

# **Conditions for Passenger Aircraft Minimum Fuel Consumption, Direct Operating Costs and Environmental Impact**

Task for a project or thesis

## **Background**

Asking for minimum fuel consumption (maximum range) and minimum costs for passenger aircraft is as old as air travel itself. Over the decades airlines tried to maximize profits by reducing costs. Textbooks about aircraft performance certainly cover flight conditions like airspeed and altitude to be selected for maximum range. However, simplifying assumptions are made: drag is represented by a simple drag polar that does not include Mach effects on induced drag and ignores wave drag. Thrust Specific Fuel Consumption (TSFC) is considered constant (ignoring especially the important dependency on airspeed). When it comes to minimizing costs, textbooks limit the topic to a general discussion without attempting a calculation. Global warming comes from emitting CO<sub>2</sub> and – even more so – water into the atmosphere. Water emitted at high altitudes has a large contribution to aviation's global warming share. Nevertheless, water shows its detrimental effects on climate change only if it becomes visible as contrails or aviation induced cirrus clouds – and this depends on flight altitude. Very important, it has to be distinguished, if a given aircraft is considered, or if the optimization takes place during aircraft design, where aircraft parameters (like wing area) are still to be determined. Answers available in the open are missing and (potential) answers behind closed doors do not help society. Simple yet reliable and tested models exist to describe aerodynamics, engines, cost structures and the atmosphere with sufficient accuracy to produce answers to the research question generated against this background.

## **Task**

Determine optimum flight parameters (for a given aircraft) and geometric parameters (for an aircraft to be designed) to minimize fuel consumption, Direct Operating Costs (DOC) and environmental impact. Follow these steps:

- Start with a review to show what exists (or rather does not exist) on the topic. Define your terms (MRC, LRC, ECON, ...).
- Quickly review and reference the models for aerodynamics (Oswald factor, wave drag), TSFC (Herrmann, Roux), DOC (AEA, TU Berlin) and for the atmosphere (see JOHANNING) as used in the lectures accompanying this task at HAW Hamburg.
- Combine the individual models to one model for fuel minimization (based on Specific Air Range), DOC and environmental impact in an Excel table.
- Visualize your results in contour plots (with the output shown as isolines) of the variables lift coefficient ( $C_L$ ) versus cruise Mach number ( $M$ ).
- Find optimum flight parameters ( $M$  and  $C_L$  respectively altitude,  $h$ ) for minimum fuel, DOC and environmental input a) for a given aircraft and b) for aircraft design, depending on aircraft mass,  $m$  or wing loading,  $m/S$  as well as other input parameters and boundary conditions.
- Discuss your results and make recommendations.

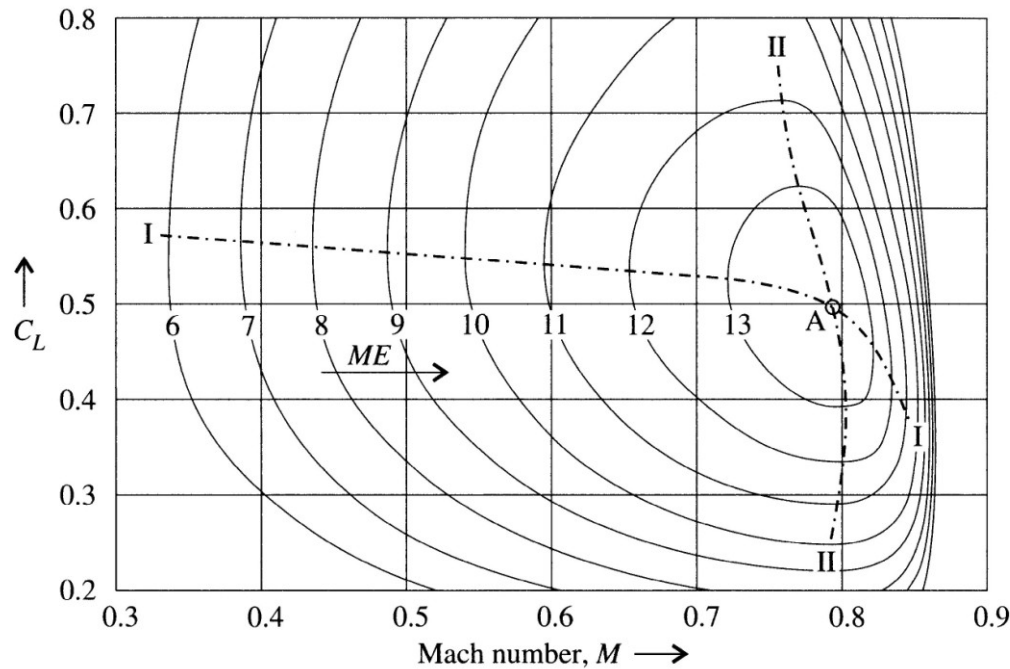
The report has to be written in English based on German or international standards on report writing.

# Help to Get Started

Use:

YOUNG, Trevor: Performance of the Jet Transport Airplane. Chichester, UK : Wiley, 2018

This is a start as far as a good textbook can go. I have copied Fig. 13.6 to show the contour plot as drawn for maximizing range or minimizing fuel consumption.



Traditionally it is started with maximizing Specific Air Range (SAR),  $r_a$  with units of m/kg. I would favor the inverse  $f = 1/r_a$  with units of kg/m (or kg/km or kg/100 km or 1/100 km).  $f$  is called fuel consumption and  $f' = f/m$  with mass,  $m$  is the (mass) specific fuel consumption per kg.

$$r_a = VE/(cmg) \text{ and } f = cmg/(VE) \text{ hence } f' = cg/(VE)$$

Written with respect to Mach number,  $M$  and speed of sound  $a$ :

$$r_a = aME/(cmg) \text{ and } f = cmg/(aME) \text{ hence } f' = cg/(ME)$$

Instead of plotting  $ME$  or  $1/ME$  we can plot  $cg/(ME)$  or  $c/(ME)$  and take account also of the fact that  $c$  alias Thrust Specific Fuel Consumption (TSFC) is not constant (but rather a function of speed or Mach number).

$$E = C_L/C_D \text{ and } C_D = C_{D0} + C_{D, wave}(M) + C_L^2/(Ae(M)) \quad .$$

Lift coefficient,  $C_L$  follows from the lift equation with input of mass,  $m$  (for aircraft analysis) or wing loading,  $m/S_W - S_W$  is wing area – (for aircraft design) and altitude,  $h$ . The use of  $C_L$  is efficient, because it combines the effect of mass and altitude in one variable.

$$C_L = \frac{2m}{\rho(h)M^2 a_0^2 \theta(h) S} \quad \text{or} \quad C_L = \frac{2m/S}{\rho(h)M^2 a_0^2 \theta(h)}$$

$$\text{with } \theta(h) = \frac{T}{T_0}$$

$a_0$  is the speed of sound at sea level and  $T_0 = 288.15$  K is the ISA temperature at sea level.

In aircraft design at given (large)  $M$ ,  $C_L$  tends to come out initially smaller than optimum. For this reason altitude has to be increased, which in the troposphere decreases density  $\rho$  and decreases temperature,  $T$ , but in the stratosphere only decreases density,  $\rho$ . Wing loading  $m/S$  cannot be increase more than the limit given by landing requirements coming from the limited landing distance equivalent to a limited approach speed.

In performance calculation wing area  $S$  is given and cruise altitude follows from the particular aircraft mass at each time and during cruise depending on the amount of fuel already burned.

In order to avoid contrails and aviation induced cirrus clouds, aircraft could fly lower. This would mean that they have to fly slower. This measure is called Cruise Speed Reduction (CSR). CSR is not very popular:

- 1.) An increase in altitude in the troposphere will produce lower inlet air temperature which reduces the specific fuel consumption.
- 2.) An increase in altitude requires increased engine RPM to provide cruise thrust and the specific fuel consumption reduces as normal rated RPM is approached.  
([http://code7700.com/aero\\_range\\_performance.htm](http://code7700.com/aero_range_performance.htm))

Furthermore:

- 3.) Time dependent costs are increased due to longer flight time.

However, thrust is reduced at altitude. This means in aircraft design a larger engine (with respect to sea level thrust) and as such a heavier engine has to be installed to be able to provide the thrust requirements at altitude. Thrust at altitude is a lot less than sea level thrust, but it cannot be increased without increasing turbine inlet temperature, which is limited due to temperature limits of turbine materials.

## 13.6 Best Cruise Speeds and Cruise Altitudes

### 13.6.1 Maximum Range Cruise Speed

The maximum range will be achieved by operating the airplane, throughout the cruise, at the condition that yields the maximum SAR. The flight speed that results in the greatest SAR is known as the *maximum (max.) range cruise* (MRC) speed. The MRC speed for an airplane depends on its gross weight, the cruising altitude, and the ambient air temperature (WAT). Operating at the MRC speed minimizes the fuel needed to cover a given cruise distance (in still air). When operating at a set altitude, the MRC speed will decrease a little during the cruise as the airplane gets lighter—this is illustrated in Figure 13.9.

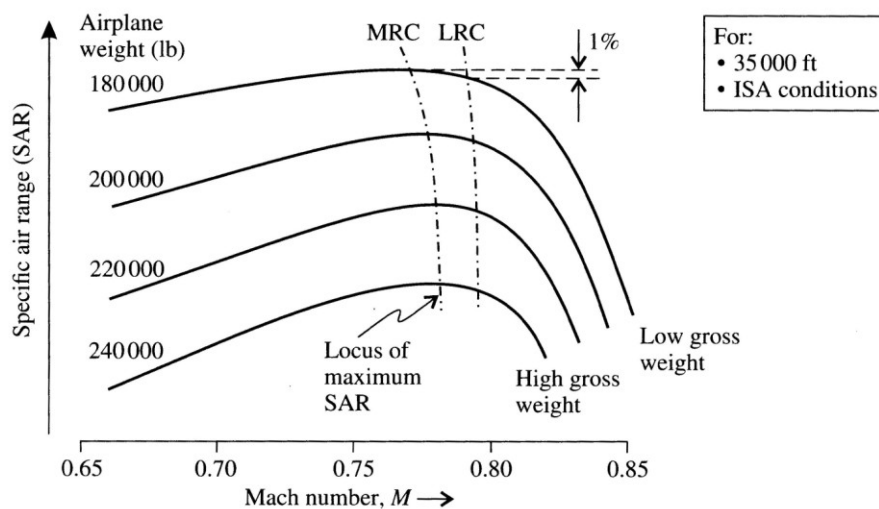
### 13.6.2 Economy Cruise Speed

In practice, airlines normally fly a little faster than the MRC speed, sacrificing a small increase in fuel usage, to obtain a shorter cruise time. This makes sense as the trip cost can be reduced. The *direct operating cost* (see Section 18.2) associated with a particular flight can be expressed as the sum of three terms: a fixed cost, fuel cost, and time-dependent cost. Time-dependent costs—which can include all or part of the crew, maintenance, and lease costs—are reduced by flying faster. The cruise speed that would result in the lowest operating cost is thus faster than the MRC speed (which is the speed that would result in the lowest fuel cost).

The cruise speed that would result in the lowest operating cost is called the *economy cruise speed*, which is usually abbreviated as the ECON speed. The calculation of the ECON speed is based on the airline's individual cost structure, which is quantified in terms of a *cost index* (see Section 18.3) determined for the particular airplane type and route.

### 13.6.3 Long Range Cruise Speed

A simplified approach to that described in Section 13.6.2, which is widely used and does not require knowledge of the airline's costs, is based on the *long range cruise* (LRC) speed. The LRC



**Figure 13.9** Maximum range cruise (MRC) and long range cruise (LRC) speeds (example for a mid-size, twin-engine airliner).

speed is determined by allowing a 1% reduction from the peak SAR and selecting the faster of the two possible speeds [19], as illustrated in Figure 13.9.

As LRC speeds depend solely on the airplane's performance characteristics—that is, without considering costs associated with an individual operator—LRC speeds can be determined by the airplane manufacturer. In fact, this is done as a matter of routine. LRC data are published in such documents as the Flight Crew Operating Manual (FCOM)—for example: Mach number, indicated airspeed, true airspeed, engine setting, and fuel flow would typically be tabulated for selected values of airplane weight, altitude, and temperature (WAT).

#### 13.6.4 Comparison of Cruise Speeds

Figure 13.10 illustrates how the fixed cost, time cost, fuel cost, and total cost vary with Mach number in cruise. The Mach numbers for the three reference cruise speeds (i.e., MRC, LRC, and ECON) are indicated as  $M_{MRC}$ ,  $M_{LRC}$ , and  $M_{ECON}$ , respectively.  $M_{MRC}$  corresponds to the minimum fuel cost and  $M_{ECON}$  to the minimum total cost.  $M_{LRC}$  is typically 2–5% faster than  $M_{MRC}$  for jet transport airplanes [20, 21].  $M_{ECON}$  is always faster than  $M_{MRC}$  when time-dependent costs are included, and, for most commercial passenger operations,  $M_{ECON}$  is a little slower than  $M_{LRC}$  (as illustrated in Figure 13.10). Note that there is little value in flying slower than  $M_{MRC}$  as both fuel-dependent and time-dependent costs increase with reducing speed. The airplane's certified maximum operating Mach number ( $M_{MO}$ ) sets an upper limit to the allowable cruise speed.

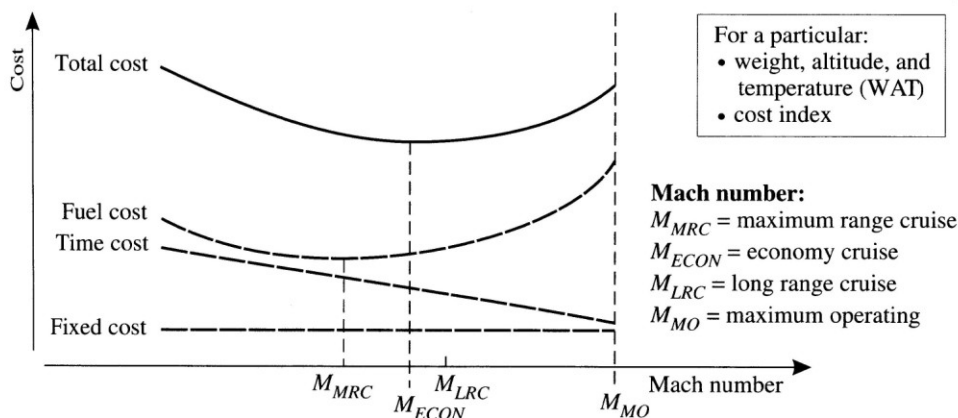


Figure 13.10 Characteristic (reference) Mach numbers for cruise.

#### 13.6.5 Optimum Altitude

An airplane's *optimum altitude* is that altitude which would result in the greatest SAR for a particular airplane weight, air temperature, and Mach number. Optimum altitude data are determined by flight testing and published in the airplane's FCOM for selected speeds (e.g., MRC, LRC) and temperatures (e.g., ISA, ISA + 10 °C, ISA + 20 °C). A common feature of such data—for many airplane types—is that the optimum altitude for a given cruise Mach number tends to increase in an almost linear fashion with reducing airplane weight.

The most efficient cruise—that is, a cruise that will consume the least fuel—corresponds to the airplane flying a cruise-climb along the locus of points representing the altitude optima. Due to flight level restrictions (imposed by ATC), jet transport airplanes routinely fly a stepped approximation of the cruise-climb, climbing to higher cruise altitudes as fuel is burned.