

# Memo

Aero\_M\_FuelCostsDueToAircraftSystems  
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**To:**

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## Fuel Costs due to Aircraft Systems

Fuel costs are differentiated by means of their physical origin. This approach helps to pinpoint the origin of fuel costs and allows to effectively find measures to reduce fuel consumption. Causes of fuel consumption due to aircraft systems, subsystems, or single parts are:

- fuel costs due to transportation of *fixed mass* (mass that does not vary in flight)
- costs due to mechanical *power off-takes* from the engines (e.g. by electrical generators)
- fuel costs due to *bleed air off-takes*,
- fuel costs due to *ram air off-takes*,
- fuel costs due to *additional drag* caused by the presents of aircraft systems, subsystems, or single parts (e.g. due to drain masts).

In addition to the fuel necessary for the above physical causes X, fuel is needed to carry the fuel for causes X during later flight intervals. The fuel needed to carry fuel is calculated just as calculating fuel used for fixed mass.

The calculation fuel required during flight interval  $i$  is done by a summation over time intervals from the last interval  $n$  back to interval  $i$ . Summation takes place backwards from landing to take-off.



## D. Scholz: Fuel Costs due to Aircraft Systems – Calculated from Small Time Intervals

$$C_{F, SYS} = C_{F, mf} + C_{F, P} + C_{F, B} + C_{F, R} + C_{F, D}$$

due to: fixed mass, power off-takes from the engines,  
bleed air off-takes, ram air off-takes, additional drag

$$C_{F, X} = m_{fuel, X} \cdot P_F \cdot n_{t, a}$$

$m_{fuel, X}$  mass of fuel consumed due to **cause X** (*mf, P, B, R, D*) during the whole flight

$P_F$  price of fuel

$n_{t, a}$  number of flights (trips, *t*) per year (annum, *a*)



The fuel consumption is calculated for 7 **flight phases**  $i$  :

$i = 1$ , engine start,

$i = 2$ , taxi,

$i = 3$ , take-off,

$i = 4$ , climb,

$i = 5$ , cruise,

$i = 6$ , descent,

$i = 7$ , landing, taxi, engine shut down.

**Here alternative approach:**

The fuel consumption is calculated for many very small time intervals.

All **consumptions are added up** for total fuel consumed.

**Summation over time intervals** from interval  $n$  back to interval  $i$  yields required fuel mass in interval  $i$

Note: Summation backwards from landing to take-off!

$$m_{fuel,j,X,f} = \sum_{i=n}^j \dot{m}_{fuel,i,X,f} \cdot \Delta t \quad \text{fuel due to cause } X \text{ directly: } f$$

$$m_{fuel,j,X,m} = \sum_{i=n}^j \dot{m}_{fuel,i,X,m} \cdot \Delta t \quad \text{fuel due to fuel mass due to } X: m$$

$$m_{fuel,j,X} = m_{fuel,j,X,f} + m_{fuel,j,X,m}$$

$$m_{fuel,X} = m_{fuel,1,X} = m_{fuel,1,X,f} + m_{fuel,1,X,m}$$

Fuel consumption due to **fixed mass**  $m_{i,mf}$  during flight phase  $i$

$$\dot{m}_{fuel,i,mf} = m_{i,mf} \cdot SFC_i \cdot g \cdot \left( \frac{\cos \gamma_i}{L/D_i} + \sin \gamma_i \right)$$

Fuel consumption due to **transported fuel mass (fuel for later time intervals)**  $m_{fuel,i,X,m}$  during flight phase  $i$

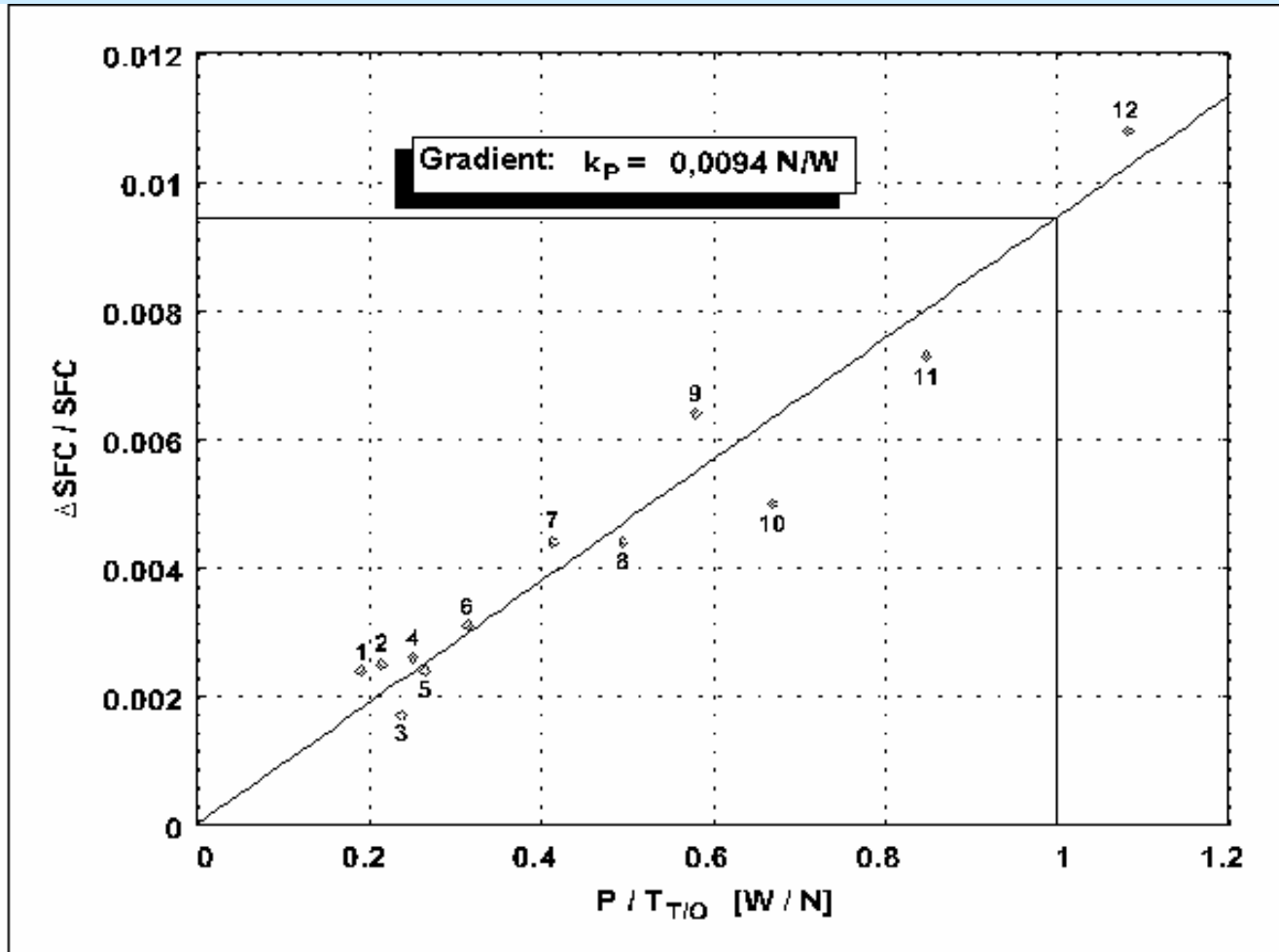
$$\dot{m}_{fuel,i,X,m} = m_{fuel,i,X,f} \cdot SFC_i \cdot g \cdot \left( \frac{\cos \gamma_i}{L/D_i} + \sin \gamma_i \right)$$

Fuel consumed due to **power off-takes**  $P_i$  during flight phase  $i$

$$(SFC)_P = \frac{k_P \cdot SFC_i \cdot m_{A/C} \cdot g}{n_E \cdot T_{T/O}} \cdot \left( \frac{\cos \gamma_i}{L/D_i} + \sin \gamma_i \right)$$

$T_{T/O}$  take-off thrust (one engine)

$n_E$  number of engines



Gradient  
 $k_p$   
for power  
off-takes

$k_p$  from  
Gasturb-  
Examples  
(AHLEFELDER):  
0.0116 N/W

Fuel consumption due to **power off-takes**  $P_i$  during flight phase  $i$

$$\dot{m}_{fuel,i,P,f} = P_i \cdot (SFC)_P$$

$(SFC)_P$	Mittelwert:	0,097 kg/kWh (SCHOLZ)
	A300:	0,125 kg/kWh (DECHOW)
	A400M:	0,167 kg/kWh (BRIX)
	Gasturb-Examples:	0,176 kg/kWh (AHLEFELDER)



Fuel consumption due to **bleed air off-takes** during flight phase  $i$   
(following SAE AIR 1168/8)

$$\dot{m}_{fuel,i,B,f} = k_B \cdot T_{tb} \cdot \dot{m}_B$$

$\dot{m}_B$  bleed air mass flow

$T_{tb}$  turbine inlet temperature

$$k_B = 3.015 \cdot 10^{-5} 1/K$$

New approach:

$$\dot{m}_{fuel,i,B,f} = k_B \cdot T_{tb} \cdot \dot{m}_B = k_B^* \cdot \dot{m}_B$$



Fuel consumption due to **bleed air off-takes** during flight phase  $i$   
(following AHLEFELDER)

$$\dot{m}_{fuel,i,B,f} = k_B^* \cdot \dot{m}_B$$

$$k_B^* \quad 0,0335 \quad (\text{AIR 1168/8})$$

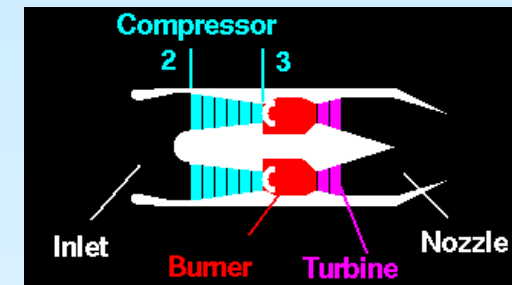
$$k_B^* = k_{BB} \left( \frac{p_3}{p_2} \right)^y \quad \frac{p_3}{p_2} \quad \text{is compressor (overall) pressure ratio}$$

CFM56-5C: 37,4

$$k_{BB} : 4,99 \cdot 10^{-3} \quad (\text{at relative enthalpy of 0,63})$$

$$y : 0,475$$

$$k_B^* \quad 0,028 \quad (\text{AHLEFELDER, CFM56-5C})$$





It makes sense to **consider bleed air off-takes** also as **power off-takes**. The compressor increases temperature,  $T$  and pressure,  $p$  at the same time. For simplicity we call now drop the \*. Summing up:

$k_B^* = k_B = 0,028$	see above (CFM56-5C)
$\frac{p_3}{p_2} = 37,4$	compressor pressure ratio (CFM56-5C)
$k_{RE} = 0,63$	relative enthalpy

With equations from next page, the **efficiency** for **bleed air off-takes** can now be calculated with

$H = 42,5 \cdot 10^6 \text{ Nm/kg}$	heating value for JET-A1
$T_1 = T_2 = 217 \text{ K}$	Compressor entry temperature
$c_p = 1,02 \text{ kJ/kg/K}$	Specific heat at constant pressure
$\eta_B = 22\%$	efficiency for bleed air off-takes

↓ fuel flow      ↓ bleed air flow

$$(1) \quad \dot{m}_{F,B} = K_B \cdot \dot{m}_B$$

$$= K_{p,B} \cdot SFC \cdot P_B$$

$$P_B = c_p (T_3 - T_1) \cdot \dot{m}_B$$

$$= c_p \cdot K_{RE} \cdot T_1 [K_{OAPR}^{0,29} - 1]$$

↑ factor for Relative  
Enthalpy

$$(2) \quad \dot{m}_{F,B} = K_{p,B} \cdot SFC \cdot c_p \cdot K_{RE} \cdot T_1 [K_{OAPR}^{0,29} - 1] \cdot \dot{m}_B$$

Comparing (1) and (2)

$$K_B = K_{p,B} \cdot SFC \cdot c_p \cdot K_{RE} \cdot T_1 [K_{OAPR}^{0,29} - 1]$$

$$\eta_B = \frac{P_B}{\dot{m}_{F,B} \cdot H} = \frac{1}{K_{p,B} \cdot SFC \cdot H}$$

$$\eta_B = \frac{c_p \cdot K_{RE} \cdot T_1 [K_{OAPR}^{0,29} - 1]}{K_B \cdot H}$$



Fuel consumption due to **ram air off-takes**  $Q_i$  during flight phase  $i$

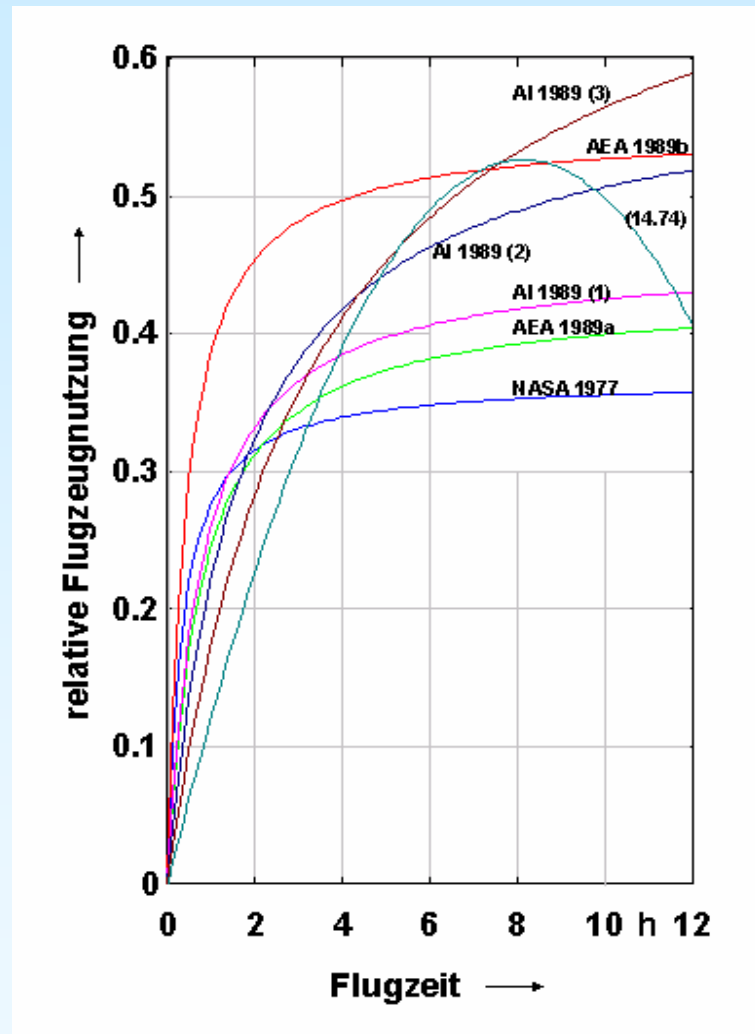
$$\dot{m}_{fuel,i,R,f} = SFC_i \cdot \rho_i \cdot Q_i \cdot v_i$$

$Q$  required air flow rate  
 $\rho$  air density;  $v$  true air speed  
 $SFC$  Specific Fuel Consumption

Fuel consumption due to **additional drag**  $D_i$  during flight phase  $i$

$$\dot{m}_{fuel,i,D,f} = SFC_i \cdot D_i$$

## Number of flights per year



## A/C DOC methods:

$$U_{a,f} = t_f \frac{k_{U1}}{t_f + k_{U2}}$$

Quelle	$k_{U1}$ h	$k_{U2}$ h
AA 1980 / NASA 77	3205	0.327
AEA 1989a	3750	0.750
AEA 1989b	4800	0.420
AI 1989 <sup>a</sup>		
$R < 1000$ nm	3994	0.754
$1000 \text{ nm} \leq R \leq$	5158	1.650
$2000 \text{ nm}$ (2)	6566	3.302
$2000 < R$ nm		

## Recommended for DOCsys

$$U_{h,f} = k_{U,A} (t_f - k_{U,B})^2 + k_{U,C}$$

$$k_{U,A} = -0.00796 \text{ 1/h}^2$$

$$k_{U,B} = 8.124 \text{ h}$$

$$k_{U,C} = 0.525$$

$$n_{t,a} = U_{a,f} / t_f$$

$t_f$  flight time

$$U_{a,f} = U_{h,f} \cdot 24 \cdot 365$$

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