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Numerical and Analytical Takeoff Field Length Calculations for Jet Aircraft

With some precision, Balanced Field Length (BFL) and Critical Engine Failure Recognition Speed (V1) can only be determined numerically with a simulation based on the integration of the differential equation describing the aircraft motion. Amazingly, a simple analytical approximation is doing quite well.

Background: The Takeoff Field Length (TOFL) is the takeoff distance of an aircraft including some margin of safety. The TOFL is by definition the greater of the Balanced Field Length (BFL) and 115% of the takeoff distance with all engines operative (TOD AEO). The BFL is determined by the condition that the distance to continue a takeoff following a failure of an engine at a critical engine failure recognition speed (go case) is equal to the distance required to abort it (stop case). It represents the worst-case scenario, since a failure at a lower speed requires less distance to abort, whilst a failure at a higher speed requires less distance to continue the takeoff. V1 during takeoff is the maximum speed at which the pilot is able to take the first action to stop the airplane (apply brakes) within the accelerate-stop distance and at the same time the minimum speed at which the takeoff can be continued to achieve the required height above the takeoff surface within the takeoff distance. V1 is called Critical Engine Failure Recognition Speed or Takeoff Decision Speed. The BFL is usually the distance that determines the TOFL for aircraft with two engines. With some precision, BFL and V1 can only be determined numerically with a calculation / simulation based on the integration of the differential equation describing the aircraft motion under BFL conditions. A simple analytical equations was found approximating a BFL calculation.

PURPOSE

The greater of two distances (Balanced Field Length or Takeoff Distance +15%) results in the Takeoff Field Length (TOFL). The TOFL is a takeoff distance with safety margins according to Certification Standards for Large Aeroplanes by EASA (CS-25) and FAA (FAR Part 25). Simple analytical approximations for the TOFL are checked against more demanding numerical simulations to determine the validity of the simple solutions and to implement adjustments for them, as necessary.

METHODOLOGY

The differential equation of the aircraft's acceleration is solved in MATLAB together with varying engine failure speeds. Analytical calculations of the Balanced Field Length by Torenbeek, Kundu, and Loftin are investigated. This includes the evaluation of statistical data.

FINDINGS

Analytical approximations deviate by 0.1% to 28.2% from the numerical solution. The most accurate analytical approximation is the simple method proposed by Loftin based on statistics. It shows deviations of less than 5.4%. The results confirm that the TOFL for jets with four engines is determined by the Takeoff Distance +15%, while for jets with two engines, the Balanced Field Length is decisive for TOFL (Figure 1).

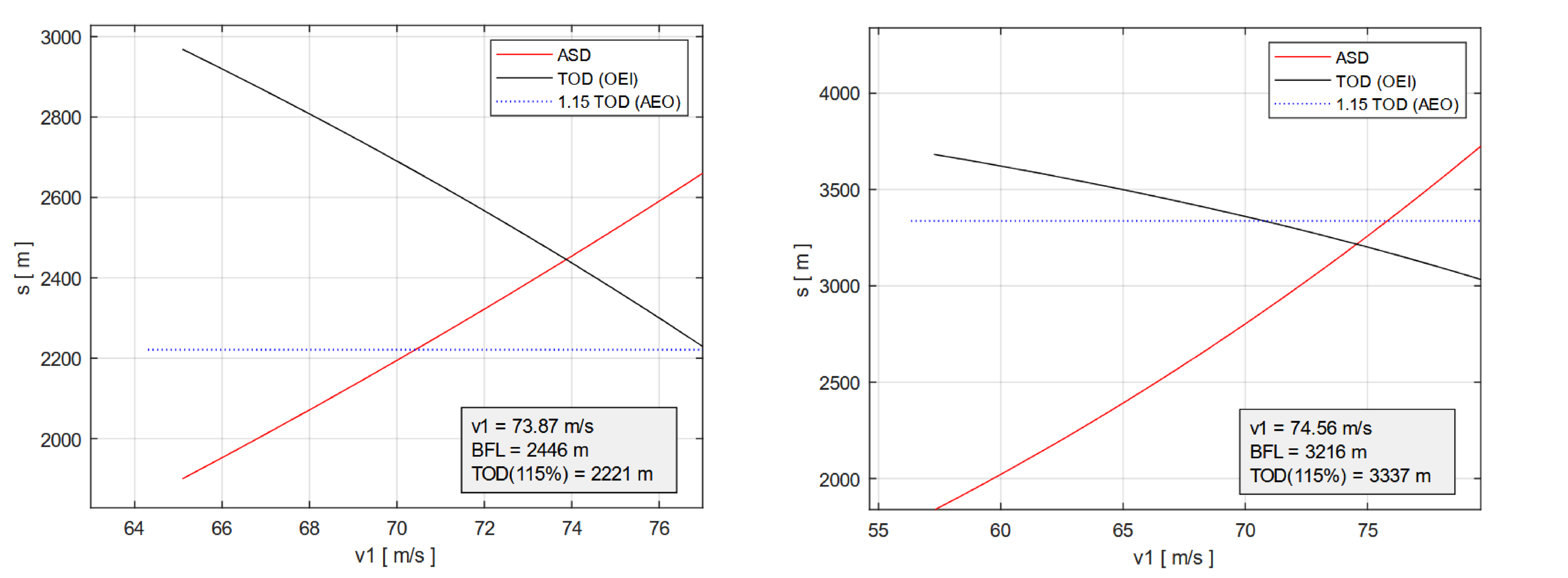


Figure 1: **TOFL is the greater of**
a) BFL: Intersection of ASD (stop case) and TOD (OEI) (go case) and
b) 1.15 TOD with AEO.
Left: A320, 2 engines, m = 78 t, T0 = 117.9 kN, confi 1+F, H = 0 ft: TOFL = BFL > 1.15 TOD (AEO)
Right: A340, 4 engines, m = 271 t, T0 = 138.8 kN, confi 1+F, H = 0 ft: TOFL = 1.15 TOD (AEO) > BFL

RESEARCH LIMITATIONS

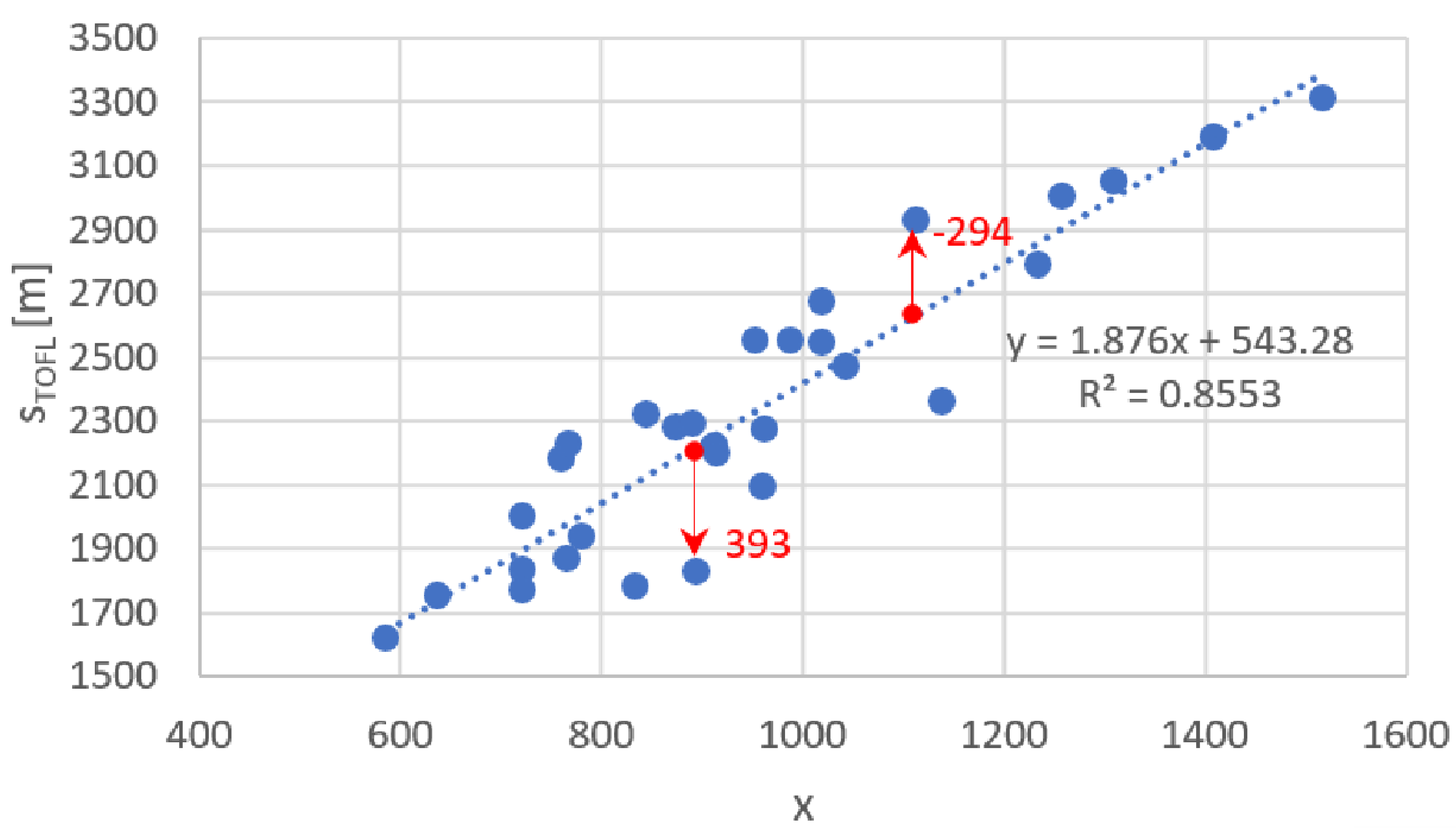
Simplifying assumptions had to be made e.g., regarding rotation time and speed, flap geometry, and asymmetric drag. While ground distances were solved numerically from acceleration and deceleration, air distance and rotation distance had to be determined analytically.

PRACTICAL IMPLICATIONS

A reliable and tested analytical procedure is useful for quick aircraft performance estimates and to include an inverse TOFL method into aircraft preliminary sizing (Figure 2).

ORIGINALITY

This seems to be the first report to provide a systematic check of available analytical approximations for the TOFL in comparison with a numerical solution.



$$s_{TOFL} = 1.876\,x + 543.28 \qquad [\mathrm{m}]$$

$$x = \frac{1}{\sigma \cdot C_{LmaxTO}} \cdot \frac{m_{MTO}/S_W}{T_{TO}/(m_{MTO} \cdot g)} \qquad [\mathrm{kg/m^2}]$$

Figure 2: **Values for the parameters in the equation and for the TOFL were taken from Jenkinson (2001) for jet aircraft with 2 and 4 engines. The linear regression has a coefficient of determination (R^2) of 0.8553. The absolute error is shown. The maximum relative error is 18%. A comparison with numerical results for the Airbus A320 and A340 shows a maximum relative error of 5.4%. σ is relative density, ρ/ρ_0.**

All details in the Bachelor Thesis of Lucht (2022):
https://nbn-resolving.org/urn:nbn:de:gbv:18302-aero2022-06-15.018
Program and Data (Excel / MATLAB) at Harvard Dataverse:
https://doi.org/10.7910/DVN/QX3MAH

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