11 Empennage Sizing

In Section Empennage General Design, the areas of the horizontal and vertical tailplanes were calculated merely with the aid of tail volume coefficients. The tail lever arms were estimated as a percentage of the length of the fuselage. This initial estimate was necessary as a starting value for the iteration to be carried out here and in order to have tail parameters available for calculating mass and center of gravity. In this Section Empennage Sizing, the tail parameters will be calculated from stability and control requirements.

11.1 Horizontal Tailplane Sizing

Horizontal tailplane sizing according to control requirement

The horizontal tailplane sizing is based on the moment equilibrium around the lateral axis. The sum of the moments around the center of gravity gives

$$M_{CG} = M_W + L_W \cdot x_{CG-AC} + M_F + M_E + M_N - L_H \cdot \left(l_H - x_{CG-AC}\right) + M_H \ . \quad (11.1)$$

Designations from equation (11.1) are explained in **Fig. 11.1** in connection with **Fig. 11.2**. In addition:

M_F : The pitching moment of the fuselage (F) can be estimated according to **DATCOM 1978** (4.2.2). The DATCOM method is comparatively time-consuming and is not presented here. M_F is positive. For control, disregarding M_F is a conservative estimate.

M_E : The pitching moment through the engines (E) is caused:

1. by the fact that the thrust vector does not go through the center of gravity;
2. by a propeller force perpendicular to the propeller shaft upward or downward, caused when the flow against the propeller is not directly from the front, but at an angle to the propeller shaft;
3. by a change in the dynamic pressure over part of the wing;
4. by a change in the lift at the wing, also resulting in a change in the pitching moment caused by the wing;
5. by a change in the dynamic pressure on the horizontal tailplane;
6. by a change in the angle of attack of the horizontal tailplane.

In this case only Effect 1 will be taken into consideration. The pitching moment through the engine will therefore be described in simplified form as $M_E = -T \cdot z_E$. Effects 2 to 6 are dealt with in **DATCOM 1978** (4.6).

M_N : The pitching moment through the nacelles is not taken into account here; a calculation is possible if the lift and pitching moment of the nacelles are calculated with the methods for fuselages.

M_H : The moment around the y axis on the horizontal tailplane is comparatively small and is therefore not taken into account here. A calculation is, however, possible with the methods also used for wings.

It is also important to bear in mind that the pitching moment can change near to the ground due to:

- a change in the lift (or negative lift) on the horizontal tailplane due to the ground effect;
- a change in the lift on the wing due to the ground effect.

A calculation is possible according to **DATCOM 1978** (4.7.3).

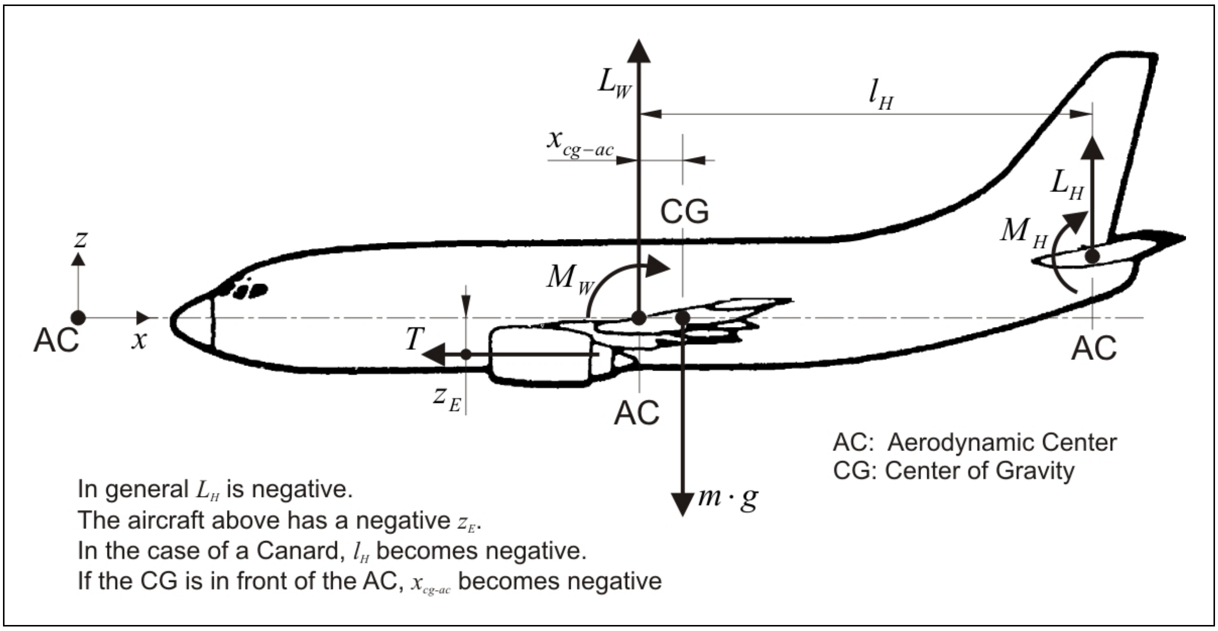

Fig. 11.1 Forces, moments and lever arms to calculate pitching moment

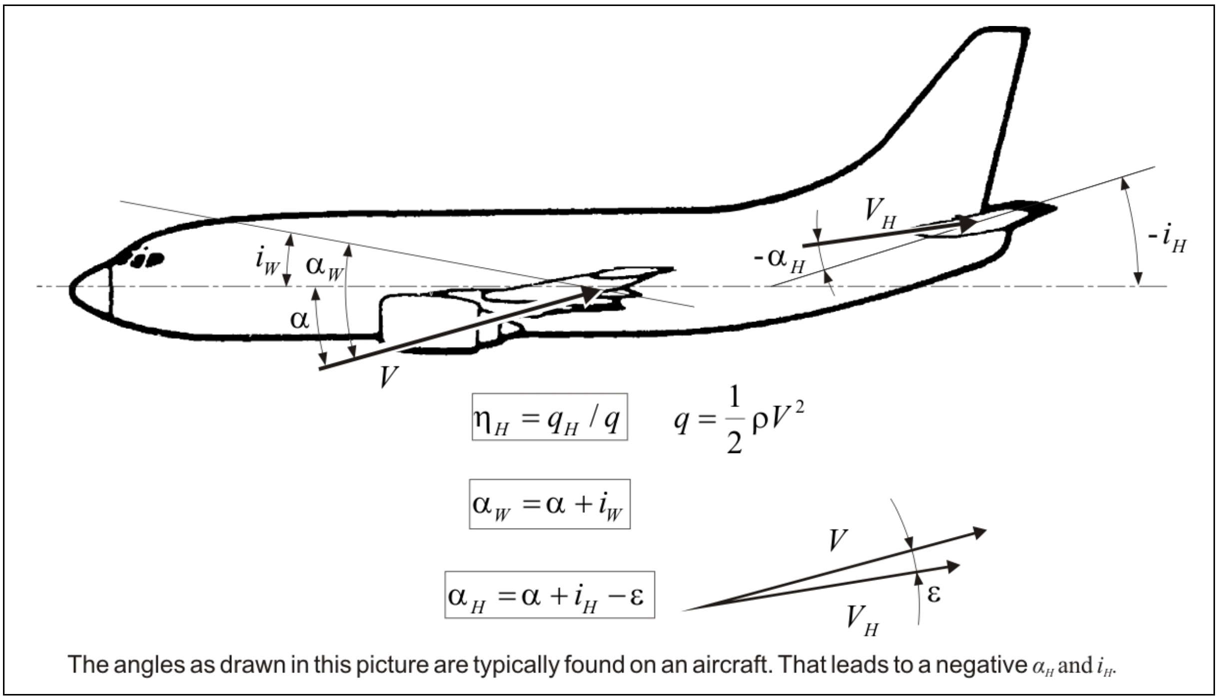

Fig. 11.2 Angles and flow velocities to calculate pitching moment

Taking into account the omissions discussed above, equation (11.1) is simplified to

$$M_{CG} = M_W + L_W \cdot x_{cg-ac} - T \cdot z_E - L_H \cdot \left(l_H - x_{CG-AC} \right) \quad . \tag{11.2}$$

The aircraft's total lift with the dynamic pressure of the free flow q is

$$L = C_L \cdot q \cdot S_W \quad . \tag{11.3}$$

The lift on the wing is

$$L_W = C_{L,W} \cdot q \cdot S_W \quad . \tag{11.4}$$

The lift on the horizontal tailplane is

$$L_H = C_{L,H} \cdot q_H \cdot S_H \quad . \tag{11.5}$$

L_H is negative (negative lift) if $C_{L,H}$ is negative. This is the case if (as usual) the angle of attack on the horizontal tailplane α_H is negative. The dynamic pressure q_H is less than the dynamic pressure on the wing. The reason for this is a delay in the flow caused by the wing drag. The reduction in dynamic pressure is expressed by the dynamic pressure ratio.

$$\eta_H = \frac{q_H}{q} \quad . \tag{11.6}$$

The pitching moment coefficient of the wing is related to wing area and mean aerodynamic chord (MAC)

$$M_W = C_{M,W} \cdot q \cdot S_W \cdot c_{MAC} \quad . \tag{11.7}$$

Similarly,

$$M_{CG} = C_{M,CG} \cdot q \cdot S_W \cdot c_{MAC} \quad . \tag{11.8}$$

Analogous to equation (11.7) we write

$$M_E = C_{M,E} \cdot q \cdot S_W \cdot c_{MAC} = -T \cdot z_E \quad . \tag{11.9}$$

Therefore

$$C_{M,E} = \frac{-T \cdot z_E}{q \cdot S_W \cdot c_{MAC}} \quad . \tag{11.10}$$

The aircraft's total lift is

$$L = L_W + L_H \quad . \tag{11.11}$$

In this simplified view the lift caused by the fuselage is only taken into account indirectly, in that the wing area S_W according to Section 7 also includes the area between the airfoils *in the* fuselage. With equations (11.3) to (11.6) the following applies

$$C_L = C_{L,W} + C_{L,H} \cdot \eta_H \cdot \frac{S_H}{S_W} \tag{11.12}$$

or

$$C_{L,W} = C_L - C_{L,H} \cdot \eta_H \cdot \frac{S_H}{S_W} \tag{11.13}$$

By inserting equations (11.3) to (11.7) in (11.2) and dividing the equation by $q \cdot S_W$ gives:

$$C_{M,CG} \cdot c_{MAC} = C_{M,W} \cdot c_{MAC} + C_{L,W} \cdot x_{cg-ac} + C_{M,E} \cdot c_{MAC} - \eta_H \cdot \frac{S_H}{S_W} \cdot C_{L,H} \cdot \left(l_H - x_{CG-AC}\right) \tag{11.14}$$

Now we divide the equation by c_{MAC}, multiply out, and simplify the notation at the same time by inserting the abbreviation

$$\overline{x_{CG-AC}} = \frac{x_{CG-AC}}{c_{MAC}} \tag{11.15}$$

$$C_{M,CG} = C_{M,W} + C_{L,W} \cdot \overline{x_{CG-AC}} + C_{M,E} - C_{L,H} \cdot \eta_H \cdot \frac{S_H}{S_W} \cdot \frac{l_H}{c_{MAC}} + C_{L,H} \cdot \eta_H \cdot \frac{S_H}{S_W} \cdot \overline{x_{CG-AC}} \qquad (11.16)$$

We insert $C_{L,W}$ according to equation (11.13) and require that for a **moment equilibrium** $M_{CG} = 0$ or $C_{M,CG} = 0$ must apply. This then gives

$$0 = C_{M,W} + C_L \cdot \overline{x_{CG-AC}} - C_{L,H} \cdot \eta_H \cdot \frac{S_H}{S_W} \cdot \overline{x_{CG-AC}} + C_{M,E}$$
$$- C_{L,H} \cdot \eta_H \cdot \frac{S_H}{S_W} \cdot \frac{l_H}{c_{MAC}} + C_{L,H} \cdot \eta_H \cdot \frac{S_H}{S_W} \cdot \overline{x_{CG-AC}} \qquad (11.17)$$

In summary, equation (11.17) gives

$$0 = C_{M,W} + C_L \cdot \overline{x_{CG-AC}} + C_{M,E} - C_{L,H} \cdot \eta_H \cdot \frac{S_H}{S_W} \cdot \frac{l_H}{c_{MAC}} \quad . \qquad (11.18)$$

Note that in this equation the term $\frac{S_H}{S_W} \cdot \frac{l_H}{c_{MAC}} = C_H$ occurs, which is already familiar from Section 9.

The (dimensionless) **horizontal tailplane area** S_H / S_W arising from control requirements depends on the center-of-gravity position according to equation (11.18) and is **described by** the **straight line** $S_H / S_W = a \cdot \overline{x_{CG-AC}} + b$. The parameters *a* and *b* can be taken from the following complete equation:

$$\frac{S_H}{S_W} = \frac{C_L}{C_{L,H} \cdot \eta_H \cdot \frac{l_H}{c_{MAC}}} \cdot \overline{x_{CG-AC}} + \frac{C_{M,W} + C_{M,E}}{C_{L,H} \cdot \eta_H \cdot \frac{l_H}{c_{MAC}}} \quad . \qquad (11.19)$$

Parameters in equation (11.19):

$\overline{x_{CG-AC}}$	is calculated according to equation (11.15).
$C_{M,E}$	is calculated according to equation (11.10). $C_{M,E}$ is positive for engines under the wing and negative for rear engines.
η_H	assumes values between 0.85 and 0.95. A typical mean value is **0.9**. A calculation of η_H is possible with **DATCOM 1978** (4.4.1).
$C_{L,H}$	$C_{L,H}$ could be calculated according to equation (11.27). However, to do this, the flow against the horizontal tailplane and the incidence angle of the horizon-

tal stabilizer would have to be known. It is simpler for the empennage sizing to establish a value for $C_{L,H}$. With $C_{L,H} = -0{,}5$ a value is established that takes into account that the horizontal tailplane produces negative lift and, with the low figure of 0.5, it is ensured that the horizontal tailplane is not stalled.

Calculation of the remaining parameters is dealt with in Section 11.2 *Coefficients for horizontal tailplane sizing*. The following **qualitative statements** are already possible:

$C_{M,W}$ is negative; the amount of $C_{M,W}$ is greater with the flaps extended.

$C_L \cdot \overline{x_{CG-AC}}$ C_L is positive in any event. $C_L \cdot \overline{x_{CG-AC}}$ can be positive and becomes greater the further the center of gravity moves backward and the greater the lift coefficient. The shifting of the center of gravity to the rear is however (as will be shown later) subject to limits due to stability requirements.

$C_{L,H} \cdot l_H$ $C_{L,H}$ is negative for conventional empennage placement behind the wing, but positive in the case of the canard. In both cases the term $C_{L,H} \cdot l_H$ is negative but delivers a positive moment.

An examination of the **parameters** shows that ***a*** will be negative, as a rule, and ***b*** will be positive. These prior considerations and Section 9.1 lead to **critical flight states**:

- For engines below the center of gravity, i.e. $z_E < 0$ (for example, engines under the wing) a critical flight state is: **landing approach, maximum flap position, foremost center-of-gravity position**.
- For engines above the center of gravity, i.e. $z_E > 0$ (e.g. rear engines) a critical flight state is: **missed approach, maximum flap position, foremost center-of-gravity position**.

Dimensioning could **also** be influenced by the following, according to Section 9.1: 1.) rotation during take off, 2.) flare during landing, 3.) control with trimmed horizontal stabilizer. The flight cases 1.) and 2.) require a calculation of the pitching moment coefficients with ground effect. In 3.) extensive calculations according to JAR 25.255 have to be carried out.

Horizontal tailplane sizing according to stability requirement

The following example is intended to demonstrate that the gradient of the pitching moment coefficient $C_{M,CG}$ over the angle of attack, i.e. $\partial C_{M,CG} / \partial \alpha$ - also written as $C_{M,\alpha}$ - is responsible for the stability of the aircraft around the lateral axis. A distinction must be made between the following two cases:

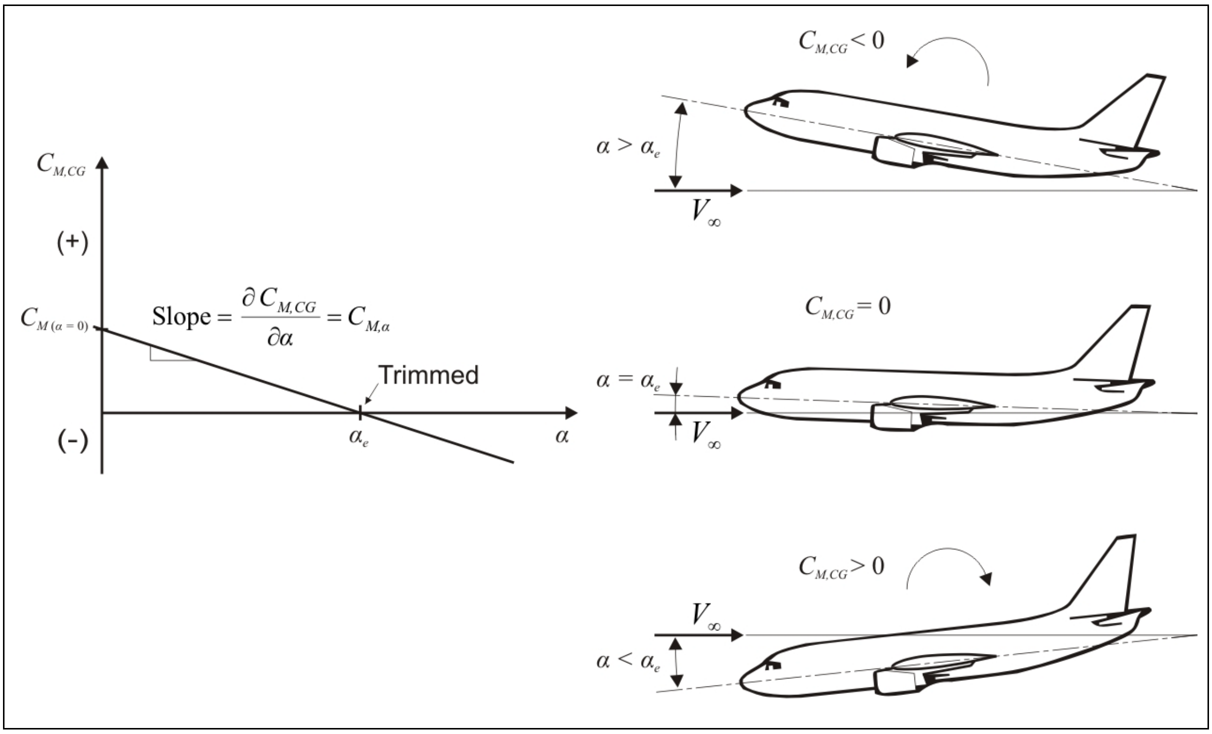

Fig. 11.3 Pitching moment coefficient as a function of angle of attack with a negative gradient. Aircraft moments with a negative gradient lead to stable flight characteristics.

1. The gradient $\partial C_{M,CG} / \partial \alpha$ is negative as shown in **Fig. 11.3**. The aircraft flies trimmed and with moment equilibrium, i.e. with $M_{CG} = 0$ or $C_{M,CG} = 0$. The angle of attack at which this moment equilibrium is achieved is (depending on the elevator and horizontal stabilizer angle) α_e (*e* stands for *equilibrium*). If the angle of attack now changes (for instance, due to minor control deflections or a gust), the moment around the center of gravity also changes according to Fig. 11.3. The result shows that in any event the moment has a direction which acts against the disturbance. The aircraft is therefore stable around the lateral axis.
2. The gradient of the pitching moment coefficient $\partial C_{M,CG} / \partial \alpha$ is positive, as demonstrated in **Fig. 11.4**. If the angle of attack now changes, the moment around the center of gravity also changes according to Fig. 11.4. The result shows that in any event the moment has a direction that increases the disturbance. The aircraft is therefore unstable around the lateral axis.

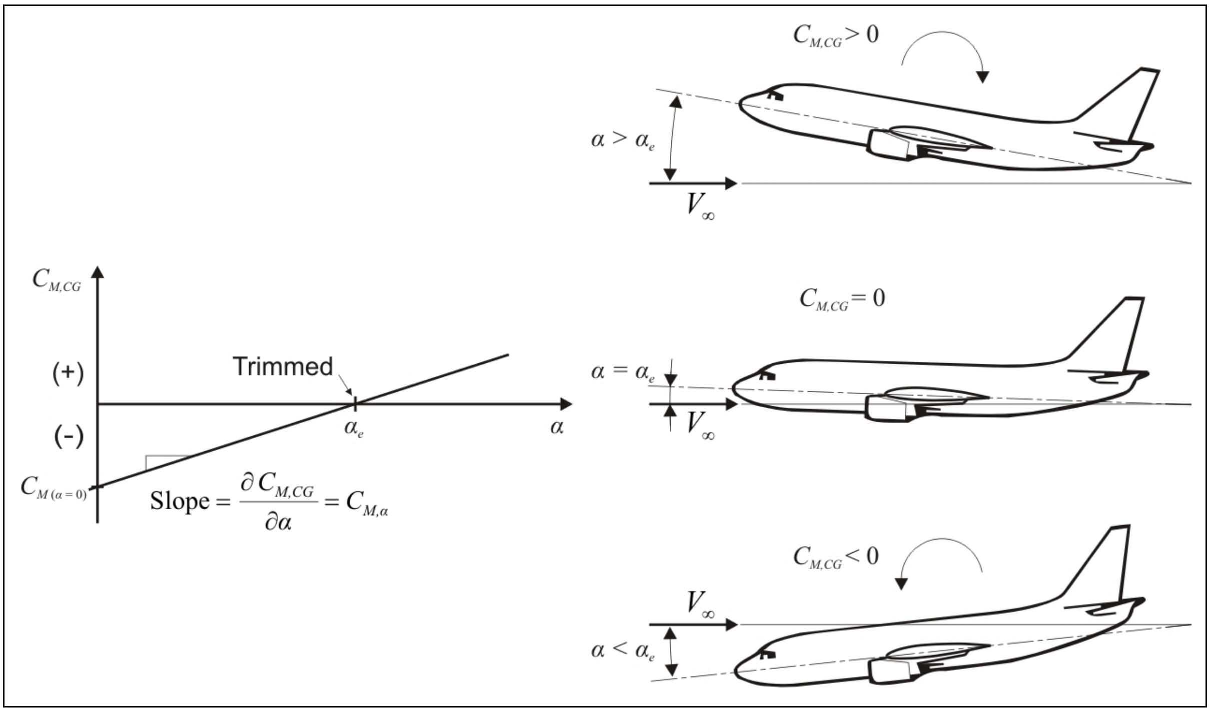

Fig. 11.4 Pitching moment coefficient as a function of angle of attack with a positive gradient. Aircraft moments with a positive gradient lead to unstable flight characteristics.

Necessary **requirement for static longitudinal stability:**

1. $\partial C_{M,CG} / \partial \alpha$ is negative.
2. $C_{M,CG}(\alpha = 0)$ is positive.

Pitching moment curve needs to resemble the curve in Fig. 11.3.

For a detailed examination of the static stability around the lateral axis we use equation (11.16) as a basis and first insert equations to calculate $C_{L,W}$ and $C_{L,H}$ (see Section 11.2).

$$C_{M,CG} = C_{M,W} + C_{L,\alpha,W}\left(\alpha + i_w - \alpha_{0,W}\right) \cdot \overline{x_{CG-AC}} + C_{M,E}$$
$$- C_{L,\alpha,H}\left(\alpha + i_H - \varepsilon - \alpha_{0,H}\right) \cdot \eta_H \cdot \frac{S_H}{S_W} \cdot \left(\frac{l_H}{c_{MAC}} - \overline{x_{CG-AC}}\right) \quad . \qquad (11.20)$$

Equation (11.20) is now derived according to α. In doing so, the following has to be taken into account:

$C_{M,W}$ the pitching moment coefficient relative to the aerodynamic center is constant by definition, and therefore also independent of the angle of attack.

$C_{L.\alpha}$ is constant for both the wing (W) and the horizontal tailplane (H) as long as the angle of attack remains in the linear range, i.e. does not get too close to the angle of attack of the maximum lift coefficient. Differing flap positions of Fowler flaps and slats demonstrate different lift gradients (see Fig. 8.1 and Fig. 8.3), but for a selected flap position $C_{L.\alpha}$ in the linear range is also independent of the angle of attack.

$C_{M,E}$ here only takes into account the influence of the engine placement in relation to the center of gravity. The engine influence – referred to above as Effect 1 – is independent of the angle of attack. The other Effects 2 to 6 partly display dependency on the angle of attack, but are disregarded here. Details can be found in **DATCOM 1978** (4.6.3).

ε the downwash angle is dependent on the lift on the wing and therefore also on the angle of attack. A calculation method for ε and $d\varepsilon / d\alpha$ can be found in Section 11.2.

η_H is dependent on the drag coefficient with zero lift and is therefore assumed to be constant as an initial approximation.

i_H the incidence angle of the horizontal tailplane is constant for an examined trimmed flight state.

i_W, $\alpha_{0,W}$, $\alpha_{0,H}$ and lever arms are constant.

With these prior considerations the following applies:

$$C_{M,\alpha,CG} = C_{L,\alpha,W} \cdot \overline{x_{CG-AC}} - C_{L,\alpha,H} \cdot \eta_H \cdot \frac{S_H}{S_W} \cdot \left(1 - \frac{\partial\varepsilon}{\partial\alpha}\right) \cdot \left(\frac{l_H}{c_{MAC}} - \overline{x_{CG-AC}}\right) \quad . \qquad (11.21)$$

We obtain an indifferent equilibrium, i.e. limiting stability, with $C_{M,\alpha,CG} = \partial C_{M,CG} / \partial\alpha = 0$

$$0 = C_{L,\alpha,W} \cdot \overline{x_{CG-AC}} - C_{L,\alpha,H} \cdot \eta_H \cdot \frac{S_H}{S_W} \cdot \left(1 - \frac{\partial\varepsilon}{\partial\alpha}\right) \cdot \left(\frac{l_H}{c_{MAC}} - \overline{x_{CG-AC}}\right) \quad . \qquad (11.22)$$

A condition for the horizontal tailplane area can be derived from this condition

$$\frac{S_H}{S_W} = \frac{C_{L,\alpha,W} \cdot \overline{x_{CG-AC}}}{C_{L,\alpha,H} \cdot \eta_H \cdot \left(1 - \frac{\partial\varepsilon}{\partial\alpha}\right) \cdot \left(\frac{l_H}{c_{MAC}} - \overline{x_{CG-AC}}\right)} \quad . \qquad (11.23)$$

As l_H / c_{MAC} is much greater than $\overline{x_{CG-AC}}$, we simplify equation (11.23) in order to obtain an idea of the interrelationships:

$$\frac{S_H}{S_W} \approx \frac{C_{L,\alpha,W}}{C_{L,\alpha,H} \cdot \eta_H \cdot \left(1 - \frac{\partial\varepsilon}{\partial\alpha}\right) \cdot \frac{l_H}{c_{MAC}}} \cdot \overline{x_{CG-AC}} \quad . \qquad (11.24)$$

The (dimensionless) **horizontal tailplane area** S_H / S_W arising from stability requirements depends, according to equation (11.23), on the center-of-gravity position and is approximately **described** with equation (11.24) **by the straight line** $S_H / S_W \approx a \cdot \overline{x_{CG-AC}}$. Parameter a can be taken from equation (11.24).

An examination of the **parameters** in equation (11.24) shows that ***a*** will be positive, as a rule.

Horizontal tailplane sizing – overall picture

Requirements with regard to the (dimensionless) horizontal tailplane area S_H / S_W can be drawn jointly in one diagram (**Fig. 11.5**) on the basis of the **control requirement** (equation (11.19)) and on the basis of the **stability requirement** (equation (11.23)). It can be seen that a necessary center-of-gravity range Δx ascertained according to Section 10 from the loading diagram requires a **minimum area of the horizontal tailplane**.

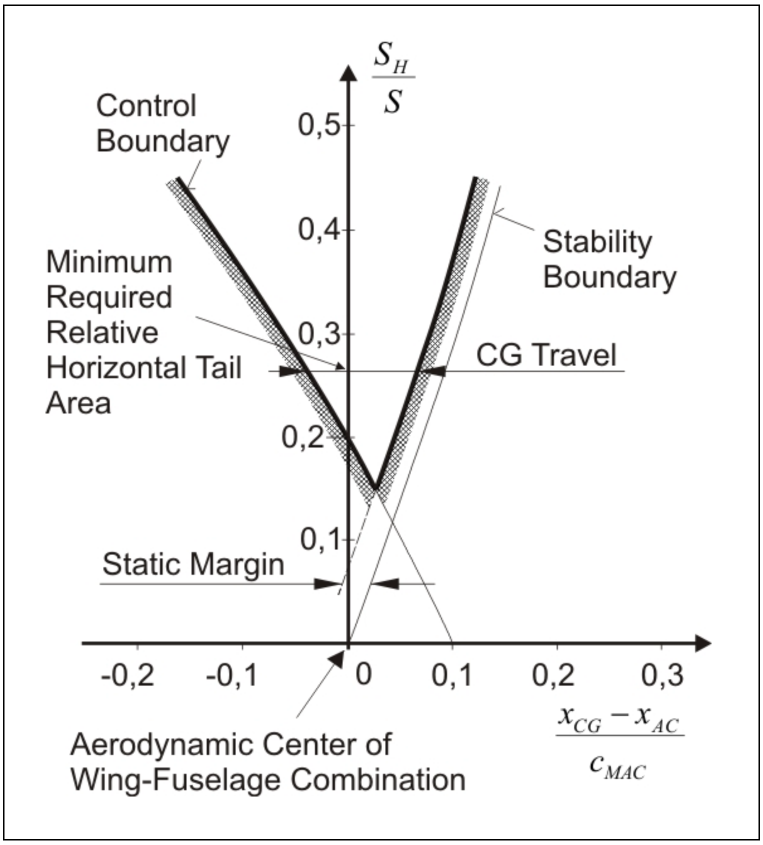

Fig. 11.5 Diagram to determine the minimum required relative horizontal tail area considering controllability and stability requirements as well as the required CG-range Δx

Fig. 11.5 also shows that the rear center-of-gravity position has to comply with a "safety margin" separating it from the natural stability limit. This **static margin** is stated in **Table 11.1**.

The connection with the pitching moment coefficient $\partial C_{M,CG} / \partial \alpha = C_{M,\alpha}$ - the initially discussed measure of stability – is given by

$$C_{M,\alpha} = -C_{L,\alpha} \cdot (\text{static margin}) \quad . \tag{11.25}$$

"static margin" is stated as a percentage of the mean aerodynamic chord, MAC, with values between 0 and 1. $C_{L,\alpha}$ is the wing's lift gradient. Target values of the static margin for the aircraft design are given in the form of the pitching moment coefficient $C_{M,\alpha}$ in **Fig. 11.6**.

If **engine effects** 2. to 6. (see Section 11.1) are disregarded, it must be expected (according to **Raymer 1989**) that the **static margin** will be **reduced** by:
4% - 10% in the case of propeller aircraft;
1% - 3% in the case of jets.

Table 11.2 summarizes the **process for sizing the horizontal tailplane**. Sizing of the horizontal tailplane also means establishing the aircraft's center of gravity, including moving the wing in relation to the fuselage, should this be necessary.

Table 11.1 Required static margins of different models of aircrafts (**Roskam II**)

airplane category	**static margin**
homebuilts	10% MAC
single engine propeller driven airplanes	10% MAC
twin engine propeller driven airplanes	10% MAC
agricultural airplanes	10% MAC
business jets	5% MAC
regional turbopropeller driven airplanes	5% MAC
jet transports	5% MAC
military trainers	5% MAC
fighters (with natural stability)	5% MAC
military patrol and transport airplanes	5% MAC
flying boats, amphibious and float airplanes	5% MAC
supersonic cruise airplanes	5% MAC

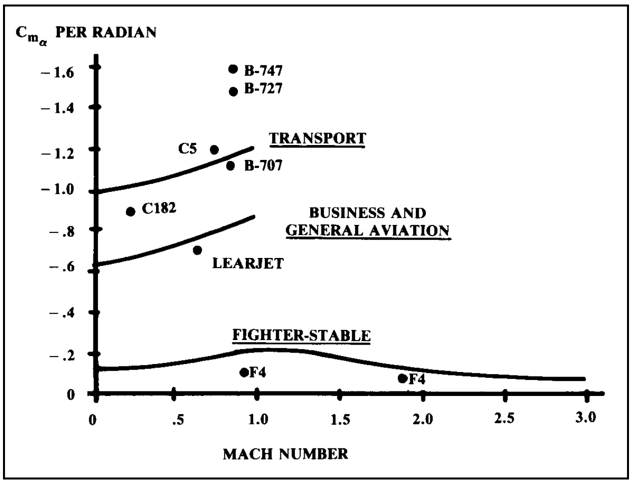

Fig. 11.6 Typical pitching moment coefficients $C_{M,\alpha}$ for aircraft design predefinition of static margin (**Raymer 1989**)

Table 11.2 Procedure for horizontal tail sizing

<u>Step 1</u> <u>(Section 9)</u>		
l_H	horizontal tail lever arm	approximated
C_H	horizontal tail volume coefficient	statistic
S_H	horizontal tail area	calculated from l_H and C_H
<u>Step 2</u> <u>(Section 10)</u>		
m_H	horizontal tail mass	calculated from statistic equation
x_{CG}	aircraft CG 1st action	calculated
x_{CG}	aircraft CG 2nd action	adjust wing position in a way that
		$x_{CG,LEMAC}$ = 0.25 c_{MAC}
		or choose another reasonable value
Δx	CG-range	from loading sheet
<u>Step 3</u> <u>(Section 11)</u>		
S_H	horizontal tail area	from diagram in Fig. 11.5
S_H	<u>*Comparison*</u>	with S_H from step 1:
	if	difference greater then 10%,
	then	calculate new m_H (see section 10).
$x_{CG,aft}$	aft CG position	from diagram in Fig 11.7
$x_{CG,aft}$	<u>*Comparison*</u>	with $x_{CG,aft}$ from loading sheet:
	if	difference less then 3%,
	then	**END**
	else	adjust wing position (see Section 10).
l_H	horizontal tail lever arm	new calculation from fuselage geometry
↵	go back to the beginning of step 3	

11.2 Parameters for Horizontal Tailplane Sizing

Aerodynamic center

The aerodynamic center (AC) for wings with $A \geq 5$ and $\varphi_{25} \leq 35°$ corresponds to the aerodynamic center of the airfoil. The aerodynamic center of the airfoil is roughly $0.25 \cdot C_{MAC}$. More precise figures are contained in **Fig. 11.7**.

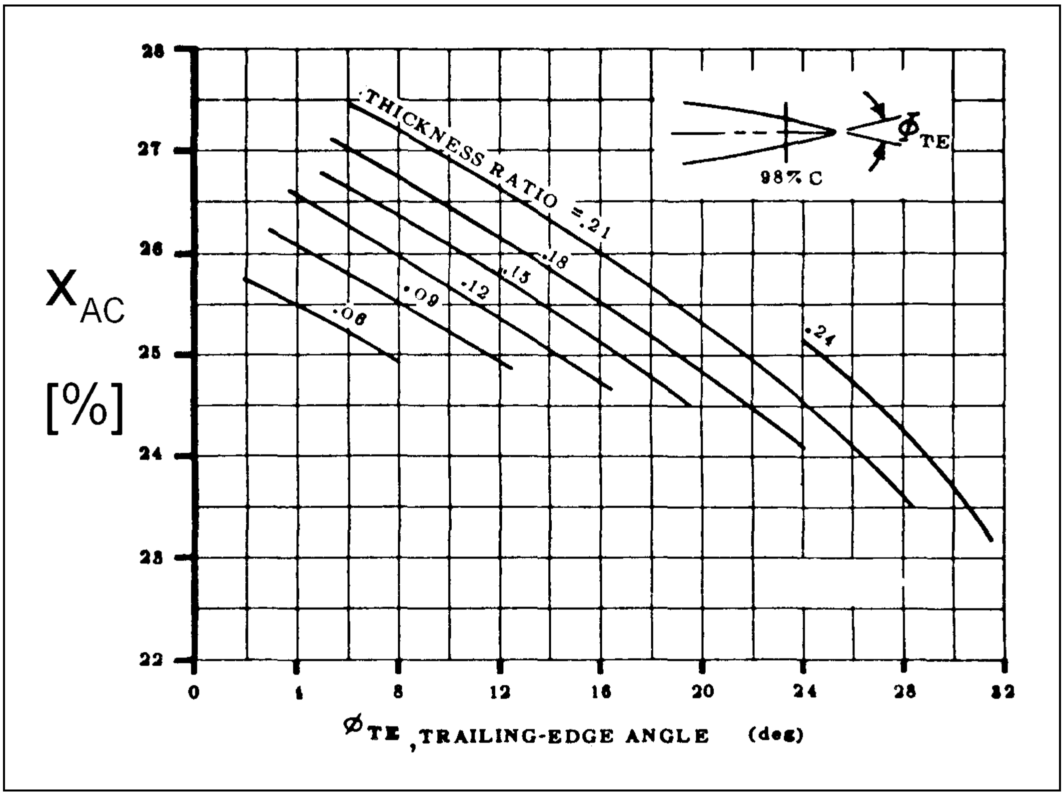

Fig. 11.7 Influence of trailing-edge angle and relative airfoil thickness on aerodynamic center x_{AC} / C_{MAC} at subsonic speeds (**DATCOM 1978**)

Lift coefficient

The lift coefficient of wings and horizontal tailplane is simply calculated from the lift gradient and the angle of attack measured on the basis of the zero lift angle of attack α_0. In the case of cambered airfoils α_0 is negative. In the case of the horizontal tailplane, the downwash angle ε caused by the wing must also be taken into account.

$$C_{L,W} = C_{L,\alpha,W} \cdot \left(\alpha + i_W - \alpha_{0,W}\right) \quad . \tag{11.26}$$

$$C_{L,H} = C_{L,\alpha,H} \cdot \left(\alpha + i_H - \varepsilon - \alpha_{0,H}\right) \quad . \tag{11.27}$$

It is important to ensure that no lift coefficients larger than the maximum lift coefficient are calculated with the equations. The maximum lift coefficient for wings and empennages, including the slats, flaps and elevators, can be calculated using the methods outlined in Section 8. Empennages should never be operated with a lift coefficient close to the maximum lift coefficient. For safety reasons, sufficient reserve for additional lift must always be available here.

Zero lift angle of attack for a wing

According to (**DATCOM 1978**) (4.1.3.1), the zero-lift angle of attack of a wing $\alpha_{0,W}$ (and, of course, empennages) with linear wing twist is estimated from

$$\alpha_{0,W} = \alpha_0 + \frac{\Delta\alpha_0}{\varepsilon_t} \cdot \varepsilon_t \cdot \frac{\alpha_{0,at\ M}}{\alpha_{0,at\ M=0.3}} \quad . \tag{11.28}$$

α_0 is the zero lift angle of attack of the airfoil. The value must be taken from airfoil catalogs, such as **Abbott 1959**. In the case of symmetrical airfoils $\alpha_0 = 0$. In the case of cambered airfoils α_0 can assume values of up to approximately -4°.

$\frac{\Delta\alpha_0}{\varepsilon_t}$ Change in the zero lift angle per angular unit of the wing twist

$\frac{\alpha_{0,at\ M}}{\alpha_{0,at\ M=0.3}}$ Mach number correction.

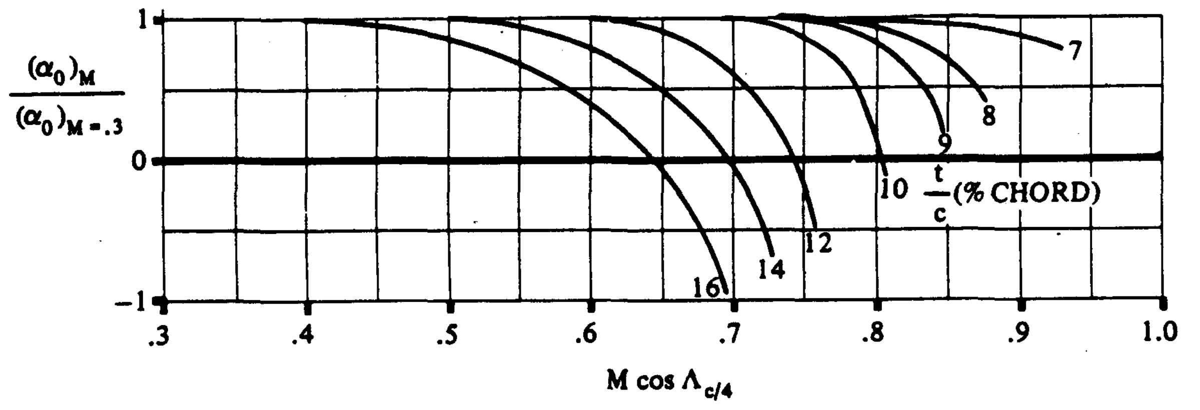

Fig. 11.8 Mach number correction to calculate wing zero lift angle (**DATCOM 1978**)

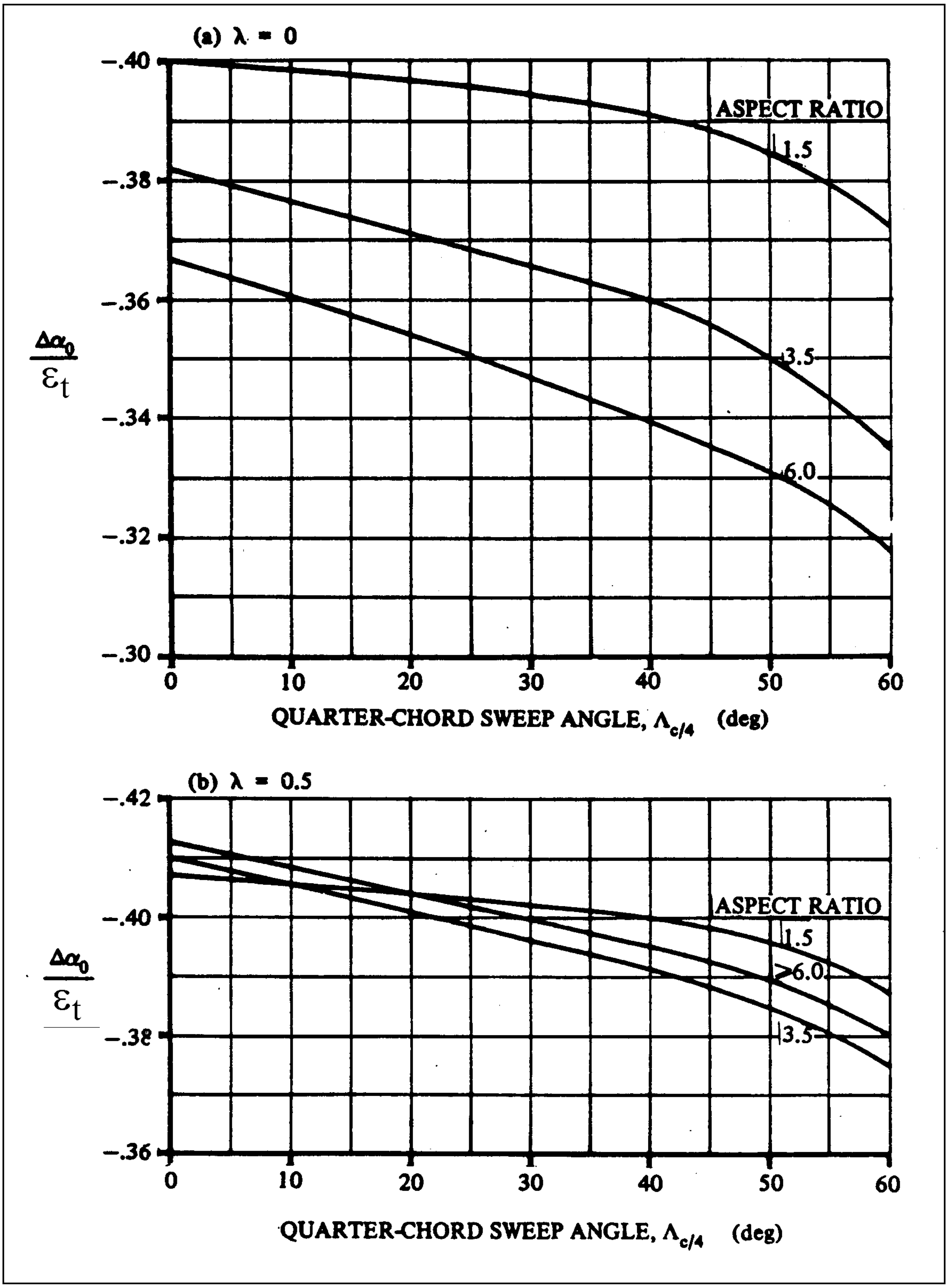

Fig. 11.9a Influence of linear wing twist on the wing zero lift angle (**DATCOM 1978**) - Fig. 1 out of 2

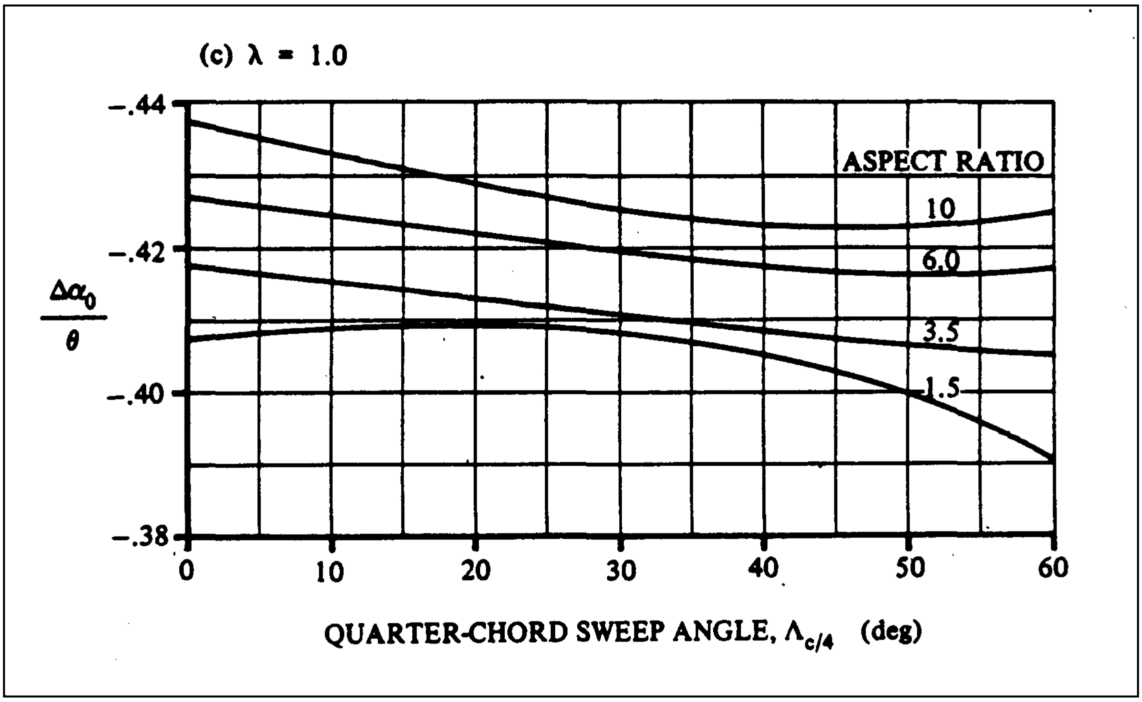

Fig. 11.9b Influence of linear wing twist on the wing zero lift angle (**DATCOM 1978**) - Fig. 2 out of 2

Downwash angle

According to **Dubs 1987**, the downwash angle ε in rad is calculated from the following for wings with elliptical lift distribution

$$\varepsilon = \frac{C_{L,W}}{\pi \cdot A} \cdot \left[\left(1 - \frac{C_{L,W}}{\sqrt{C_{L,W}^{\ 2} + 1}} \right) \cdot \delta_1 + \frac{C_{L,W}}{\sqrt{C_{L,W}^{\ 2} + 1}} \cdot \delta_2 \right] \quad . \tag{11.29}$$

δ_1 and δ_2 are additional parameters that can be taken from **Fig.11.10**. For typical aircraft geometries the square brackets produce a value of 1.75. If the lift distribution is more complete in the middle of the wing (e.g. in the case of extended landing flaps), then the downwash angle on the horizontal tailplane is up to 10% greater. On the other hand, if the lift distribution is more complete on the wing tips, the downwash angle can be up to 15% less on the horizontal tailplane. The corresponding DATACOM method (4.4.1) for calculating the downwash angle is considerably more complex than this DUBS method.

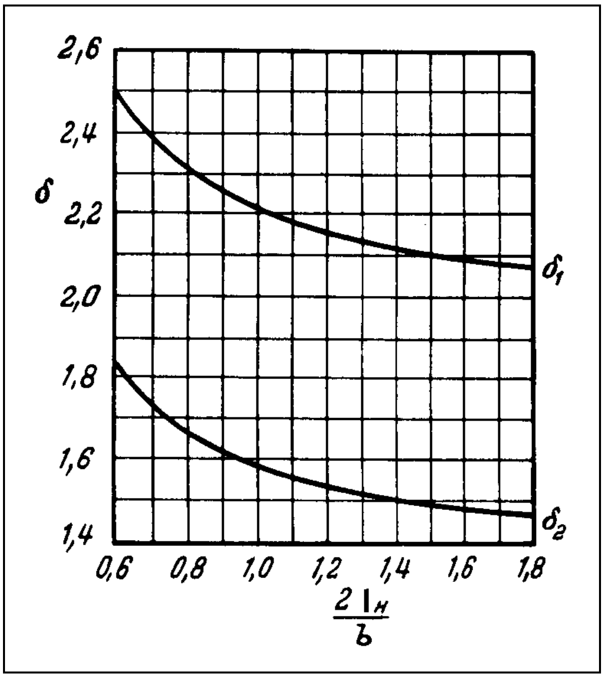

Fig. 11.10 Diagram for calculation of horizontal tail downwash angle with the help of horizontal tail lever arm l_H and wing span b (**Dubs 1987**)

Pitching moment of the airfoil at the aerodynamic center

The pitching moment coefficient of the airfoil in relation to the aerodynamic center $c_{M,0}$ must be taken from airfoil catalogs such as **Abbott 1959**. For symmetrical airfoils $c_{M,0}$ = 0. In the case of cambered airfoils $c_{M,0}$ can assume values up to approximately - 0.1.

If flaps are extended, the size of the pitching moment increases (i.e. becomes more negative) and the following applies:

$$c_{M,0,flaped} = c_{M,0} + \Delta c_M \quad . \qquad (11.30)$$

The following applies to **Fowler flaps or slotted flaps**:

$$\Delta c_M = \Delta c_{L,flaped} \cdot \left(\frac{x_{AC}}{c_{MAC}} - \frac{x_{CP}}{c_{MAC}} \left(\frac{c'}{c} \right) \right) \quad \text{with} \quad \frac{x_{CP}}{c_{MAC}} = 0.44 \quad . \qquad (11.31)$$

The following applies to **plain flaps**:

$$\Delta c_M = \Delta c_{L,flaped} \cdot \left(\frac{x_{AC}}{c_{MAC}} - \frac{x_{CP}}{c_{MAC}} \right) \quad \text{mit} \quad \frac{x_{CP}}{c_{MAC}} = -0.25 \cdot \frac{c_F}{c} + 0.5 \ . \qquad (11.32)$$

$\Delta c_{L,flaped}$ Increase in the maximum lift coefficient of an airfoil due to Fowler, slotted or plain flap.

$\frac{x_{AC}}{c_{MAC}}$ Position of the aerodynamic center in relation to the length of the mean aerodynamic chord.

$\frac{x_{CP}}{c_{MAC}}$ Position of the center of pressure CP in relation to the length of the mean aerodynamic chord c_{MAC}. See equations (11.31) and (11.32).

c'/c see **Fig. 11.11**.

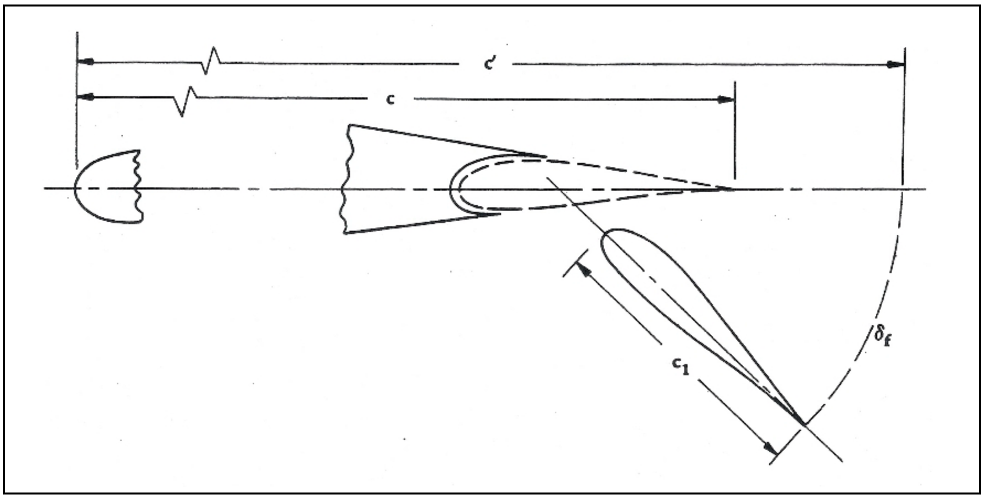

Fig. 11.11 Definition of Fowler and slotted-flaps geometry (**DATCOM 1978**)

Pitching moment of the wing at the aerodynamic center

The pitching moment coefficient of the wing in relation to the aerodynamic center is calculated here according to **DATCOM 1978** (4.1.4.1). In doing so, the pitching moment coefficient of a "mean" wing section must be used as a basis for the calculation. If the flaps have been retracted, the pitching moment coefficient $c_{M,0}$ from the airfoil catalog is used. If the flaps on the wing have been extended, the pitching moment coefficient of the airfoil $c_{M,0,flap}$ according to equation (11.30) should be used in the calculation in this case (pursuant to the suggestion from **Raymer 1989**). This approach assumes that the flaps extend over the full span. Should this not be the case, the amount of the wing pitching moment will be overestimated accordingly. If the wing pitching moment is too great, this represents a conservative estimate in the design of the horizontal tailplane. **DATCOM 1978** (6.1.5.1) contains more precise calculation methods for changing the pitching moment coefficient caused by flaps and slats.

The pitching moment coefficient of the wing in relation to the aerodynamic center is

$$C_{M,W} = \left[c_{M,0,flaped} \cdot \frac{A \cdot \cos^2 \cdot \varphi_{25}}{A + 2\cos\varphi_{25}} + \left(\frac{\Delta c_{m,0}}{\varepsilon_t} \right) \cdot \varepsilon_t \right] \cdot \frac{\left(c_{m,0}\right)_M}{\left(c_{m,0}\right)_{M=0}} \quad . \tag{11.33}$$

$\left(\frac{\Delta c_{m,0}}{\varepsilon_t} \right)$ Change to the pitching moment coefficient with respect to the aerodynamic center per degree of linear wing twist according to **Fig. 11.12**.

ε_t Wing twist between wing root and wing tip in degree. If the incidence angle decreases toward the wing tip, ε_t is negative (see also Section 7.3). A linear distribution of the twist is assumed.

$\frac{\left(c_{m,0}\right)_M}{\left(c_{m,0}\right)_{M=0}}$ Mach number influence according to **Fig. 11.13**.

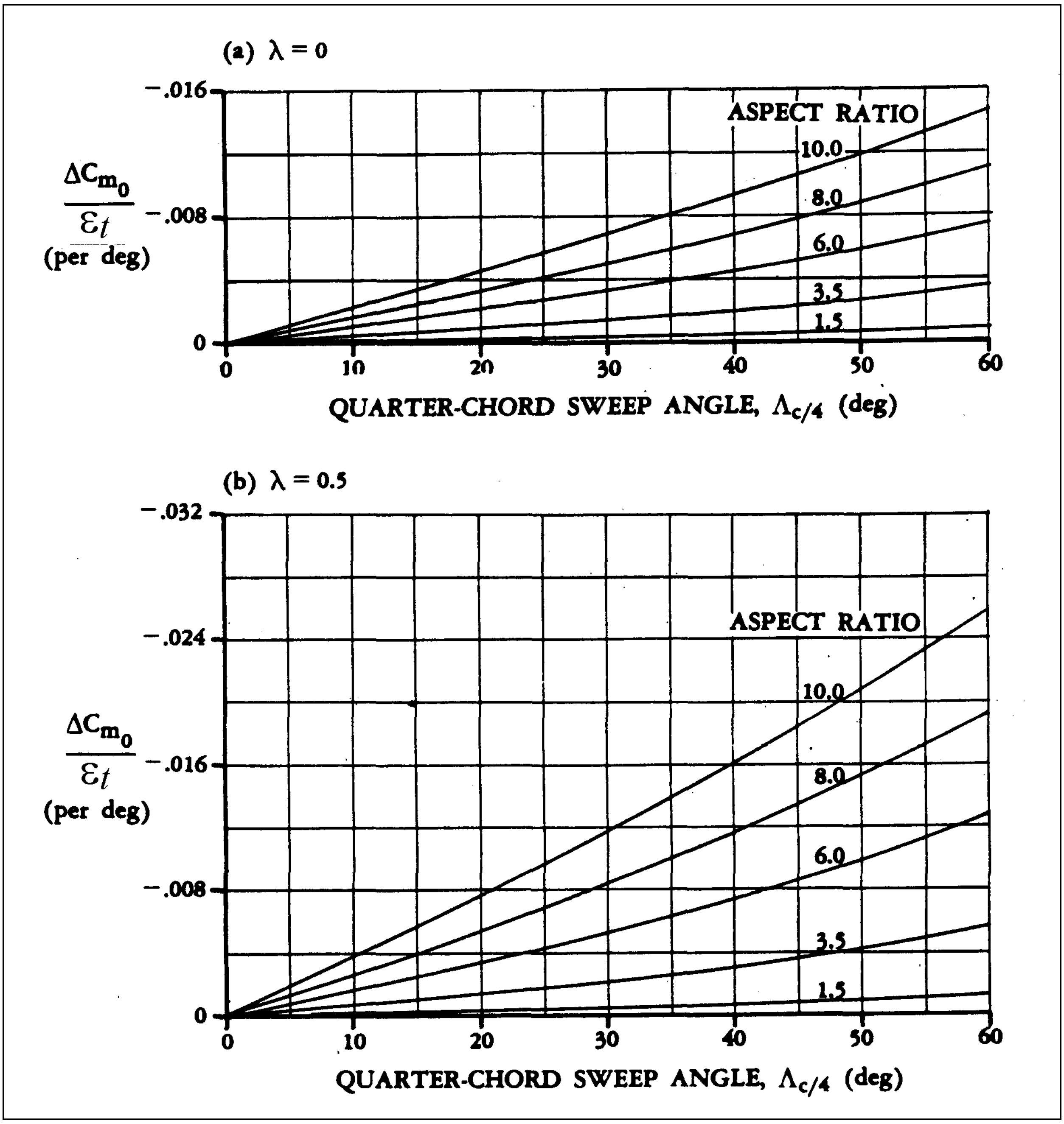

Fig. 11.12a Influence of linear wing twist on pitching moment coefficient with respect to the wing aerodynamic center. $\Lambda_{c/4}$ stands for quarter-chord sweep angle φ_{25}. (**DATCOM 1978**) - Fig. 1 out of 2

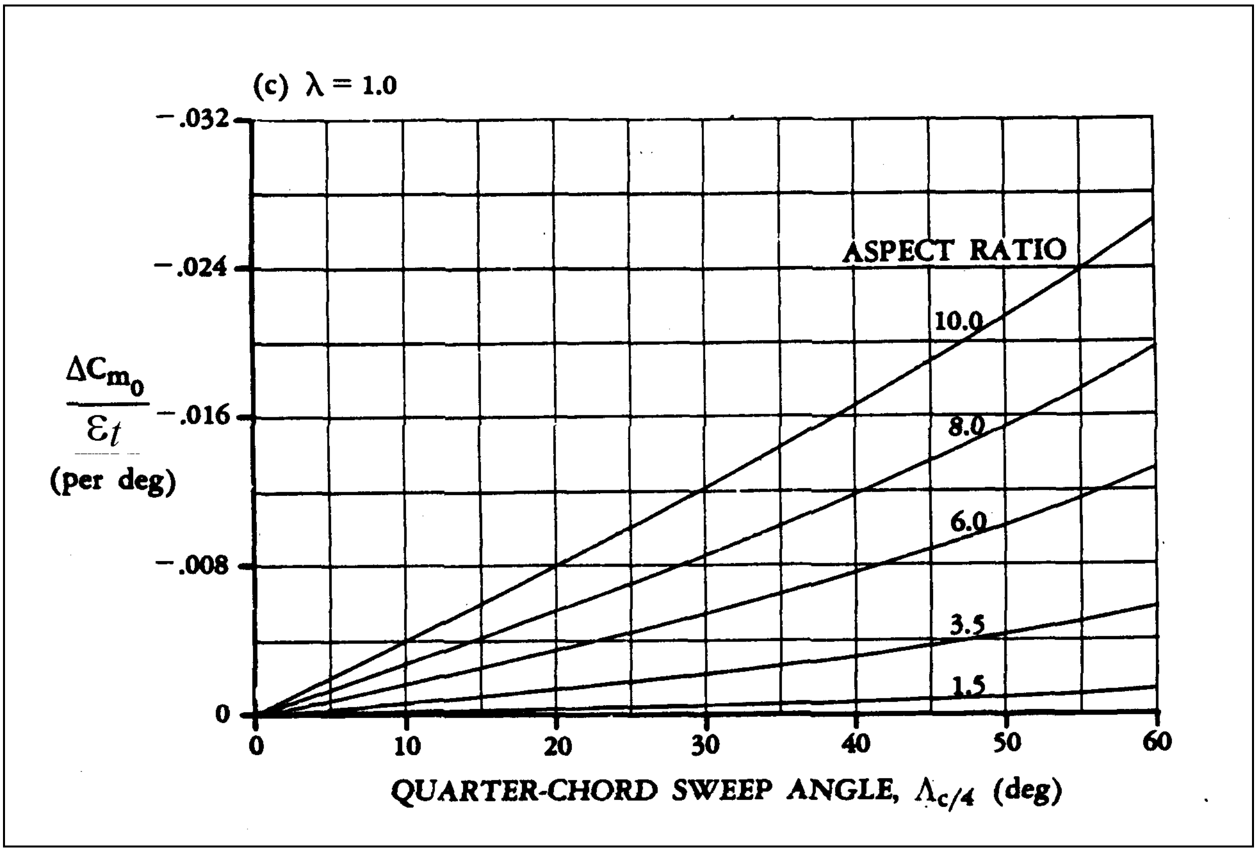

Fig. 11.12b Influence of linear wing twist on pitching moment coefficient with respect to the wing aerodynamic center. $\Lambda_{c/4}$ stands for quarter-chord sweep angle φ_{25} (**DATCOM 1978**) - Fig. 2 out of 2

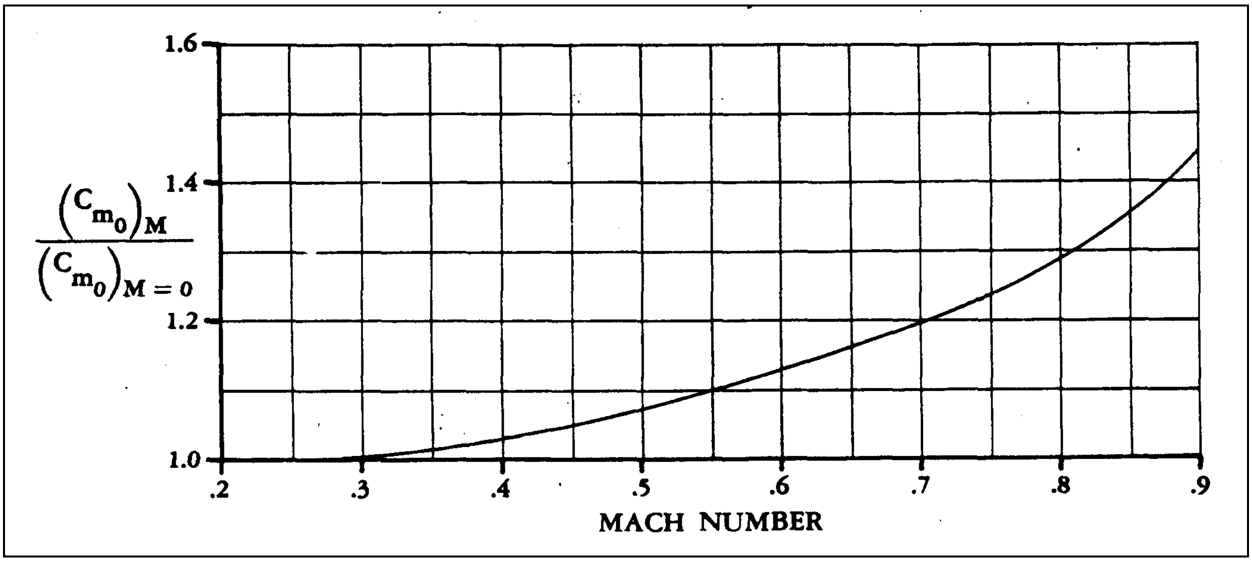

Fig. 11.13 Mach number correction to calculate pitching moment coefficient with respect to the wing aerodynamic center (**DATCOM 1978**)

Downwash gradient $\partial\varepsilon / \partial\alpha$

The average downwash gradient $\partial\varepsilon / \partial\alpha$ on the horizontal tailplane behind a wing is calculated here according to **DATCOM 1978** (4.4.1 Method 2). This method only applies to angles of attack where $\partial\varepsilon / \partial\alpha$ is a linear function of α - i.e. only "small" to "normal" angles of attack.

$$\frac{\partial\varepsilon}{\partial\alpha} = 4.44 \cdot \left[k_A \cdot k_\lambda \cdot k_H \cdot \sqrt{\cos\varphi_{25}} \right]^{1.19} \cdot \frac{\left(C_{L,\alpha}\right)_M}{\left(C_{L,\alpha}\right)_{M=0}} \quad (11.34)$$

Factor: wing aspect ratio
$$k_A = \frac{1}{A} - \frac{1}{1 + A^{1.7}} \quad (11.35)$$

Factor: wing taper
$$k_\lambda = \frac{10 - 3\lambda}{7} \quad (11.36)$$

Position factor, horizontal tailplane
$$k_H = \frac{1 - \left|\frac{z_H}{b}\right|}{\sqrt[3]{\frac{2 l_H}{b}}} \quad (11.37)$$

$\frac{\left(C_{L,\alpha}\right)_M}{\left(C_{L,\alpha}\right)_{M=0}}$ with $C_{L,\alpha}$ calculated from equation (7.24)

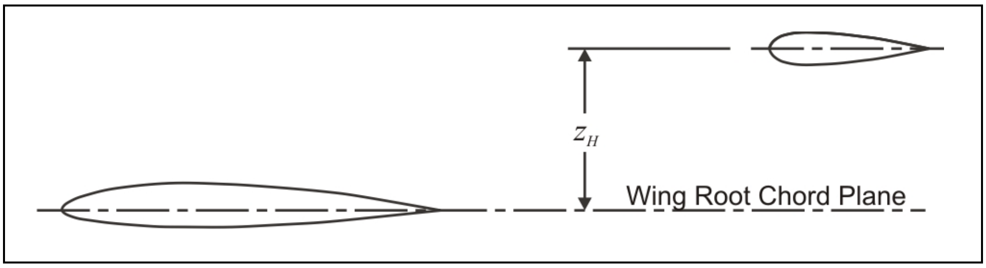

Fig. 11.14 Definition of z_H used in equation (11.34) according to **Roskam VI**

11.3 Vertical Tailplane Sizing

Vertical tailplane sizing according to control requirement

The dimensioning flight case for the rudder of a multi-engine aircraft is engine failure during take-off, as a rule. The **active engine** positioned symmetrically in relation to the failed engine causes a moment

$$N_E = \frac{T_{TO}}{n_E} \cdot y_E \quad . \tag{11.38}$$

Here, y_E is the distance between the failed engine and the plane of symmetry. n_E is the number of engines and T_{TO} / n_E is the take-off thrust of *one* engine. The **failed engine** causes drag, which can be determined as follows:

Propeller aircraft with fixed pitch propeller:	$N_D = 0.75 \cdot N_E$
Propeller aircraft with variable pitch propeller:	$N_D = 0.25 \cdot N_E$
Jet with rotating fan (windmilling) and low by-pass ratio (BPR):	$N_D = 0.15 \cdot N_E$
Jet with rotating fan (windmilling) and high by-pass ratio (BPR):	$N_D = 0.25 \cdot N_E$.

$$N_E + N_D = N_V \tag{11.39}$$

JAR 25.149 **Minimum control speed**

(b) VMC is the calibrated airspeed, at which, when the critical engine is suddenly made inoperative, it is possible to maintain control of the aeroplane with that engine still inoperative, and maintain straight flight with an angle of bank of not more than 5º.

(c) VMC may not exceed 1·2 VS ...

In the certification regulations JAR 25.149(b), flight with an angle of bank of not more than 5° is admissible to compensate for the moment of a failed engine. Thus, a small sideslip angle can be built up. The sideslip angle creates a transverse force on the vertical tailplane. This transverse force is used to compensate for the moment created by the engine failure. Here it is assumed that the **compensating moment is created solely by a rudder deflection** and that the aircraft is flying without a sideslip angle. **DATCOM 1978** contains methods to calculate the moment caused by the rudder

$$N_V = \frac{1}{2} \rho V_{MC}^{\;2} \cdot \delta_F \left[\frac{c_{L,\delta}}{\left(c_{L,\delta}\right)_{theory}} \right] \cdot \left(c_{L,\delta}\right)_{theory} \cdot K' \cdot K_\Lambda \cdot S_V \cdot l_V \quad . \tag{11.40}$$

According to JAR 25.149(c), the minimum control speed V_{MC} [1] is the minimum speed required to retain control of the aircraft in the event of engine failure. The following should apply to the required rudder deflection:

$$\delta_F \leq 25° \tag{11.41}$$

As a rule, aircraft are designed so that the following applies:

$$V_{MC} = 1.2 \cdot V_S \tag{11.42}$$

where V_S is the stall speed in the respective configuration of the aircraft. In summary, this estimate gives the required vertical tailplane area as follows:

$$S_V = \frac{N_E + N_D}{\frac{1}{2}\rho V_{MC}^{\ 2} \cdot \delta_F \left[\frac{c_{L,\delta}}{\left(c_{L,\delta}\right)_{theory}}\right] \cdot \left(c_{L,\delta}\right)_{theory} \cdot K' \cdot K_\Lambda \cdot l_V} \quad . \tag{11.43}$$

K_Λ from Section 8. Other parameters see below in Section 11.4.

Vertical tailplane sizing according to stability requirement

The equation for the sum of the moments at the aircraft's center of gravity around the vertical axis is

$$N_{CG} = N_W + N_F - L_V \cdot l_V \quad . \tag{11.44}$$

The total moment N_{CG} at the center of gravity should be positive and therefore counteract the sideslip angle:

$$N_{CG} = C_{N,\beta} \cdot \beta \cdot q \cdot S_W \cdot b \quad . \tag{11.45}$$

The fuselage moment N_F has a destabilizing effect

$$N_F = C_{N,\beta,F} \cdot \beta \cdot q \cdot S_W \cdot b \quad . \tag{11.46}$$

The transverse force L_V caused by the vertical tailplane is

---

[1] JAR-1: "VMC" means minimum control speed with the critical engine inoperative.

$$L_V = C_{Y,\beta,V} \cdot \beta \cdot q \cdot S_V \quad . \tag{11.47}$$

The wing moment N_W has a stabilizing effect if the wing has an aft sweep. It is

$$N_W = C_{N,\beta,W} \cdot \beta \cdot q \cdot S_W \cdot b \quad .$$

An aft swept wing has a stabilizing yawing moment $C_{N,\beta,W} > 0$. However, at present there is no available method for estimating this influence of the wing. For this reason, the vertical tailplane area calculated according to equation (11.51) must be critically appraised! Owing to the stabilizing effect of the sweptback wing a smaller vertical tailplane area than the one calculated according to (11.51) might suffice! The influence of the wing is only omitted in the further calculation due to a lack of suitable data.

If all the parameters in (11.44) are inserted, this gives

$$C_{N,\beta} \cdot \beta \cdot q \cdot S_W \cdot b = C_{N,\beta,F} \cdot \beta \cdot q \cdot S_W \cdot b - C_{Y,\beta,V} \cdot \beta \cdot q \cdot S_V \cdot l_V \quad . \tag{11.48}$$

If this is divided by β, q, S_W and b, this leads to:

$$C_{N,\beta} = C_{N,\beta,F} - C_{Y,\beta,V} \cdot \frac{S_V \cdot l_V}{S_W \cdot b_W} \quad . \tag{11.49}$$

According to **Roskam II**, the following should be met for sufficient static directional stability:

$$C_{N,\beta} \geq 0.001 \ 1/\deg = 0.057 1/\mathrm{rad} \quad . \tag{11.50}$$

Thus, the minimum required (dimensionless) vertical tailplane area can be calculated:

$$\boxed{\frac{S_V}{S_W} = \frac{C_{N,\beta} - C_{N,\beta,F}}{- C_{Y,\beta,V}} \cdot \frac{b_W}{l_V} \quad .} \tag{11.51}$$

General assessment of vertical tailplane sizing

For the vertical tailplane the larger of the two areas S_V arising from the **control requirement** (equation (11.43)), on the one hand, and the **stability requirement** (equation (11.51)) on the other hand must be chosen.

11.4 Parameters for Vertical Tailplane Sizing

The rudder - a plain flap

The rudder is a plain flap and is calculated as such. Equations (11.40) and (11.43) contain the term

$$\Delta c_{L,Flap} = \delta_F \cdot \left[\frac{c_{L,\delta}}{\left(c_{L,\delta}\right)_{theory}} \right] \cdot \left(c_{L,\delta}\right)_{theory} \cdot K' \quad . \tag{11.52}$$

δ_F flap angle [rad], $c_{L,\delta}$ flap efficiency

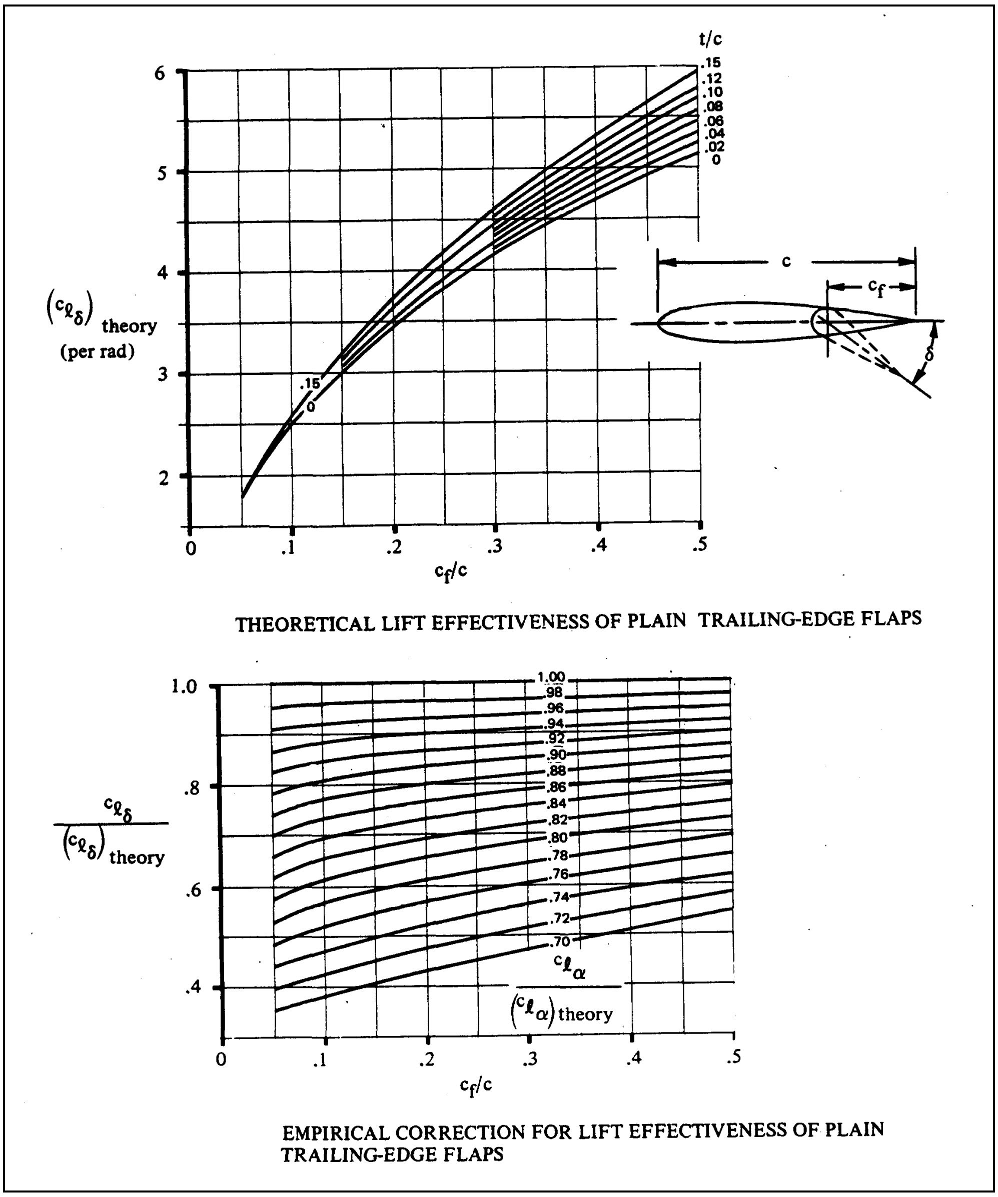

Fig. 11.15 Increase in lift (**DATCOM 1978**). $c_{L,\alpha} / \left(c_{L,\alpha}\right)_{theory}$ from Fig. 11.16

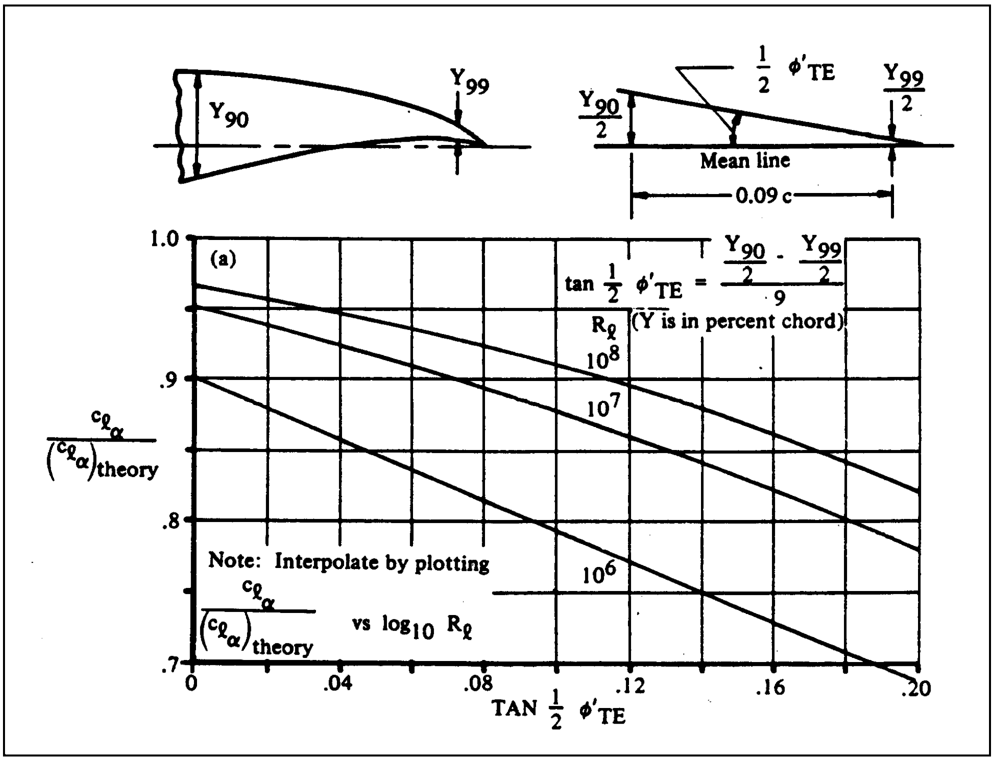

Fig. 11.16 $c_{L,\alpha} / \left(c_{L,\alpha}\right)_{theory}$ for Fig. 11.15 (**DATCOM 1978**)

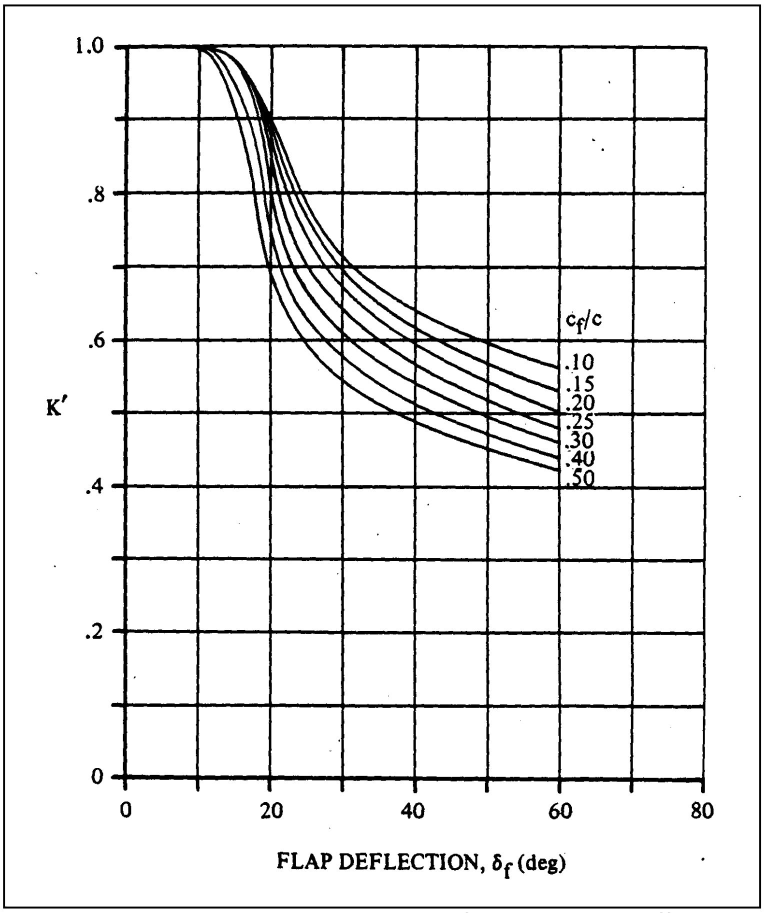

Fig. 11.17 Empirical correction for nonlinear effects at bigger flap angles (**DATCOM 1978**)

Stability coefficient $C_{N,\beta,F}$

The coefficient $C_{N,\beta,F}$ provides a yawing moment N caused by a sideslip angle β due to an aerodynamic impact on the fuselage. The coefficient is calculated according to **DATCOM 1978** (5.2.3) in combination with the wing/fuselage interference. The method can be simplified if only cylindrical fuselages are examined. It is then

$$C_{N,\beta,F} = -\frac{360}{2 \cdot \pi} \cdot k_N \cdot k_{R,l} \cdot \frac{l_F{}^2 \cdot d_F}{S_W \cdot b} \quad \text{in 1/rad} \quad . \tag{11.53}$$

k_N is available here for cylindrical fuselages in the following equation:

$$k_N = 0.01 \cdot \left[0.27 \cdot \frac{x_m}{l_F} - 0.168 \cdot \ln\left(\frac{l_F}{d_F}\right) + 0.416 \right] - 0.0005 \quad . \tag{11.54}$$

$k_{R,l}$ is also available here as a viable solution in the following equation:

$$k_{R,l} = 0.46 \cdot \log\left(\frac{\text{Re}}{10^{+6}}\right) + 1 \quad . \tag{11.55}$$

Key:

l_F fuselage length

d_F fuselage diameter

x_m length measured from the nose of the aircraft to the aircraft's center of gravity

Re, the Reynold's number of the fuselage is calculated from the speed (in cruise flight), the length of the fuselage and the kinematic viscosity

$$\text{Re} = \frac{V \cdot l_F}{\nu} . \tag{11.56}$$

Stability coefficient $C_{Y,\beta,V}$

The coefficient $C_{Y,\beta,V}$ gives a side force Y caused by a sideslip angle β due to aerodynamic impact on the vertical fin (V). The coefficient is calculated according to **DATCOM 1978** (5.3.1.1). Here, only a simplified version is presented, which omits the influences of the fuse-lage, horizontal tailplane, sidewash and reduction on the vertical fin.

$$C_{Y,\beta,V} = -\left(C_{L,\alpha}\right)_V \quad . \tag{11.57}$$

$C_{Y,\beta,V}$ therefore relates to the area of the vertical tailplane S_V in this case. The calculation of $C_{L,\alpha}$ was already presented in Section 7.