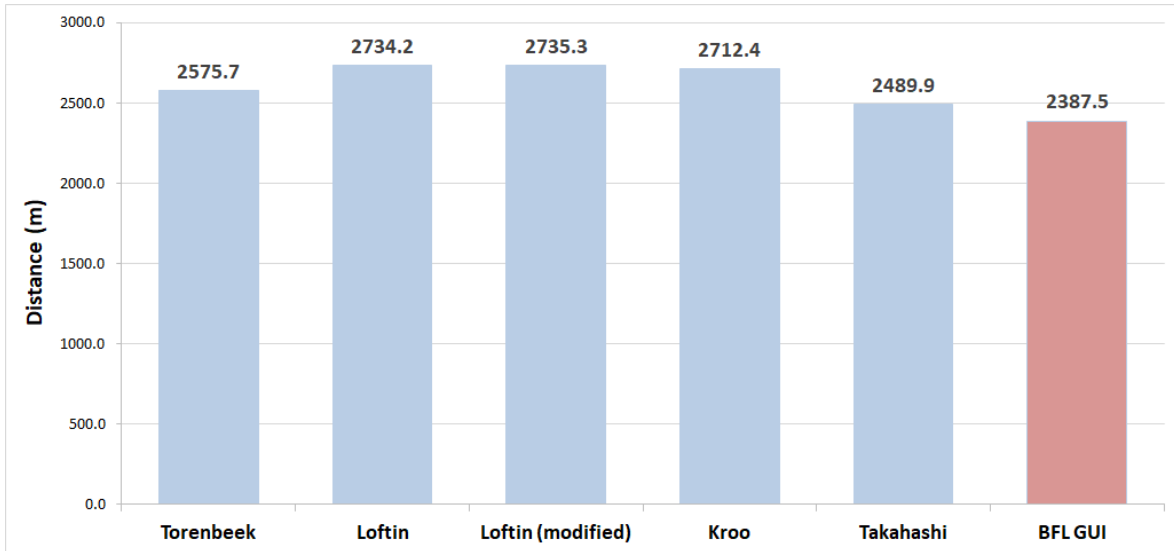


Figure 6.8 TOFL comparison for the Boeing 777-300ER, MTOW,SL,ISA

Embraer E190

The BFL Calculator gives 1716 m for the E190 at MTOW. Figure 6.9 shows the ASD and OEI-TOD curves.

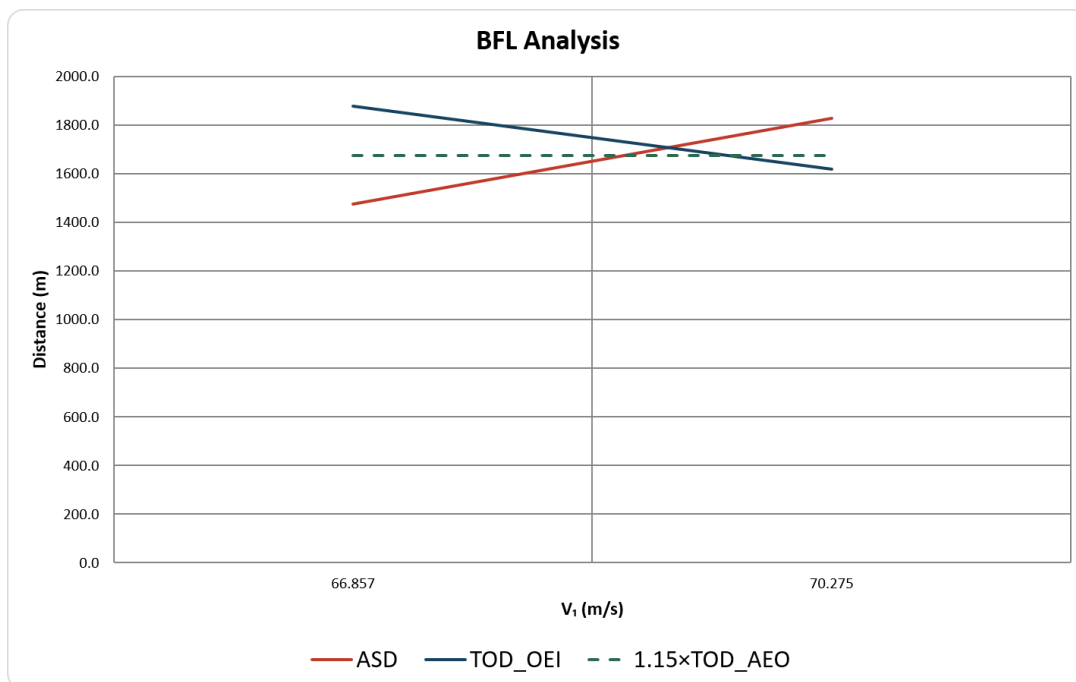


Figure 6.9 ASD and OEI-TOD vs. V_1 , E190, MTOW, SL, ISA

Among the empirical methods, Torenbeek's estimate of 1704 m is closest to the analytical result, differing by less than 1%. Standard Loftin gives the largest underestimate at 1571 m, 144 m below, while modified Loftin and Kroo overestimate at 1803 m and 1905 m. Takahashi (2025) underestimates by 10.5% at 1528 m, consistent with his own finding that regional jets are underrepresented in the regression. This spread reflects the fact that the Loftin and Kroo datasets were built primarily around larger transport aircraft, leaving the regional jet category less well represented. Figure 6.10 shows the comparison.

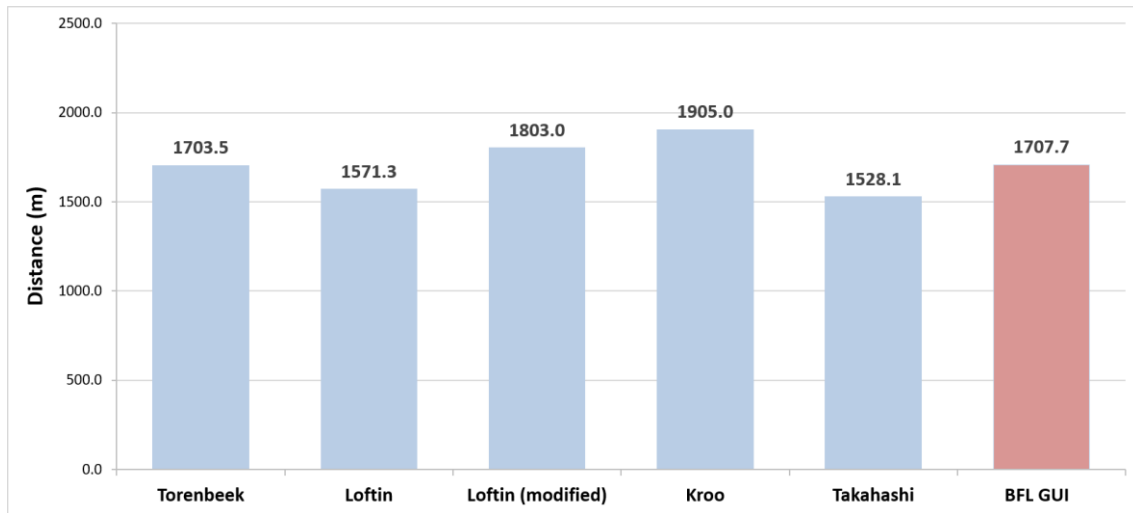


Figure 6.10 TOFL comparison for the Embraer E190, MTOW, SL, ISA

Bombardier CRJ-900

The BFL Calculator gives 1898 m for the CRJ-900 at MTOW. Figure 6.11 shows the ASD and OEI-TOD curves.

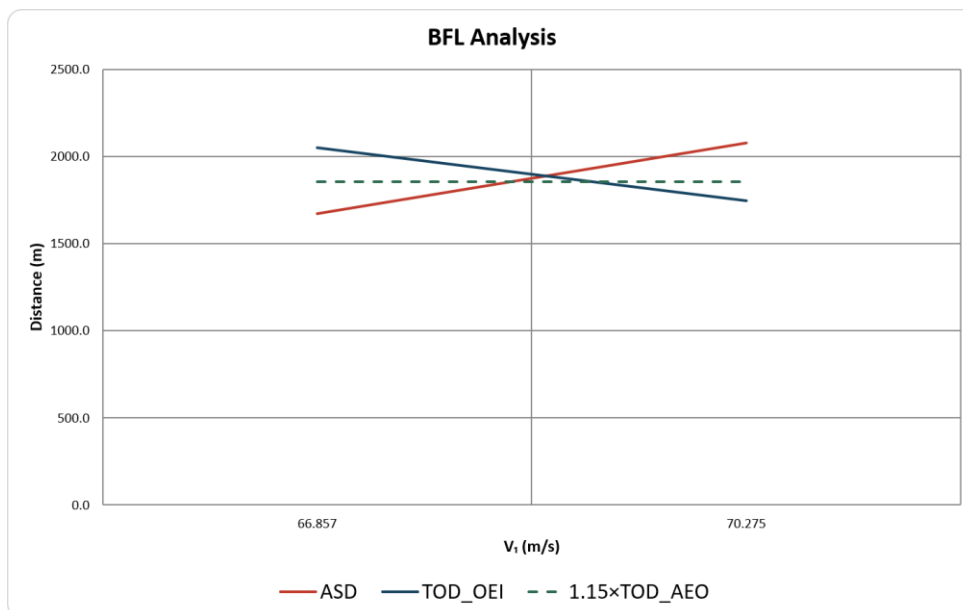


Figure 6.11 ASD and OEI-TOD vs. V_1 , CRJ-900, MTOW, SL, ISA

The Torenbeek and standard Loftin methods give 1784 m and 1743 m, both 6 to 8% below the analytical result. The modified Loftin method gives 1941 m and the Kroo method gives 2127 m. The Takahashi (2025) method gives 1670 m, 11.5% below the analytical result, the largest deviation among the six aircraft for this method. The pattern is similar to the E190 case, with a wider spread among the empirical methods than for the narrowbody types. Figure 6.12 shows the comparison with the empirical methods.

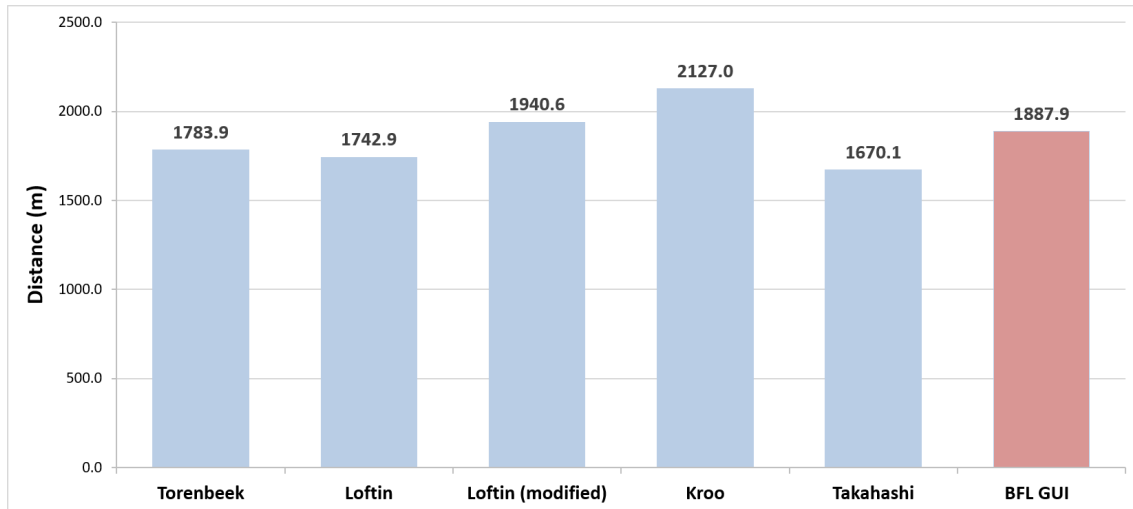


Figure 6.12 TOFL comparison for the Bombardier CRJ-900, MTOW, SL, ISA

Overall Assessment

Across all six aircraft, Torenbeek shows the smallest average absolute deviation from the BFL Calculator at 4.25%, closely followed by modified Loftin at 4.60%. Takahashi (2025) averages 8.45%, standard Loftin 7.15%, and Kroo consistently overestimates, averaging 12.19%, the largest of the five methods. Takahashi (2025) underestimates TOFL for five of six aircraft, the exception being the B777-300ER, with the largest deviations for the E190 and CRJ-900, the two types with the smallest wing loading, consistent with the reduced representation of these categories in the underlying regression, as Takahashi himself notes. These trends align with Lucht (2022), who finds no single empirical method performs consistently well across all aircraft types. The BFL Calculator applies the same equations of motion to all aircraft, producing consistent results across the full size range.

Table 6.6 Empirical method deviations

Aircraft	Torenbeek [%]	Loftin [%]	Loftin mod. [%]	Kroo [%]	Takahashi [%]
A320-200	-5.54	-2.85	+0.62	+12.67	-10.08
A340-300	+0.98	+6.91	+1.27	+6.53	-5.03
B737-800	-3.77	-2.97	+2.78	+16.11	-9.23
B777-300ER	+9.44	+14.52	+14.57	+13.61	+4.29
E190	-0.25	-7.99	+5.58	+11.55	-10.52
CRJ-900	-5.51	-7.68	+2.79	+12.66	-11.54

7 Summary and Conclusions

This thesis presents an analytical method for calculating the Balanced Field Length of jet transport aircraft, implemented in a Microsoft Excel spreadsheet with a graphical user interface. The method is based on the closed-form solution of the equation of motion integral derived by Lucht (2022), which replaces the step-wise numerical integration originally performed in MATLAB. The Oswald span efficiency factor is computed using the improved method of Niță and Scholz (2012) instead of the formulation used by Lucht (2022). The tool is validated against the numerical results of Lucht (2022) for the Airbus A320-200 and A340-300, and applied to four additional transport aircraft: the Boeing 737-800, Boeing 777-300ER, Embraer E190, and Bombardier CRJ-900.

The analytical closed-form integration method reproduces the numerical results of Lucht (2022) with deviations between -2.3% and $+4.60\%$ for the A320-200 and A340-300 respectively. These deviations are within the range considered acceptable for preliminary design tools and are comparable to the best empirical method evaluated by Lucht (2022), the modified Loftin method, which achieves deviations of up to 5.4% . The residual differences between the analytical tool and the numerical reference are most likely attributable to the choice of Oswald factor method: this thesis uses Niță and Scholz (2012), while Lucht (2022) uses a different formulation, and ground roll accounts for the majority of both governing distances. Isolating the exact contribution of any remaining factors would require access to Lucht's (2022) intermediate MATLAB values, which are not available.

The governing distance behaves as expected for each aircraft type. For all two-engine aircraft in the sample, the BFL governs the TOFL, since the 50% thrust reduction following an engine failure makes the OEI take-off distance the critical constraint. For the Airbus A340-300, the TOFL is governed by the factored AEO distance $1.15 \cdot s_{TOD,AEO}$ in all cases evaluated, which is the expected behavior for a four-engine aircraft where the thrust loss is only 25%.

Method 2 of Niță and Scholz (2012) is used for all results. Under Method 1, three of the six aircraft fall below the CS-25.121(b) minimum climb gradient for certified aircraft; under Method 2, only the A320-200 does, and only marginally (2.25% against the 2.4% minimum), a residual attributed to the single-point climb-angle evaluation used in the geometric air-distance model rather than a full flight-path integration.

Among the empirical methods evaluated across all six aircraft, the Torenbeek method shows the smallest average absolute deviation from the BFL Calculator, at 4.25% , closely followed by the modified Loftin method at 4.60% . The standard Loftin method averages 7.15% , the Takahashi (2025) method 8.45% , and the Kroo method consistently overestimates, with a mean absolute deviation of 12.19% . No single empirical method performs well across all aircraft

types and sizes, which confirms the finding of Lucht (2022) and highlights the advantage of the physics-based approach implemented in the BFL Calculator.

Compared with the five empirical methods, the BFL Calculator gives a TOFL of 2390 m for the A320-200, within the spread of the five predictions (2149 m to 2693 m). Applied to the six aircraft studied, Torenbeek and the modified Loftin method show the closest average agreement, while Kroo consistently overestimates and Takahashi (2025) consistently underestimates.

The Excel tool developed in this thesis makes the BFL calculation accessible to anyone with Microsoft Excel, without requiring MATLAB or any other numerical software. All intermediate results are visible and traceable, and the formula explanations embedded in the spreadsheet allow any result to be verified against the equations presented in this thesis. The tool includes reference data for six aircraft types and can be extended to any jet transport aircraft for which the required input parameters are available.

8 Recommendations

The following recommendations are made for future work based on the limitations and findings of this thesis.

The current tool is limited to dry runway conditions and does not account for wind components or runway slope. The braking coefficient $\mu_B = 0.35$ follows Lucht (2022) and represents a standard dry runway value. If the tool is to be used for comparison with certified flight manual data, a higher value consistent with ESDU dry braking data would reduce the positive bias observed in Section 6.3. Adding a headwind and tailwind correction following CS-25.105, and a runway slope term in the equation of motion, would extend the applicability of the tool to a wider range of real dispatch scenarios without significant additional complexity.

The asymmetric drag and windmill drag of the failed engine are not modeled separately for the four additional aircraft in the sample, since the required geometric parameters are not publicly available. A systematic method to estimate these drag increments from basic engine geometry, similar to the approach used by Lucht (2022) for the A320 and A340, would improve the accuracy of the OEI calculations for aircraft beyond the two validation types. The tool currently uses a single consolidated input cell for the additional OEI drag, which the user must fill in manually when the data are available. A structured sub-model for windmill drag and asymmetric drag, based on engine bypass ratio and nacelle geometry, would make this input more traceable and reduce the risk of omission.

The Oswald factor is computed using Method 2 of Niță and Scholz (2012) for all aircraft in the sample. However, the conditions under which Method 1 gives physically admissible climb gradients and under which it does not are not fully characterized in this thesis. A more systematic investigation of the sensitivity of the BFL to the Oswald factor method across a wider range of aircraft geometries would be valuable.

The comparison with published TOFL data in Chapter 6 is limited to the A320-200 and A340-200 from the Takahashi (2025) dataset, as the other aircraft do not appear in that reference. A more comprehensive validation using certified flight manual data for the B737-800, B777-300ER, E190, and CRJ-900 would strengthen this thesis's conclusions. Certified Balanced Field Length values for these types are freely available in the manufacturers' Documents for Airport Planning, offering a more direct comparison than the empirical equations used here.

Finally, the BFL Calculator could be integrated into an aircraft preliminary sizing tool to allow automatic TOFL estimation during the sizing loop. The inverse BFL problem, finding the required thrust-to-weight ratio or wing loading for a given target TOFL, would also be a useful addition.

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