



Technical Note

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Summary: Past experience from Regioliner predevelopment has shown that means for simple and quick simulation are needed to study aircraft dynamics affected by the mechanic and electronic design of the flight control system. This Technical Note describes two possible approaches and serves as their documentation. Simulation programming was done with the Engineering Analysis System EASY5. Future simulation demands can be met by using the linear or nonlinear representation of aircraft dynamics as presented here, extended by a controller or by specific actuator characteristics. The final EASY5 simulation (aircraft dynamics plus flight control system under consideration) will benefit from the many EASY5 features for system analysis.

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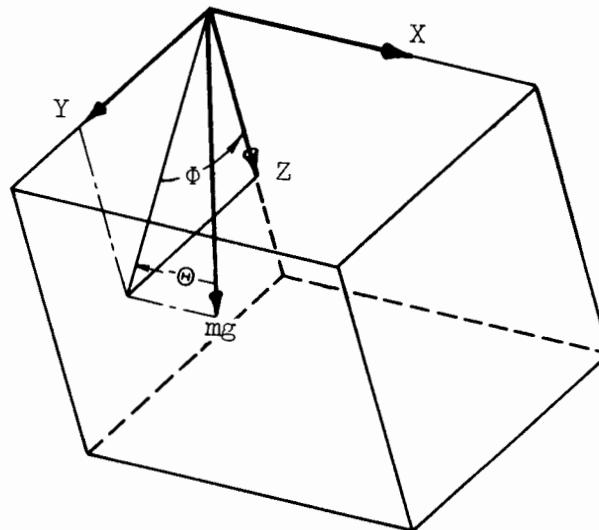
Table 1 Location of input and output in EASY5 simulation of linear aircraft simulation

Table 2 Eigenvalues calculated by EASY5 for 'test' aircraft from Ref. 1

Table 3 Eigenvalues calculated by EASY5 for DC3 with data from Ref. 1

Symbols

Note: Most symbols are explained in the text.



Orientation of gravity vector with X, Y, Z body-fixed axis system.

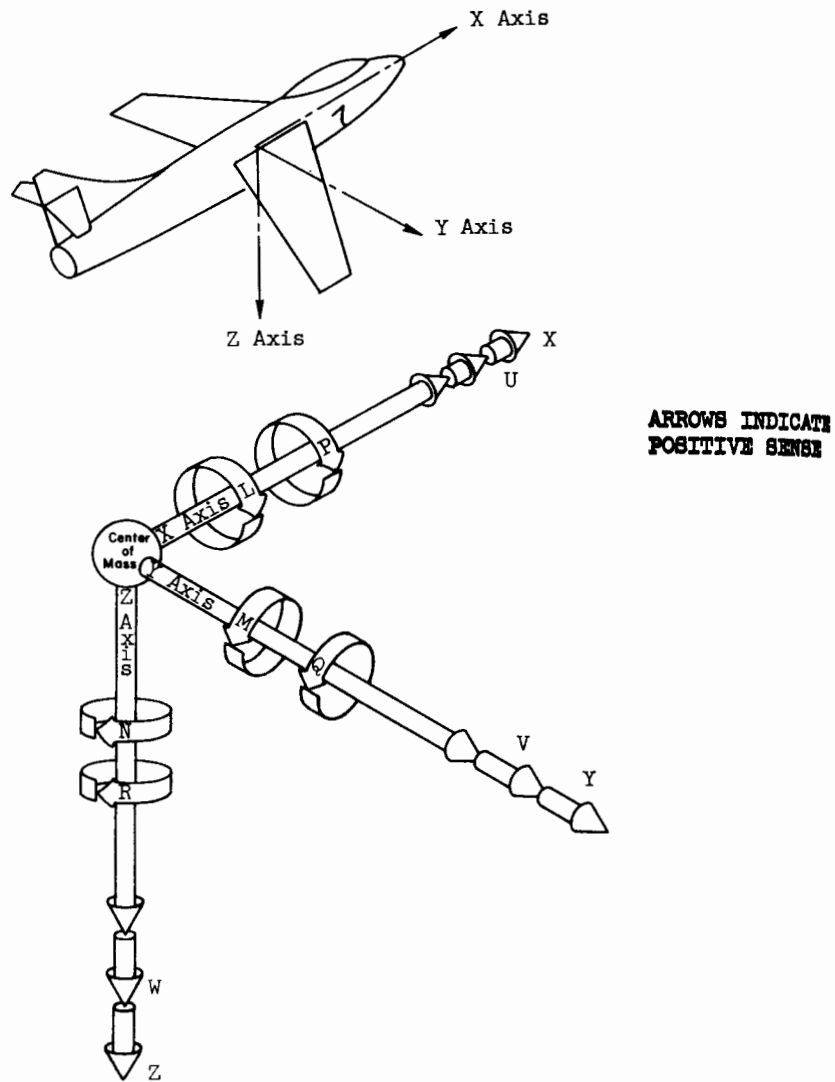


Fig. 4-1. Vehicle-fixed axis system and notation.

	Velocities	Applied forces and moments	Distances
Forward	U	X	x
Side	V	Y	y
Vertical	W	Z	z
Roll	P	L	
Pitch	Q	M	
Yaw	R	N	

Lateral dimensional stability derivative parameters (stability or body axis systems).

Quantity ^a	In terms of basic stability derivatives		Nondimensional
	Dimensional	Unit	
Y_v	$\frac{1}{mU} \frac{\partial Y}{\partial \beta}$	$\frac{1}{\text{sec}}$	$\frac{\rho SU}{2m} C_{Y\beta}$
$Y_{\dot{v}}$	$\frac{1}{mU} \frac{\partial Y}{\partial \dot{\beta}}$	$\frac{1}{\text{rad}}$	$\frac{\rho S b}{4m} C_{Y\dot{\beta}}$
Y_r^*	$\frac{1}{mU} \frac{\partial Y}{\partial r}$	$\frac{1}{\text{rad}}$	$\frac{\rho S b}{4m} C_{Yr}$
Y_p^*	$\frac{1}{mU} \frac{\partial Y}{\partial p}$	$\frac{1}{\text{rad}}$	$\frac{\rho S b}{4m} C_{Yp}$
$Y_{\dot{\delta}}^*$	$\frac{1}{mU} \frac{\partial Y}{\partial \dot{\delta}}$	$\frac{1}{\text{rad-sec}}$	$\frac{\rho SU}{2m} C_{Y\dot{\delta}}$
N_{β}	$\frac{1}{I_x} \frac{\partial N}{\partial \beta}$	$\frac{1}{\text{sec}^2}$	$\frac{\rho SU^2 b}{2I_x} C_{N\beta}$
$N_{\dot{\beta}}$	$\frac{1}{I_x} \frac{\partial N}{\partial \dot{\beta}}$	$\frac{1}{\text{sec}}$	$\frac{\rho SU^2 b^2}{4I_x} C_{N\dot{\beta}}$
N_r	$\frac{1}{I_x} \frac{\partial N}{\partial r}$	$\frac{1}{\text{sec}}$	$\frac{\rho SU^2 b^2}{4I_x} C_{Nr}$
N_p	$\frac{1}{I_x} \frac{\partial N}{\partial p}$	$\frac{1}{\text{sec}}$	$\frac{\rho SU^2 b^2}{4I_x} C_{Np}$
$N_{\dot{\delta}}$	$\frac{1}{I_x} \frac{\partial N}{\partial \dot{\delta}}$	$\frac{1}{\text{rad-sec}^2}$	$\frac{\rho SU^2 b^2}{2I_x} C_{N\dot{\delta}}$
L_{β}	$\frac{1}{I_y} \frac{\partial L}{\partial \beta}$	$\frac{1}{\text{sec}^2}$	$\frac{\rho SU^2 b}{2I_y} C_{L\beta}$
$L_{\dot{\beta}}$	$\frac{1}{I_y} \frac{\partial L}{\partial \dot{\beta}}$	$\frac{1}{\text{sec}}$	$\frac{\rho SU^2 b^2}{4I_y} C_{L\dot{\beta}}$
L_r	$\frac{1}{I_y} \frac{\partial L}{\partial r}$	$\frac{1}{\text{sec}}$	$\frac{\rho SU^2 b^2}{4I_y} C_{Lr}$
L_p	$\frac{1}{I_y} \frac{\partial L}{\partial p}$	$\frac{1}{\text{sec}}$	$\frac{\rho SU^2 b^2}{4I_y} C_{Lp}$
$L_{\dot{\delta}}$	$\frac{1}{I_y} \frac{\partial L}{\partial \dot{\delta}}$	$\frac{1}{\text{rad-sec}^2}$	$\frac{\rho SU^2 b^2}{2I_y} C_{L\dot{\delta}}$

^a The starred derivatives arise when β_r rather than v is used as the lateral motion parameter (see Chapter 6); in general, $Y_{\dot{\beta}} = Y_{\dot{\beta}} U_{\dot{\beta}}$.

Longitudinal dimensional stability derivatives (stability axis system).

Quantity	In terms of basic stability derivatives		Nondimensional
	Dimensional	Unit	
X_u	$\frac{1}{m} \frac{\partial X}{\partial u}$	$\frac{1}{\text{sec}}$	$\frac{\rho SU}{m} (-C_D - C_{D_u})^a$
X_w	$\frac{1}{m} \frac{\partial X}{\partial w}$	$\frac{1}{\text{sec}}$	$\frac{\rho SU}{2m} (C_L - C_{D_w})$
$X_{\dot{\delta}}$	$\frac{1}{m} \frac{\partial X}{\partial \dot{\delta}}$	$\frac{\text{ft}}{\text{rad-sec}^2}$	$\frac{\rho SU^2}{2m} (-C_{D\dot{\delta}})$
Z_u	$\frac{1}{m} \frac{\partial Z}{\partial u}$	$\frac{1}{\text{sec}}$	$\frac{\rho SU}{m} (-C_L - C_{L_u})^a$
Z_w	$\frac{1}{m} \frac{\partial Z}{\partial w}$	$\frac{1}{\text{sec}}$	$\frac{\rho SU}{2m} (-C_{L_w} - C_D)$
$Z_{\dot{w}}$	$\frac{1}{m} \frac{\partial Z}{\partial \dot{w}}$	$\frac{1}{\text{ft}}$	$\frac{\rho S c}{4m} (-C_{L\dot{w}})$
Z_q	$\frac{1}{m} \frac{\partial Z}{\partial q}$	$\frac{\text{ft}}{\text{rad-sec}}$	$\frac{\rho SU c}{4m} (-C_{Lq})$
$Z_{\dot{\delta}}$	$\frac{1}{m} \frac{\partial Z}{\partial \dot{\delta}}$	$\frac{\text{ft}}{\text{rad-sec}^2}$	$\frac{\rho SU^2}{2m} (-C_{L\dot{\delta}})$
M_u	$\frac{1}{I_y} \frac{\partial M}{\partial u}$	$\frac{1}{\text{ft-sec}}$	$\frac{\rho SU c}{I_y} (C_M + C_{M_u})$
M_w	$\frac{1}{I_y} \frac{\partial M}{\partial w}$	$\frac{1}{\text{ft-sec}}$	$\frac{\rho SU c}{2I_y} C_{M_w}$
$M_{\dot{w}}$	$\frac{1}{I_y} \frac{\partial M}{\partial \dot{w}}$	$\frac{1}{\text{ft}}$	$\frac{\rho S c^2}{4I_y} C_{M\dot{w}}$
M_q	$\frac{1}{I_y} \frac{\partial M}{\partial q}$	$\frac{1}{\text{sec}}$	$\frac{\rho SU c^2}{4I_y} C_{Mq}$
$M_{\dot{\delta}}$	$\frac{1}{I_y} \frac{\partial M}{\partial \dot{\delta}}$	$\frac{1}{\text{rad-sec}^2}$	$\frac{\rho SU^2 c}{2I_y} C_{M\dot{\delta}}$

List of References

- [1] McRuer, Ashkenas, Graham: "Aircraft Dynamics and Automatic Control", Princeton University Press, 1973.
- [2] Nelson, Robert C.: "Flight Stability and Automatic Control", McGraw Hill, 1989.
- [3] McLean, Donald: "Automatic Flight Control Systems", Prentice Hall International, 1990.
- [4] Boeing Computer Services: "EASY5 Engineering Analysis System, Reference Manual", The Boeing Company, 1988.
- [5] Boeing Computer Services: "EASY5 Engineering Analysis System, Users Guide", The Boeing Company, 1991.

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1 Simulation of Aircraft Dynamics

Simulation of aircraft dynamics is based on the solution of the governing equations of motion. Derivations of these equations are presented in standard textbooks on flight mechanics and aircraft dynamics [1], [2],[3]. The equations have the general form

$$\vec{F} = m \frac{d}{dt} \vec{v} = m \cdot (\dot{\vec{v}} + \vec{\omega} \times \vec{v})$$

$$\vec{M} = \frac{d}{dt} \vec{H} = I \dot{\vec{\omega}} + \vec{\omega} \times \vec{H}$$

H is the angular momentum, I is the inertia matrix. The aircraft is assumed to be a rigid body. The forces F can be divided into steady state forces: lift, drag, thrust, aircraft weight and additional unsteady forces e.g. aerodynamic forces due to aircraft movements about the steady state condition. The moments M can as well be divided into steady state moments and additional unsteady moments which can again be aerodynamic moments due to aircraft movements about the steady state condition. Furthermore, changes in thrust level or shifting of fuel or cargo in flight can be a source of unsteady forces and moments.

2 Linear Simulation of Aircraft Dynamics

The linear simulation assumes small perturbations about an equilibrium or trimmed condition. It can be shown that for this case, longitudinal and lateral motions are uncoupled [1], [2], [3].

2.1 Longitudinal Aircraft Dynamics

The State Equation

A linear system can be represented in state space notation. **x** is the state vector with perturbations (deviation from the steady state or equilibrium condition) of the state variables. **u** is the control vector. The control vector consists of control inputs from elevator and flaps. In this case also disturbances from gust inputs are built into the control vector.¹ **A** is called system matrix or state coefficient matrix and **B** is called control matrix. For a detailed query of the notation please refer to the List of Symbols.

$$\dot{\mathbf{x}} = \mathbf{A} \mathbf{x} + \mathbf{B} \mathbf{u}$$

$$\mathbf{x} = \begin{bmatrix} u \\ w \\ q \\ \Theta \end{bmatrix} \quad \mathbf{u} = \begin{bmatrix} \delta_E \\ \delta_F \\ u_g \\ w_g \\ q_g \end{bmatrix}$$

1. EASY5 allows only for the standard state space notation, therefore no extra disturbance term can be used.

$$A = \begin{bmatrix} X_u & X_w & 0 & -g \\ Z_u & Z_w & U_0 & 0 \\ \tilde{M}_u & \tilde{M}_w & \tilde{M}_q & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad B = \begin{bmatrix} X_{\delta_E} & X_{\delta_F} & -X_u & -X_w & 0 \\ Z_{\delta_E} & Z_{\delta_F} & -Z_u & -Z_w & -U_0 \\ \tilde{M}_{\delta_E} & \tilde{M}_{\delta_F} & -\tilde{M}_u & -\tilde{M}_w & -\tilde{M}_q \\ 0 & 0 & 0 & 0 & -1 \end{bmatrix}$$

The matrices A and B consist mainly of stability and control derivatives. The American notation is applied here which uses dimensional stability derivative parameters. These stability derivative parameters lead to very compact equations. U_0 is the steady state forward speed of the aircraft. The "tilde"-stability derivatives are defined as follows:

$$\tilde{M}_u = M_u + M_w Z_u$$

$$\tilde{M}_w = M_w + M_{\dot{w}} Z_w$$

$$\tilde{M}_q = M_q + U_0 M_{\dot{w}}$$

$$\tilde{M}_{\delta_E} = M_{\delta_E} + M_w Z_{\delta_E}$$

The Output Equation

If other values than the state variables are of interest, they can be calculated by use of the output equation. Values considered here are the angle of attack α , the flight path angle γ , the acceleration in z-direction at the center of gravity a_{zcg} and the corresponding load factor n_{zcg} .

$$\alpha = \frac{w}{U_0} \quad \gamma = \theta - \alpha \quad a_{zcg} = \dot{w} - U_0 \cdot q$$

$$n_{zcg} = \frac{a_{zcg}}{g}$$

The output equation can be written as

$$y = C x + D u$$

$$y = \begin{bmatrix} \alpha \\ \gamma \\ a_{zcg} \end{bmatrix}$$

$$u = \begin{bmatrix} \delta_E \\ \delta_F \\ u_g \\ w_g \\ q_g \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & \frac{1}{U_0} & 0 & 0 \\ 0 & \frac{1}{U_0} & 0 & 1 \\ Z_u & Z_w & 0 & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ Z_{\delta_E} & Z_{\delta_F} & -Z_u & -Z_w & -U_0 \end{bmatrix}$$

A change in aircraft height can be obtained from an integration of c.g.-acceleration in the z-direction.

$$\ddot{h} = -a_{zcg} \quad \dot{h} = \int \ddot{h} dt \quad h = \int \dot{h} dt$$

2.2 Lateral Aircraft Dynamics

Lateral aircraft dynamics follow the same mathematical approach as described for longitudinal aircraft dynamics. Refer to the List of Symbols for queries related to the notation. Again, the American notation using dimensional stability derivative parameters is used.

The State Equation

Aileron and rudder control the lateral dynamics of the aircraft. Gust inputs are again built into the control vector.

$$\dot{x} = A x + B u$$

$$x = \begin{bmatrix} \beta \\ p \\ r \\ \Phi \end{bmatrix} \quad u = \begin{bmatrix} \delta_A \\ \delta_R \\ \beta_g \\ p_g \\ r_g \end{bmatrix}$$

$$A = \begin{bmatrix} Y_v & 0 & -1 & g/U_0 \\ L_{\beta'} & L_{p'} & L_{r'} & 0 \\ N_{\beta'} & N_{p'} & N_{r'} & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} Y_{\delta_A}^* & Y_{\delta_R}^* & -Y_v & 0 & 1 \\ L_{\delta_A}' & L_{\delta_R}' & -L_{\beta}' & -L_{p}' & -L_{r}' \\ N_{\delta_A}' & N_{\delta_R}' & -N_{\beta}' & -N_{p}' & -N_{r}' \\ 0 & 0 & 0 & -1 & 0 \end{bmatrix}$$

The 'primed' and 'stared' stability derivatives are defined as follows:

$$\begin{aligned} L_{\beta}' &= L_{\beta} + \frac{I_{xz}}{I_x} N_{\beta} & N_{\beta}' &= N_{\beta} + \frac{I_{xz}}{I_z} L_{\beta} \\ L_{p}' &= L_p + \frac{I_{xz}}{I_x} N_p & N_{p}' &= N_p + \frac{I_{xz}}{I_z} L_p \\ L_{r}' &= L_r + \frac{I_{xz}}{I_x} N_r & N_{r}' &= N_r + \frac{I_{xz}}{I_z} L_r \\ L_{\delta_A}' &= L_{\delta_A} + \frac{I_{xz}}{I_x} N_{\delta_A} & N_{\delta_A}' &= N_{\delta_A} + \frac{I_{xz}}{I_z} L_{\delta_A} \\ L_{\delta_R}' &= L_{\delta_R} + \frac{I_{xz}}{I_x} N_{\delta_R} & N_{\delta_R}' &= N_{\delta_R} + \frac{I_{xz}}{I_z} L_{\delta_R} \end{aligned}$$

$$Y_{\delta_R}^* = \frac{Y_{\delta_R}}{U_0} \quad Y_{\delta_A}^* = \frac{Y_{\delta_A}}{U_0}$$

The Output Equation

Further values that could be of interest are the sideslip velocity v and the side-acceleration at the centre of gravity $a_{y_{cg}}$. They can be calculated by use of the output equation. The yaw angle Ψ and the load factor in y -direction can be calculated from yaw rate and side-acceleration as given below.

$$v = \beta \cdot U_0 \quad a_{y_{cg}} = \dot{v} - g \cdot \phi + U_0 \cdot r$$

$$y = C x + D u$$

$$y = \begin{bmatrix} v \\ a_{y_{cg}} \end{bmatrix} \quad C = \begin{bmatrix} U_0 & 0 & 0 & 0 \\ Y_v \cdot U_0 & 0 & 0 & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ Y_{\delta_A}^* \cdot U_0 & Y_{\delta_R}^* \cdot U_0 & -Y_v \cdot U_0 & 0 & U_0 \end{bmatrix}$$

$$\psi = \int r \, dt \quad n_{y_{cg}} = \frac{a_{y_{cg}}}{g}$$

2.3 Linear Aircraft Simulation with EASY5

Fig. 1 shows the linear aircraft simulation as programmed with the Engineering Analysis System EASY5. The left set of blocks is the simulation of longitudinal aircraft dynamics; the right set of blocks is the simulation of lateral aircraft dynamics. Each set of blocks has a similar architecture. The left side of each set of blocks consists of the control and gust inputs to the aircraft. FORTRAN code merely serves as a connector to the state equation. The states of the system as defined in the x -vector can be obtained from this block. The control vector is multiplied with the matrix D in the EASY5 blocks MZ01/MZ02 to $D \cdot U$. The state vector x and the vector $D \cdot U$ are fed into the output equation. The output variables as defined in the vector y above can be obtained from the EASY5 blocks LINEAR SYSTEM OUTPUT. Further calculations are performed in additional blocks. Table 1 provides a summary of inputs and outputs into and out of the system.

Table 1: Location of input and output in EASY5 simulation of linear aircraft simulation

EASY5 block	input
LINEAR SYSTEM	system matrix A , control matrix B
LINEAR SYSTEM OUTPUT	output matrix C
MZ01/MZ02	direct matrix D
GN01	earth acceleration g
GN02	earth acceleration g

EASY5 block	output
LINEAR SYSTEM	state vector x
LINEAR SYSTEM OUTPUT	output vector y
IN01	vertical speed (positive up)
IN02	height (change against steady state)
GN01	normal load factor n_{zcg}
IN03	yaw angle
GN02	load factor n_{ycg}

2.4 Results from Linear Aircraft Simulation

Reference 1 provides many sample calculations for a conventional jet-propelled aircraft with straight wings. The EASY5 simulation was checked against the results given in [1] for this ‘test’ aircraft. The EASY5 results virtually agreed exactly with the published results. This should not come as a surprise since the linear aircraft simulation as programmed here uses the same mathematical approach. The eigenvalues are given in Table 2. The other simulation results are given in Fig. 2 through 20.

Table 2: Eigenvalues calculated by EASY5 for ‘test’ aircraft from Ref. 1

mode	natural frequency ω	damping ratio ζ
short period	4.27	0.493
phugoid	0.0629	0.0717
dutch roll	1.88	0.0247
spiral mode	1/T = -0.00136	
roll subsidence	1/T = 1.78	

3 Nonlinear Simulation of Aircraft Dynamics

This nonlinear approach followed here is only nonlinear in so far as the equations of motions are concerned. In contrast, the aerodynamic forces and moments are still linearized using stability and control derivatives which are only valid for a selected operating point. From [3]:

$$\begin{aligned}
 X &= m \cdot [\dot{U} + QW - RV + g \sin \Theta] \\
 Y &= m \cdot [\dot{V} + RU - PW - g \cos \Theta \sin \Phi] \\
 Z &= m \cdot [\dot{W} + PV - QU - g \cos \Theta \cos \Phi] \\
 L &= \dot{P}I_x - I_{xz} (\dot{R} + PQ) + (I_z - I_y) QR \\
 M &= \dot{Q}I_y + I_{xz} (P^2 - R^2) + (I_x - I_z) PR \\
 N &= \dot{R}I_z - I_{xz} \dot{P} + PQ (I_y - I_x) + I_{xz} QR
 \end{aligned}$$

These equations of motion have to be solved for the highest derivative of the state variables: \dot{U} , \dot{V} , \dot{W} , \dot{P} , \dot{Q} , \dot{R} to enable integration by EASY5. With respect to the aerodynamic forces, only the linearized and perturbed forces and moments x , y , z , l , m_1 , n are considered. Note, the steady state lift force $-m g$. This aerodynamic force in the negative z -direction in an aircraft fixed reference system has to be taken into account in addition to the perturbed force to counter gravity.

$$\begin{aligned}
 \dot{U} &= \frac{x}{m} - QW + RV - g \sin \Theta \\
 \dot{V} &= \frac{y}{m} - RU + PW + g \cos \Theta \cdot \sin \Phi \\
 \dot{W} &= \frac{z}{m} - PV + QU + g \cos \Theta \cdot \cos \Phi - g \\
 \dot{P} &= \frac{1}{I_x - I_{xz}^2/I_z} \cdot \left(l + \frac{I_{xz}}{I_z} [n - PQ (I_y - I_x) - I_{xz} QR] + I_{xz} PQ - (I_z - I_y) QR \right) \\
 \dot{Q} &= \frac{1}{I_y} \cdot \left(m_1 - I_{xz} (P^2 - R^2) - (I_x - I_z) PR \right) \\
 \dot{R} &= \frac{1}{I_z - I_{xz}^2/I_x} \cdot \left(n + \frac{I_{xz}}{I_x} [l + I_{xz} PQ - (I_z - I_y) QR] - PQ (I_y - I_x) - I_{xz} QR \right)
 \end{aligned}$$

Furthermore, from the angular orientation of the gravity vector in the aircraft's fixed reference system, equations for the aircraft's pitch and roll attitude can be derived [3]:

$$\begin{aligned}
 \dot{\Theta} &= Q \cos \Phi - R \sin \Phi \\
 \dot{\Phi} &= P + R \tan \Theta \cos \Phi + Q \tan \Theta \sin \Phi
 \end{aligned}$$

There is no difference between derivatives from perturbed and unperturbed variables:

$$\begin{aligned} \dot{u} &= \dot{U} & \dot{v} &= \dot{V} & \dot{w} &= \dot{W} & \dot{\theta} &= \dot{\Theta} \\ \dot{p} &= \dot{P} & \dot{q} &= \dot{Q} & \dot{r} &= \dot{R} & \dot{\phi} &= \dot{\Phi} \end{aligned}$$

Therefore the perturbed or unperturbed derivatives of the state variables can be used for integration to yield the perturbed state variables. The unperturbed state variables are obtained by adding the initial conditions:

$$\begin{aligned} U &= \int \dot{u} dt + U_{ic} & V &= \int \dot{v} dt + V_{ic} & W &= \int \dot{w} dt + W_{ic} \\ P &= \int \dot{p} dt + P_{ic} & Q &= \int \dot{q} dt + Q_{ic} & R &= \int \dot{r} dt + R_{ic} \\ \Theta &= \int \dot{\theta} dt + \Theta_{ic} & \Phi &= \int \dot{\phi} dt + \Phi_{ic} \end{aligned}$$

The perturbed forces and moments x, y, z, l, m_1, n are calculated from stability and control derivatives:

$$\begin{aligned} x &= m \left(X_u u + X_w w + X_{\delta_E} \delta_E + X_{\delta_F} \delta_F \right) \\ y &= m \left(Y_v v + Y_{\delta_A} \delta_A + Y_{\delta_R} \delta_R \right) \\ z &= m \left(Z_u u + Z_w w + Z_{\delta_E} \delta_E + Z_{\delta_F} \delta_F \right) \\ l &= I_x \left(L_v v + L_p p + L_r r + L_{\delta_A} \delta_A + L_{\delta_R} \delta_R \right) \\ m_1 &= I_y \left(M_u u + M_w w + M_{\dot{w}} \left[\frac{z}{m} - PV + QU + g \cos \Theta \cos \Phi - g \right] \right. \\ &\quad \left. + M_q q + M_{\delta_E} \delta_E + M_{\delta_F} \delta_F \right) \\ n &= I_z \left(N_v v + N_p p + N_r r + N_{\delta_A} \delta_A + N_{\delta_R} \delta_R \right) \end{aligned}$$

3.1 Nonlinear Aircraft Simulation with EASY5

Fig. 21 shows the nonlinear aircraft simulation as programmed with the Engineering Analysis System EASY5. The left set of blocks generates inputs from elevator, flaps, rudder and ailerons. Gust inputs are omitted for sake of simplicity. The simulation itself is written in FORTRAN, split up into two EASY5 blocks for clarity. The first block calculates the aircraft aerodynamic forces and moments from stability and control derivatives as given above. The second block calculates the rate of change of the state variables. The state variables are defined as such in the

FORTRAN block and are automatically integrated by the chosen EASY5 integration algorithm by use of the corresponding rates. The DERIVATIVE OF statement is used in EASY5 to define the states:

$$\text{DERIVATIVE OF, } U = X/M - Q*W + R*V - G*\text{SIN}(\text{TH})$$

...

3.2 Results from Nonlinear Aircraft Simulation

The nonlinear simulation was checked against the results obtained from a linear simulation of the same aircraft. The aircraft used for this check was the DC3. DC3-data was taken from [1]. No differences to three significant digits were observed in the eigenvalues calculated by EASY5.² Results are listed in Table 3. It can be concluded, that under normal circumstances (small angles) there are no benefits in using the nonlinear approach. DC3-response to an elevator input is shown in Fig. 22 and 23. The simulation was done using the nonlinear approach.

Table 3: Eigenvalues calculated by EASY5 for DC3 with data from Ref. 1

Linear Simulation and Nonlinear Simulation		
mode	natural frequency ω	damping ratio ζ
phugoid	0.201	0.201
dutch roll	1.10	0.323
short period	$1/T_1 = 1.25$	$1/T_2 = 3.31$
spiral mode	$1/T = -0.043$	
roll subsidence	$1/T = 6.57$	

Note: The DC3 short period mode has two real roots

4 Example of Closed Loop Simulation of Aircraft Dynamics

Fig. 24 shows an example of an EASY5 simulation applying the linear model for longitudinal aircraft dynamics in conjunction with a controller and a hydraulic actuator. The simulation benefits from the many EASY5 features for system analysis. These are mainly:

2. EASY5 calculates the eigenvalues of a nonlinear system from the system matrix A. The state variables are at their steady state condition when the A-matrix is obtained by linearization.

**Linear Model Generation Analysis,
Transfer Function Analysis (Bode, Nichols, Nyquist),
Root Locus Analysis,
Eigenvalue Sensitivity Analysis,
Stability Margin Analysis.**

APPENDIX: HOW TO FIND STABILITY AND CONTROL DERIVATIVES

If stability and control calculations have to be done for an aircraft (the prototype) for which no stability and control derivatives exist, two general approaches are possible:

- Approach 1 Calculate derivatives from the "Aerodynamic Data Base" (for MPC75 the Aero Data Base has the Reference Number EF-T-0/014) by use of the definition of the stability and control derivatives (e.g. from Ref. 1).
- Approach 2 Estimate derivatives from the known geometry of the aircraft using empirical data presented in form of figures and equations.
- Approach 2 Use an aircraft (model) for which derivatives are known and estimate the desired derivatives of the prototype.

Approach 2 is explained in

- 1.) Roskam, J.: Methods for Estimating Stability and Control Derivatives of Conventional Subsonic Airplanes, Roskam Aviation and Eng. Corp., 1971.
- 2.) Roskam, J.: Airplane Flight Dynamics and Automatic Controls, Part I and II, Roskam Aviation and Eng. Corp., 1982.
- 3.) Roskam, J.: Airplane Design, Part IV, Roskam Aviation and Eng. Corp., 1989.

Here basic nondimensional stability and control derivatives are calculated (C_l , C_{l_a} , ...) which can be used to calculate dimensional stability and control derivative parameters (L_p , M_w , ...) from their definition as given in Ref.1 Table 4-3 and 4-4. This approach is rather cumbersome. The program AAA (Advanced Airplane Analysis, by: Design, Analysis and Research Corporation, DARCorp, 120 East Ninth (Suite2), Lawrence, Kansas 66044, USA) helps to speed up this process (which is still not easy).

If therefore stability and control derivatives are known for an aircraft with similar geometry approach 3 might lead faster to results.

Approach 3 assumes strictly speaking geometric similarity (model and prototype differ just by a constant scale factor) and dynamic similarity of the flow. For an aircraft this means that first of all the Reynolds number (Re) and the Mach number (M) must be identical for model and prototype. As known from model studies, it is only seldom possible to match all requirements. E.g. to deduce the behaviour of a larger prototype from a smaller model in the same medium, the model would need to fly at higher speed which would at the same time change the Mach number.

In the following, constant Reynolds and Mach numbers are assumed, although it is clear that these assumptions can generally not be met.

Nomenclature

model: subscript: m

prototype: subscript: p

scale factor: r = length_p/length_m

mass: m

air density: ρ

A/C speed: U

()_p / ()_m: ratio of derivatives, prototype / model
e.g. (X_u)_p / (X_u)_m

$$k1 = \frac{m_m \cdot \rho_p \cdot U_p}{m_p \cdot \rho_m \cdot U_m} \cdot r^2$$

$$k2 = \frac{m_m \cdot \rho_p}{m_p \cdot \rho_m} \cdot \left(\frac{U_p}{U_m} \right)^2 \cdot r^2$$

$$k3 = \frac{m_m \cdot \rho_p}{m_p \cdot \rho_m} \cdot r^3$$

$$k4 = \frac{m_m \cdot \rho_p \cdot U_p}{m_p \cdot \rho_m \cdot U_m} \cdot r^3$$

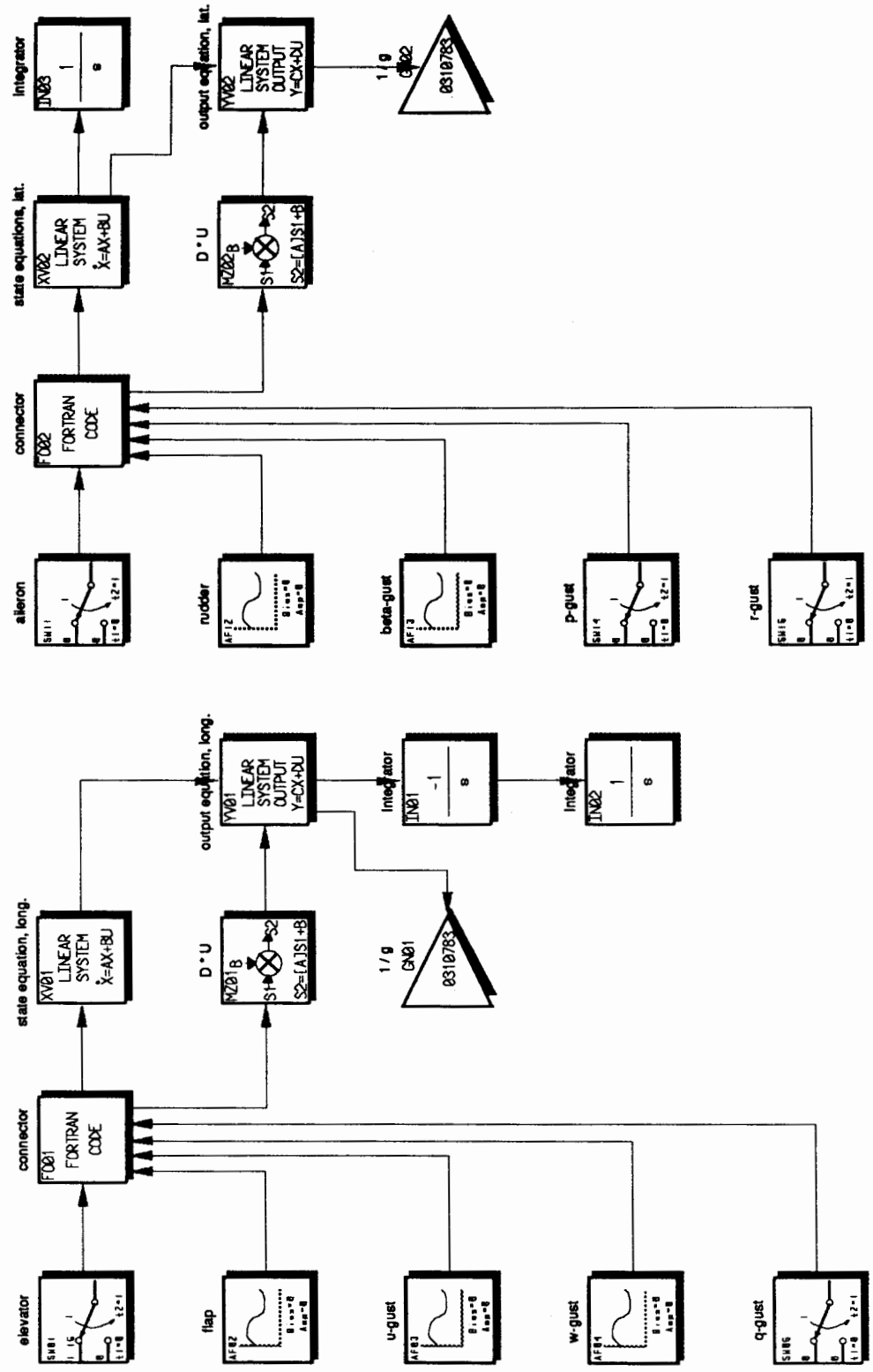
$$k5 = \frac{m_m \cdot \rho_p \cdot U_p}{m_p \cdot \rho_m \cdot U_m} \cdot r$$

$$k6 = \frac{m_m \cdot \rho_p}{m_p \cdot \rho_m} \cdot r^2$$

$$k7 = \frac{m_m \cdot \rho_p}{m_p \cdot \rho_m} \cdot \left(\frac{U_p}{U_m} \right)^2 \cdot r$$

() _p / () _m	factor	() _p / () _m	factor
X _u	k1	Y _v	k1
X _w	k1	Y _r [*]	k3
X _d	k2	Y _p [*]	k3
Z _u	k1	Y _d [*]	k1
Z _w	k1	N _b	k7
Z _w	k3	N _r	k1
Z _q	k4	N _p	k1
Z _d	k2	N _d	k7
M _u	k5	L _b	k7
M _w	k5	L _r	k1
M _w	k6	L _p	k1
M _q	k1	L _d	k7
M _d	k7		

Fig. 1: Linear Aircraft Simulation (open Loop)



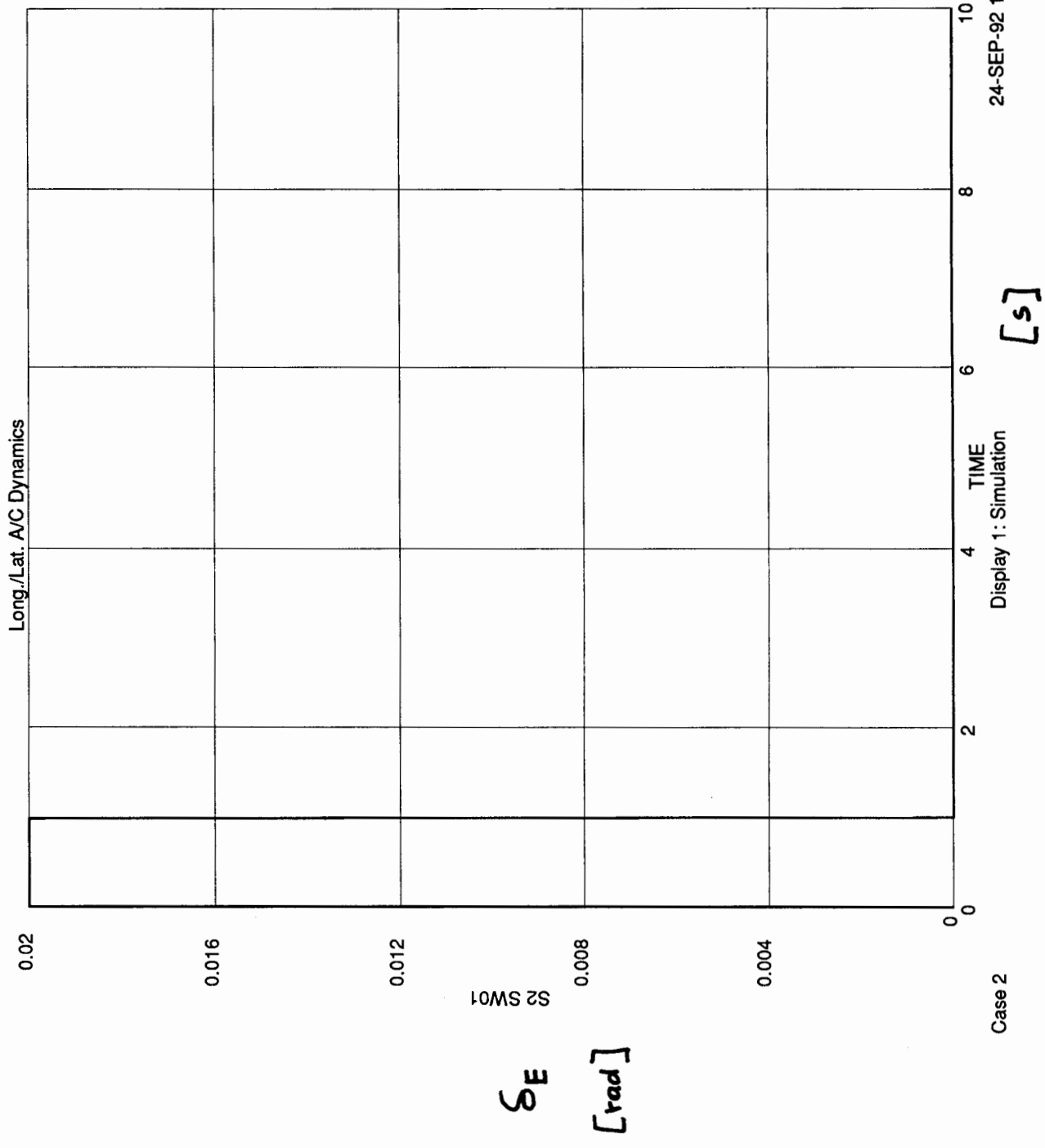
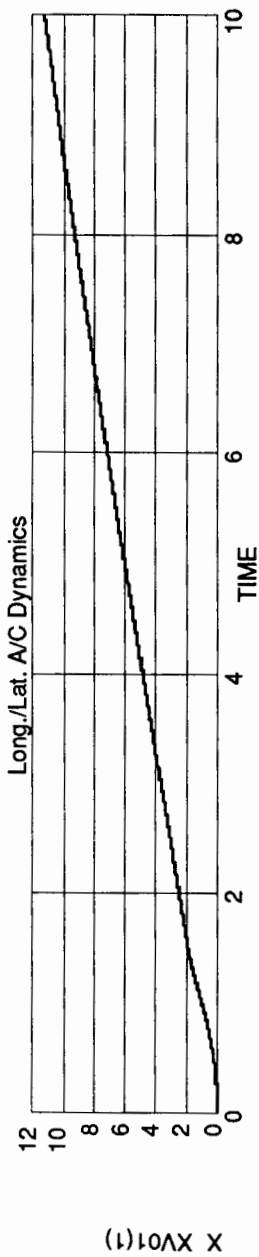


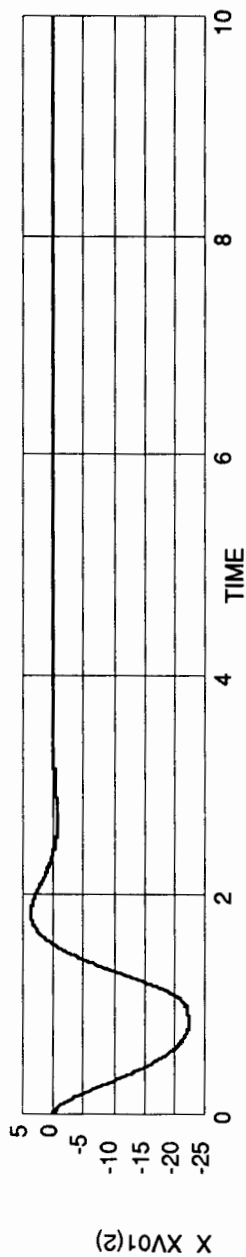
Fig. 2:

Elevator input to
'test' aircraft.

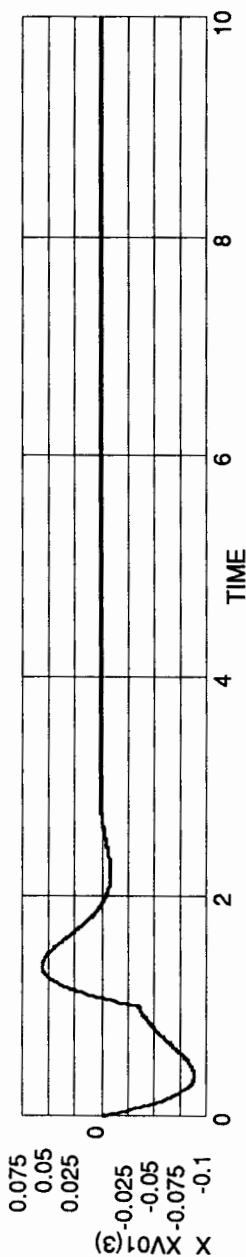
Compare to Fig. 5-3
of Ref. 1.



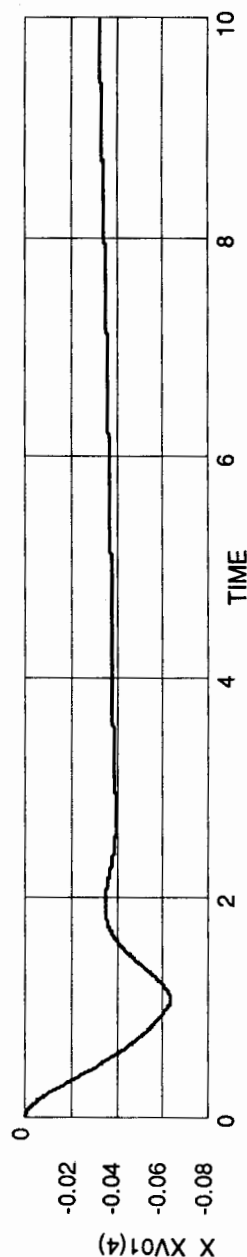
u
[ft/s]



w
[ft/s]



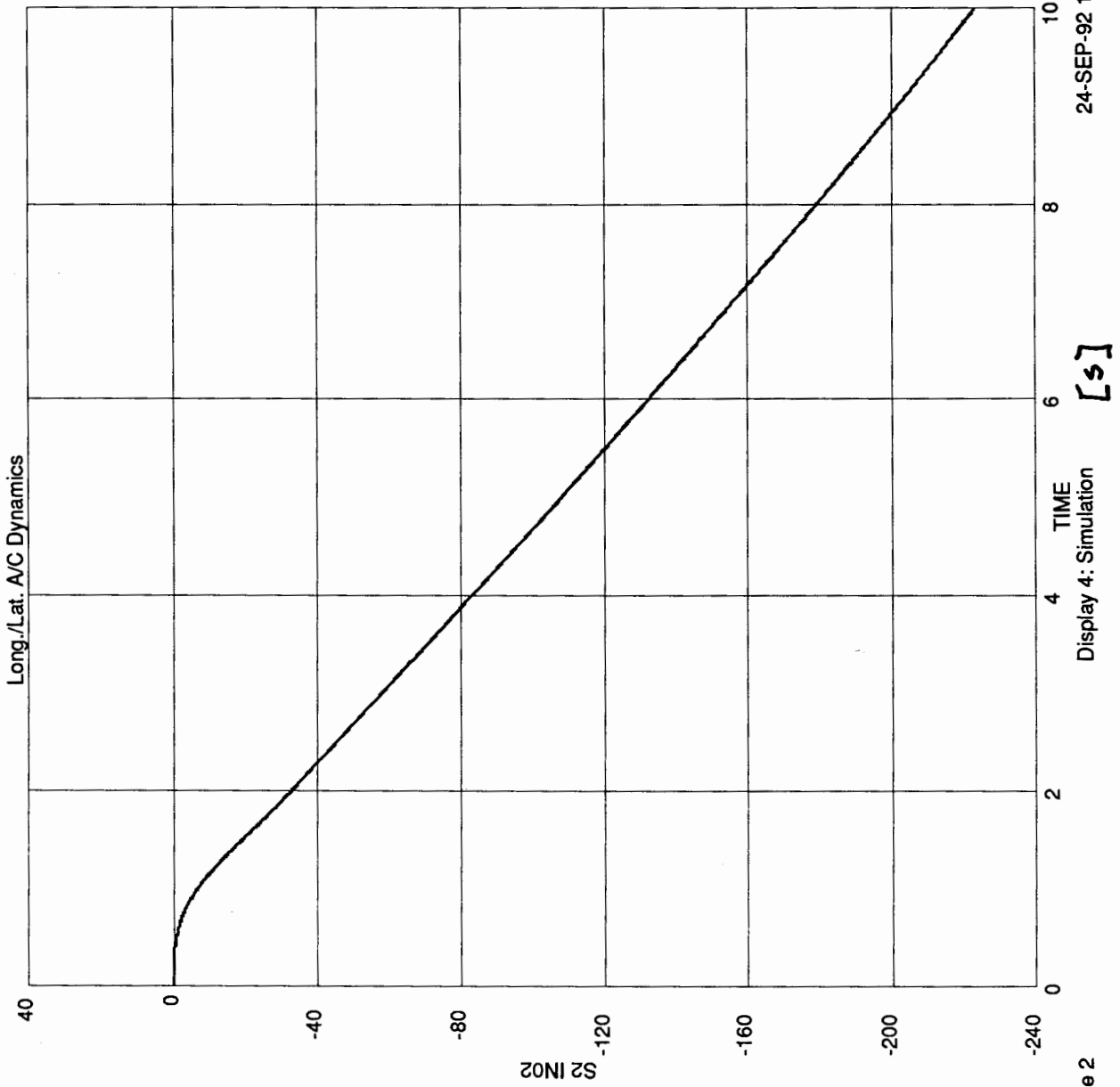
$\dot{\theta} = q$
[rad/s]



θ
[rad]

Fig. 3:

Response of
'test' aircraft
to elevator input (Fig.2).
Compare to Fig. 5-3
of Ref. 1.



Case 2

Display 4: Simulation

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Fig. 4 :

Response of
'test' aircraft
to elevator input
(Fig. 2)

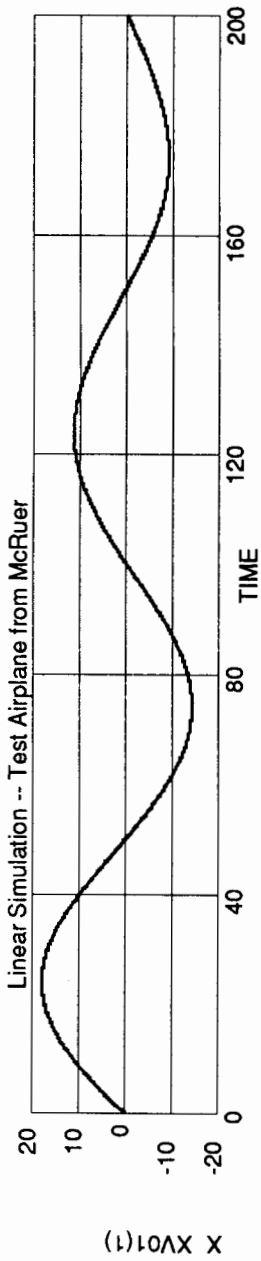
Compare to Fig. 5-3
of Ref. 1.

h
[ft]

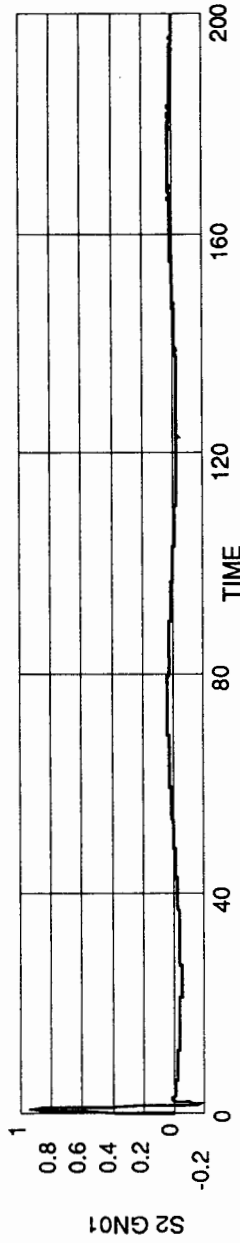
Fig. 5:

Response of
'test' aircraft
to elevator input
(Fig. 2).

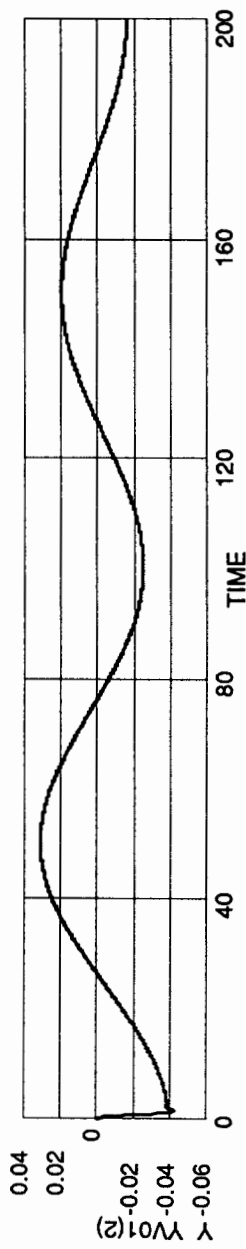
Compare to Fig. 5-3
of Ref. 1



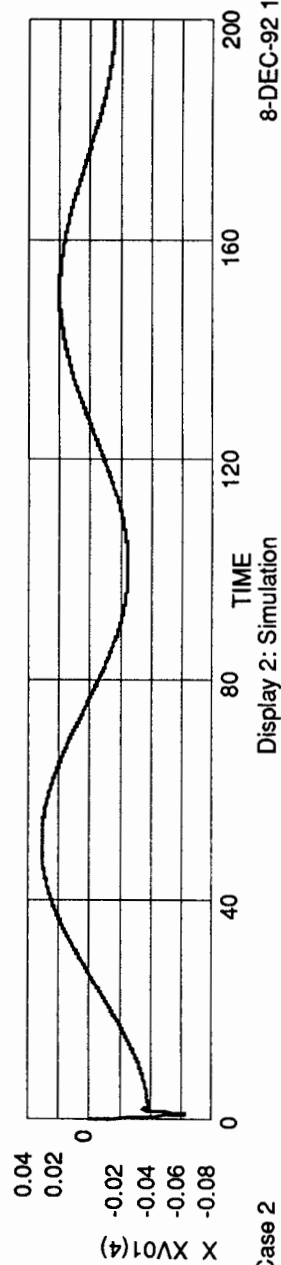
u
[ft/s]



n_{zcg}
[-]



δ
[rad]



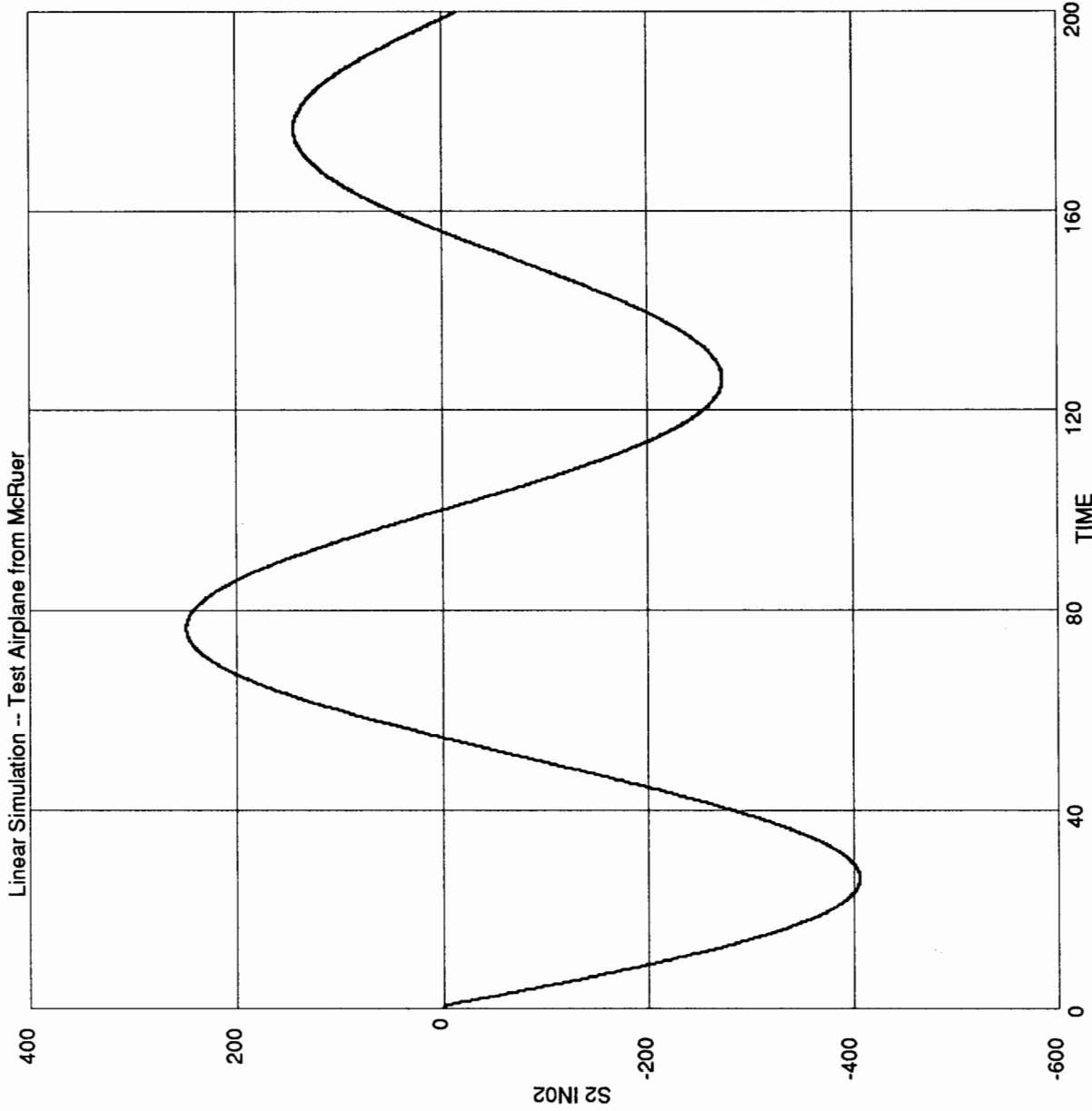
θ
[rad]

Case 2

Display 2: Simulation

8-DEC-92 14:37

Linear Simulation -- Test Airplane from McRuer



Case 2

Display 4: Simulation

8-DEC-92 14:37

Fig. 6:

Response of 'test'
aircraft to elevator
input (Fig. 2).

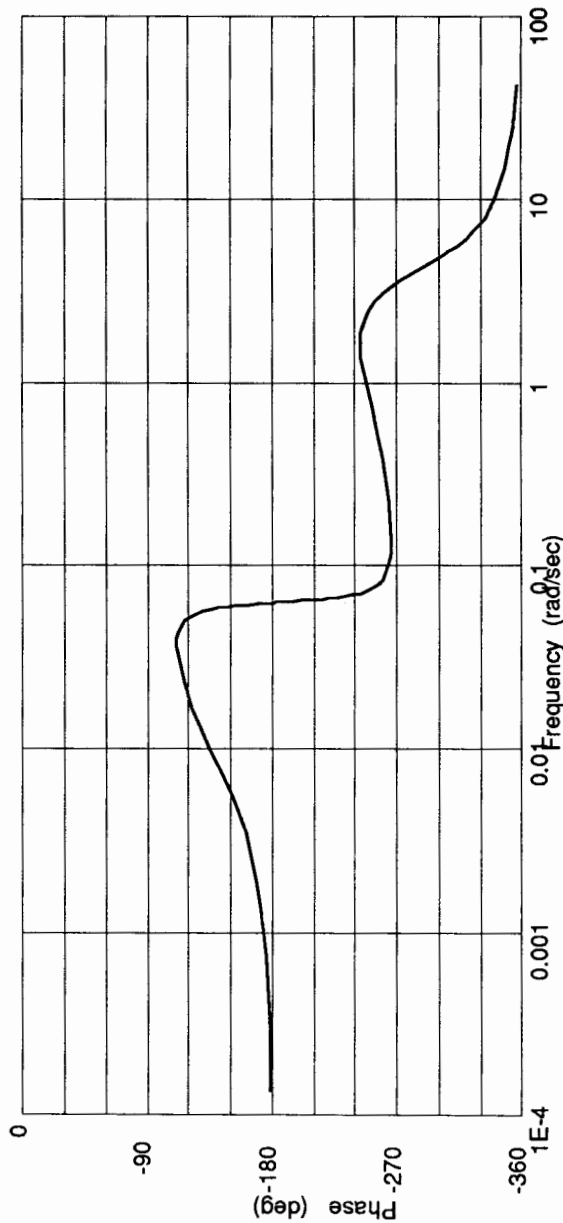
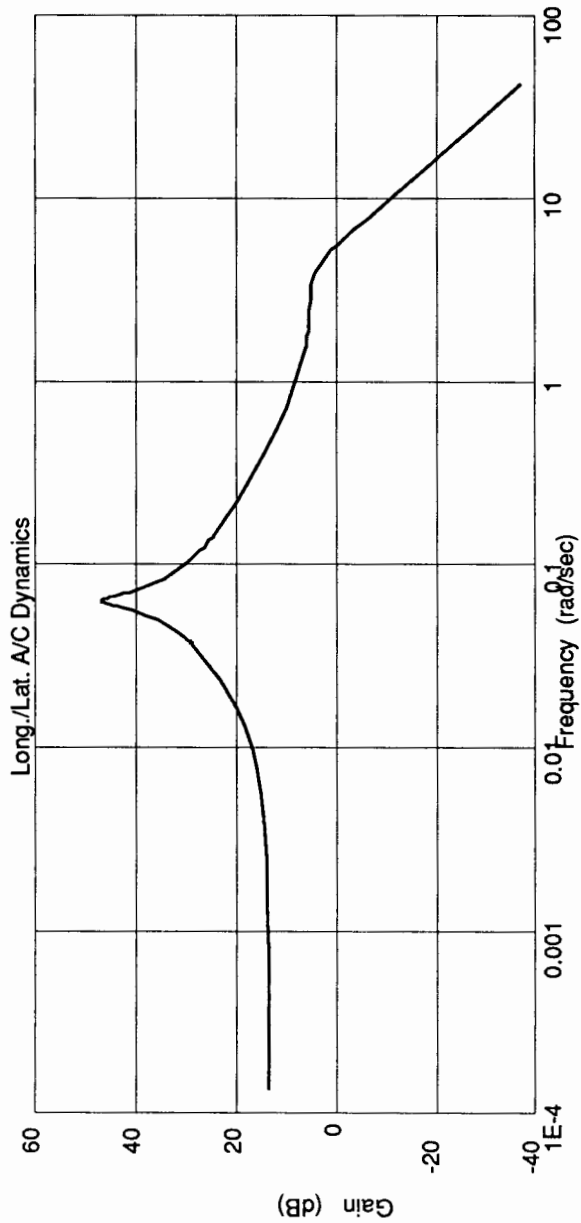
Compare to Fig. 5-3
of Ref. 1.

Fig. 7:

Bode plot

$$\frac{\Theta(s)}{\delta E(s)}$$

Compare with
Fig. 5-2 a of Ref. 1



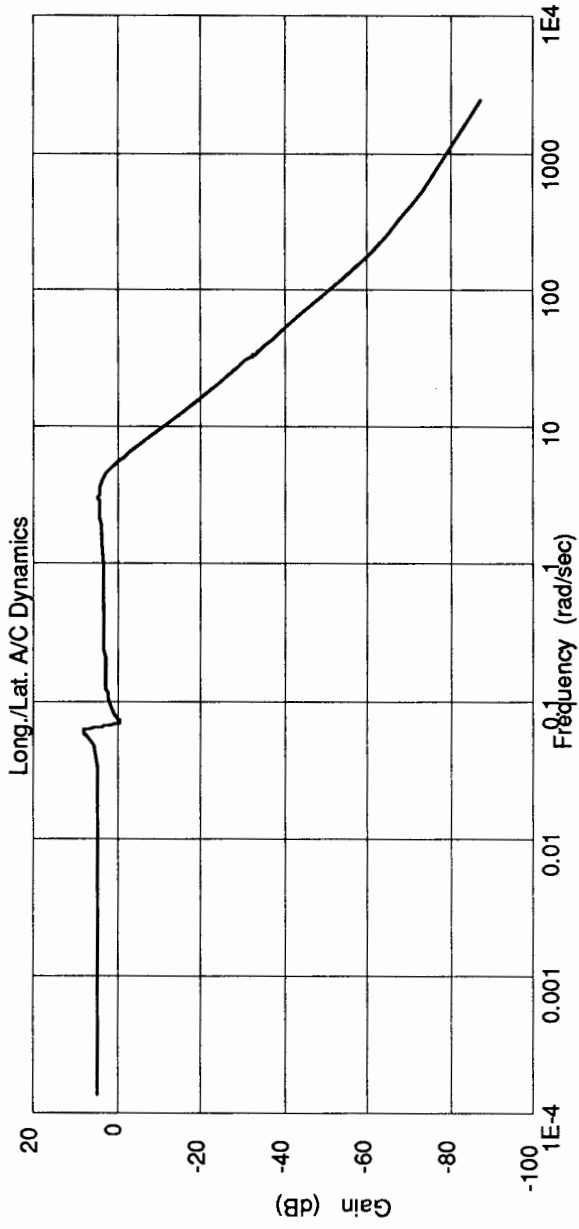


Fig. 8:

Bode plot

$$\frac{\alpha(s)}{\delta E(s)}$$

Compare with

Fig. 5-2 b of Ref. 1.

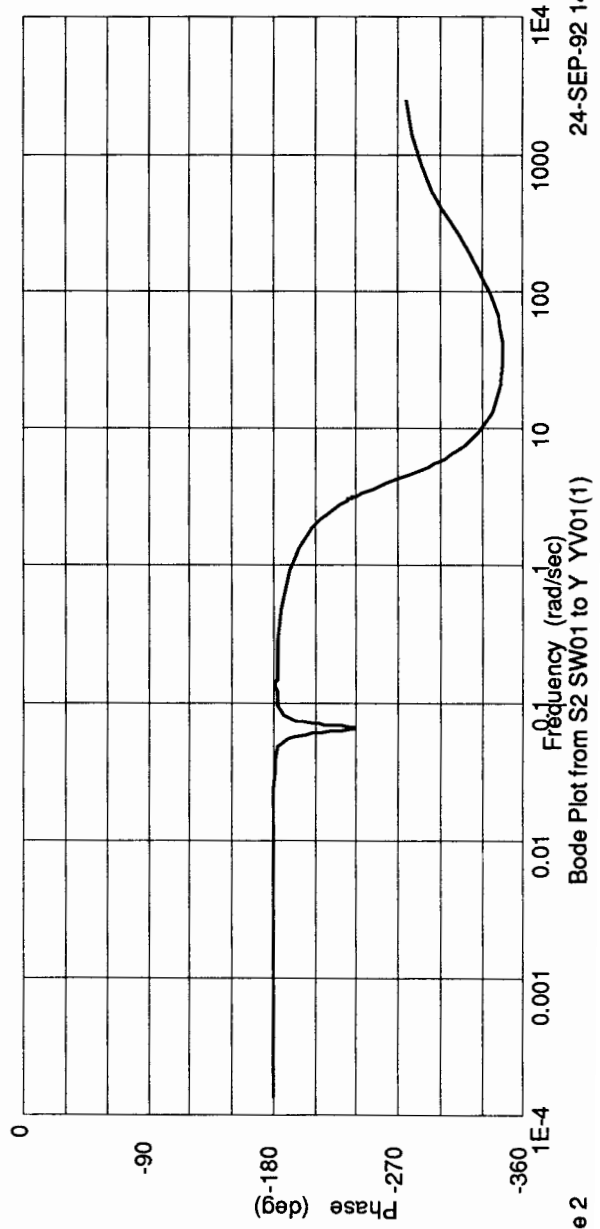


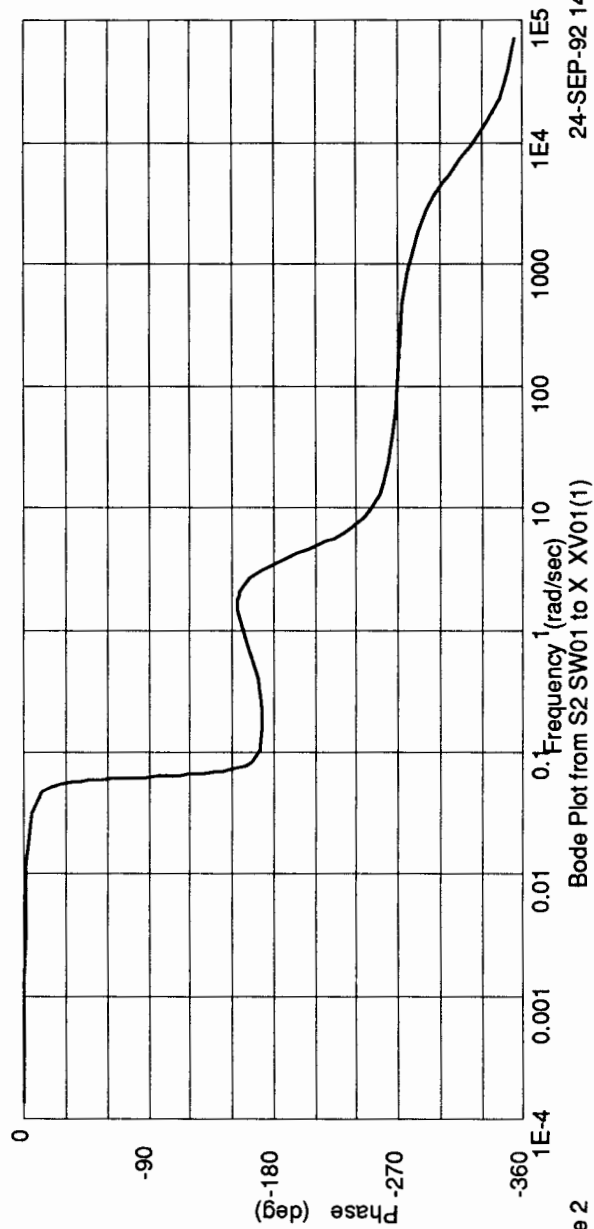
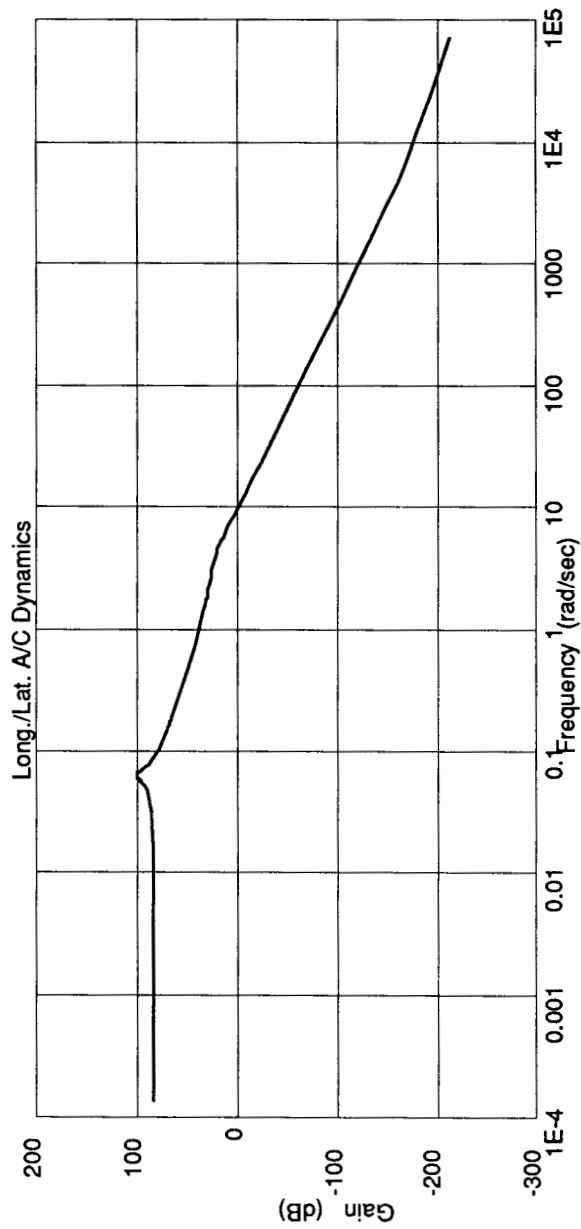
Fig. 9:

Bode plot

$$\frac{u(s)}{\delta E(s)}$$

Compare with

Fig. 5-2 c of Ref. 1.



Case 2

24-SEP-92 14:50

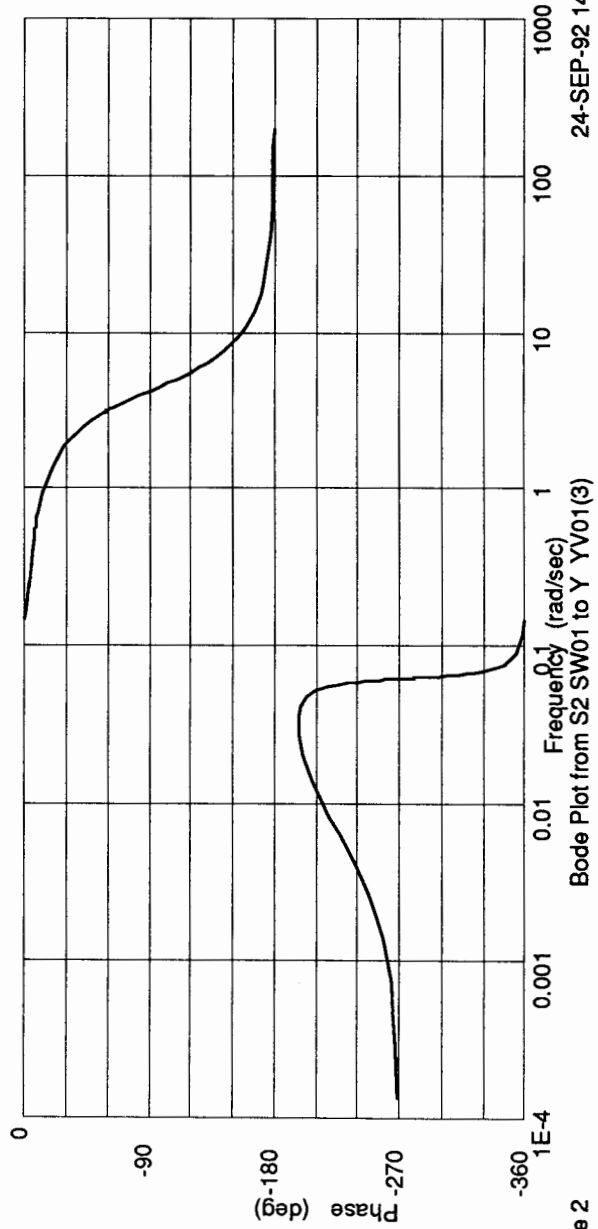
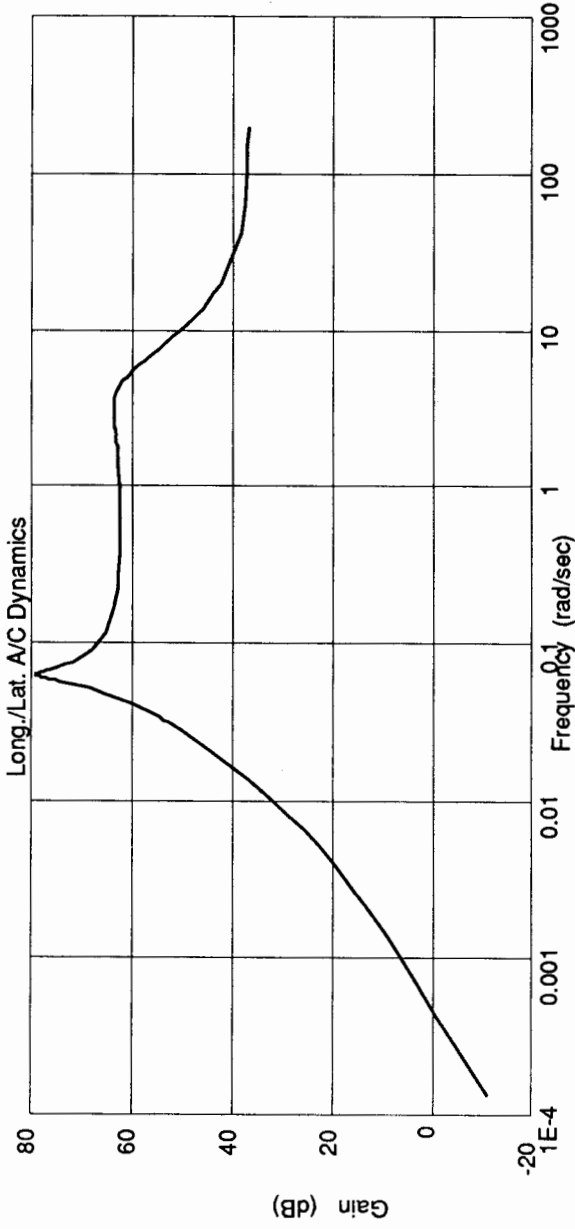
Fig. 10:

Bode plot

$$\frac{a_{2eg}(s)}{\delta E(s)}$$

Compare with

Fig. 5-2 d of Ref. 1.



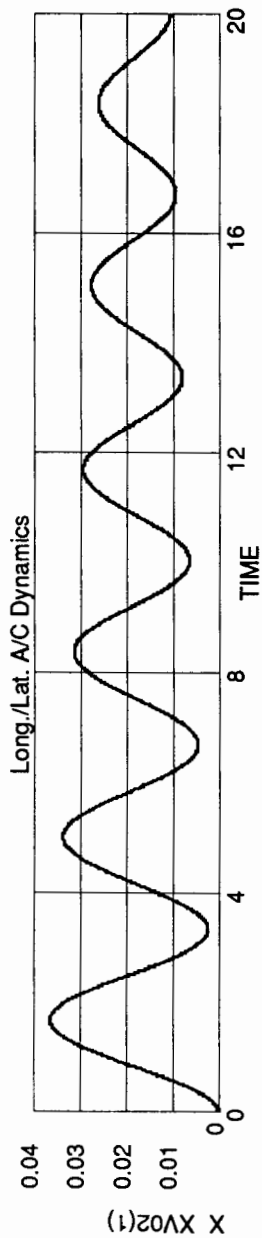
24-SEP-92 14:53

Case 2

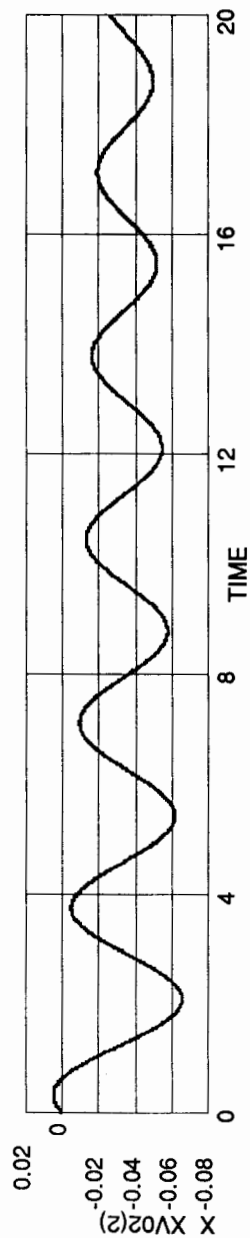
Fig. 11:

Response of 'test'
aircraft to
0.05 rad rudder step
input.

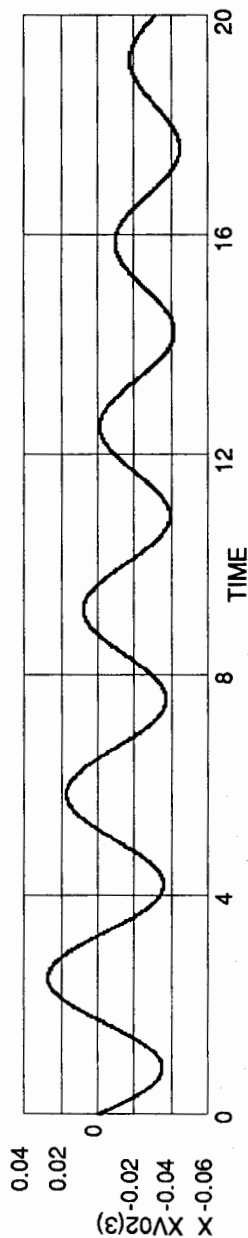
Compare to Fig. 6-3
of Ref. 1



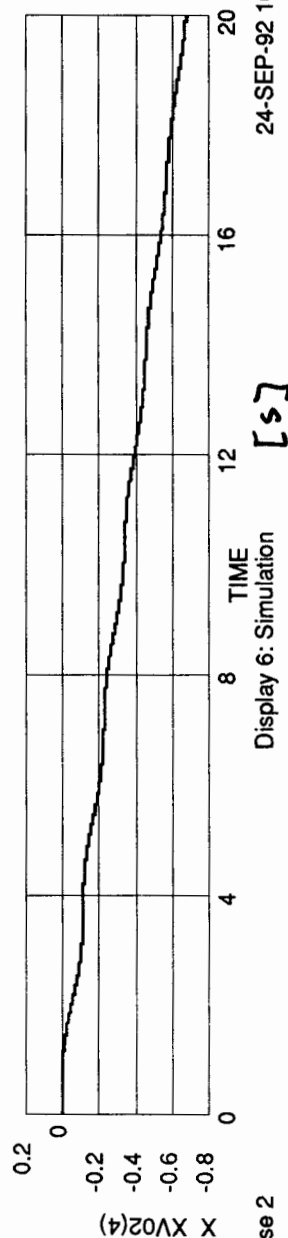
β
[rad]



p
[rad/s]



r
[rad/s]



ϕ
[rad]

Case 2

Display 6: Simulation

[s]

24-SEP-92 16:01

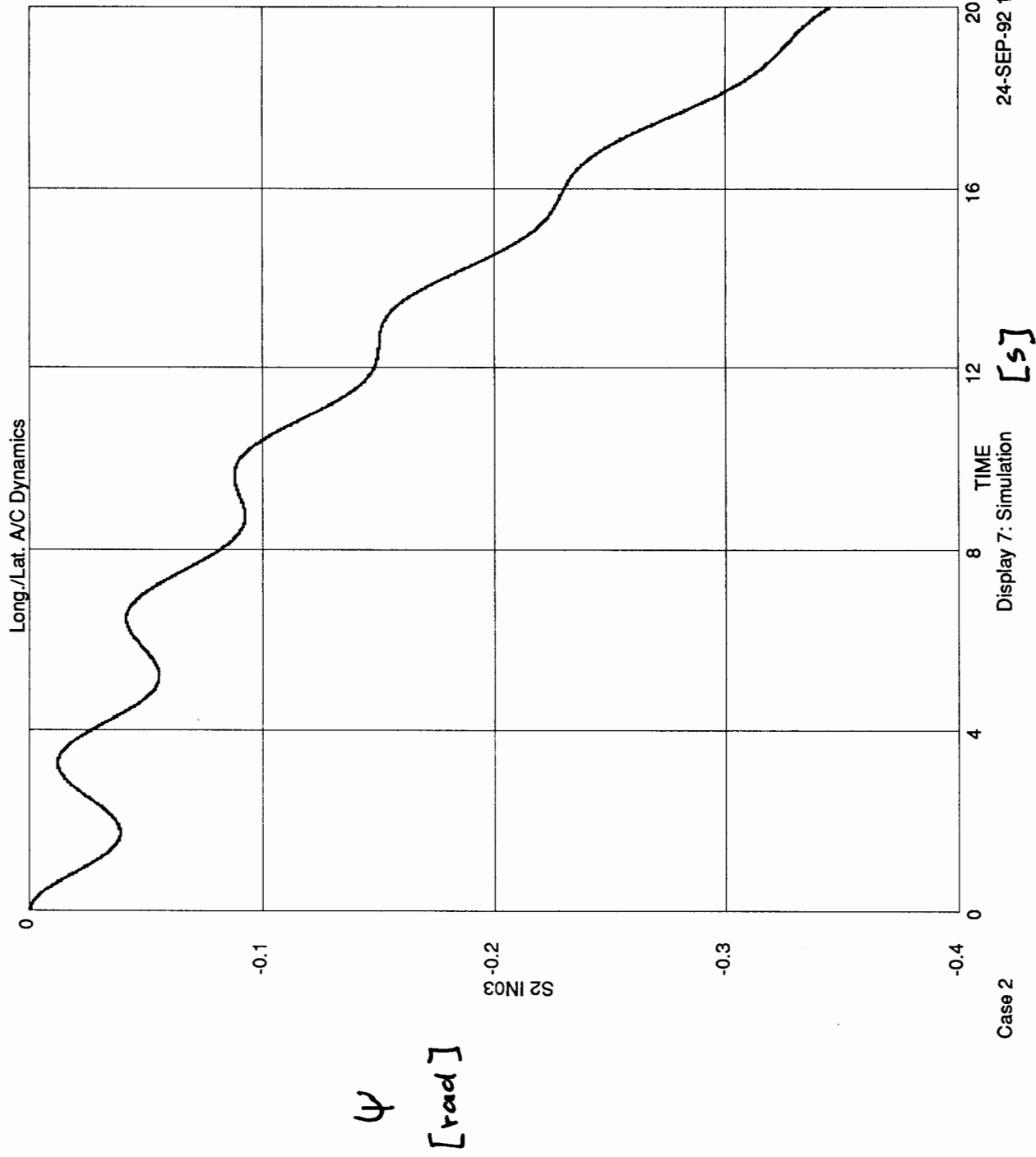
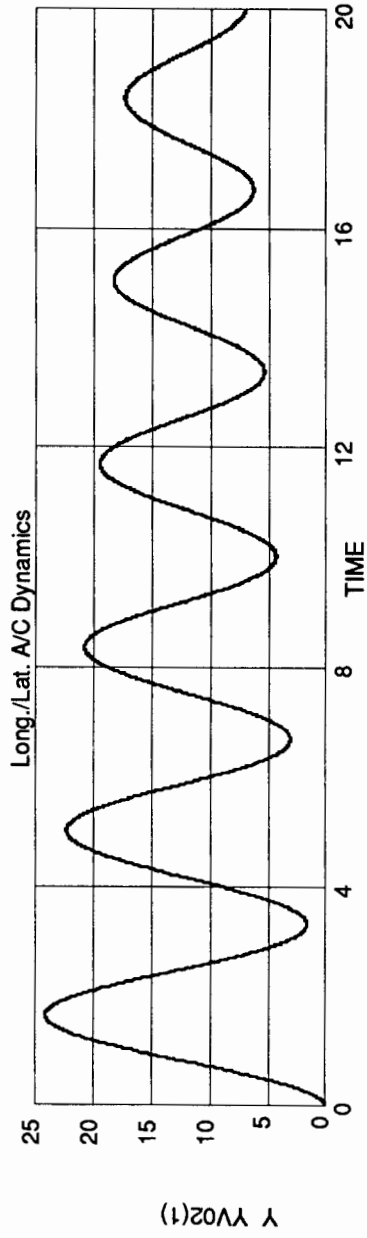


Fig. 12:

Response of 'test'
aircraft to
0.05 rad rudder step
input.

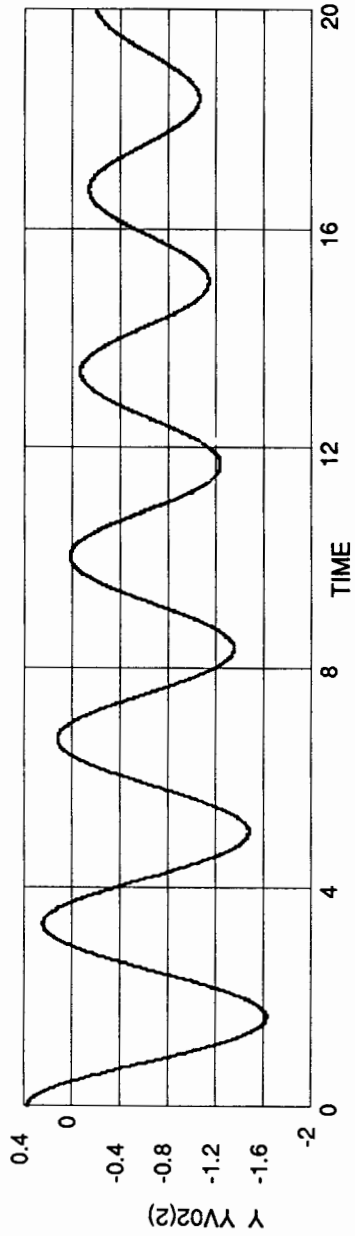
Compare to Fig. 6-3
of Ref. 1.



V
[ft/s]

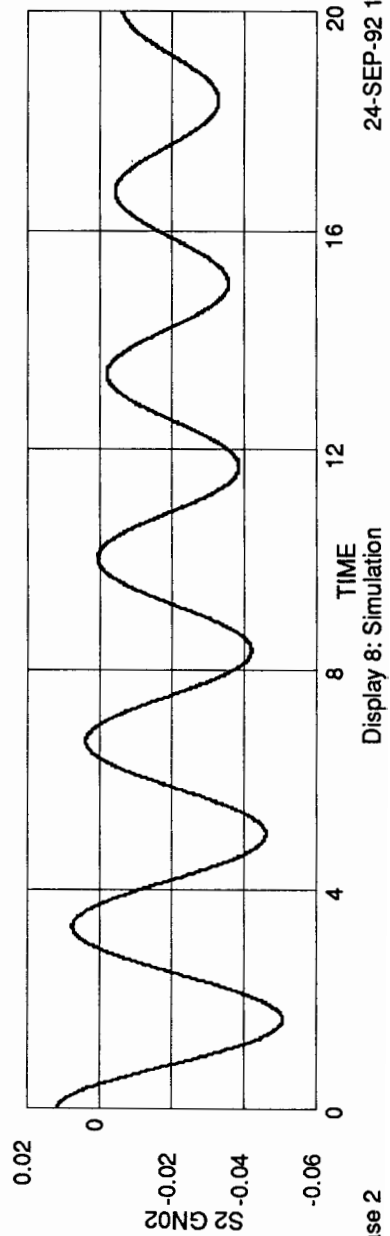
Fig. 13:

Response of 'test'
aircraft to
0.05 rad rudder step
input.



$a_{y_{cg}}$
[ft/s²]

Compare $a_{y_{cg}}$ to
Fig. G-3 of Ref. 1.



$n_{y_{cg}}$
[-]

Case 2

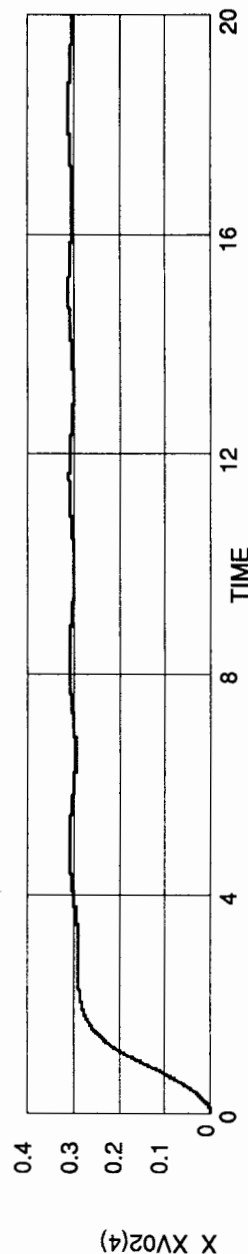
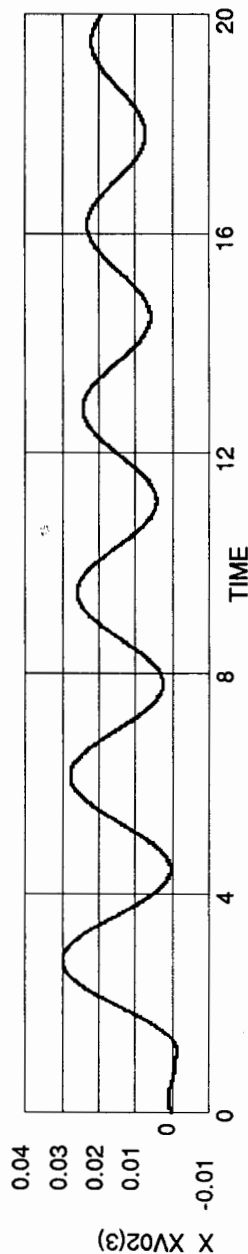
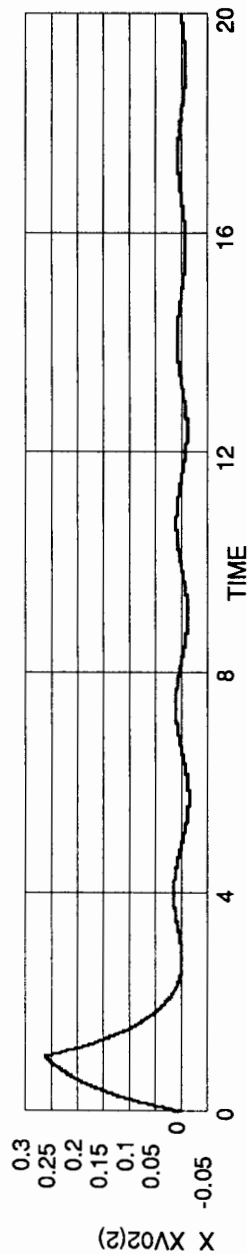
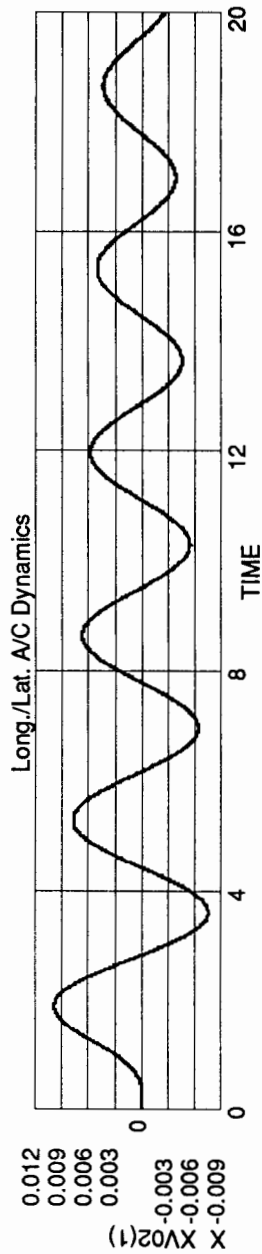
Display 8: Simulation

24-SEP-92 16:36

Fig. 14:

Response of 'test' aircraft to 1-second -0.02 rad aileron pulse input.

Compare to Fig. 6-3 of Ref. 1.



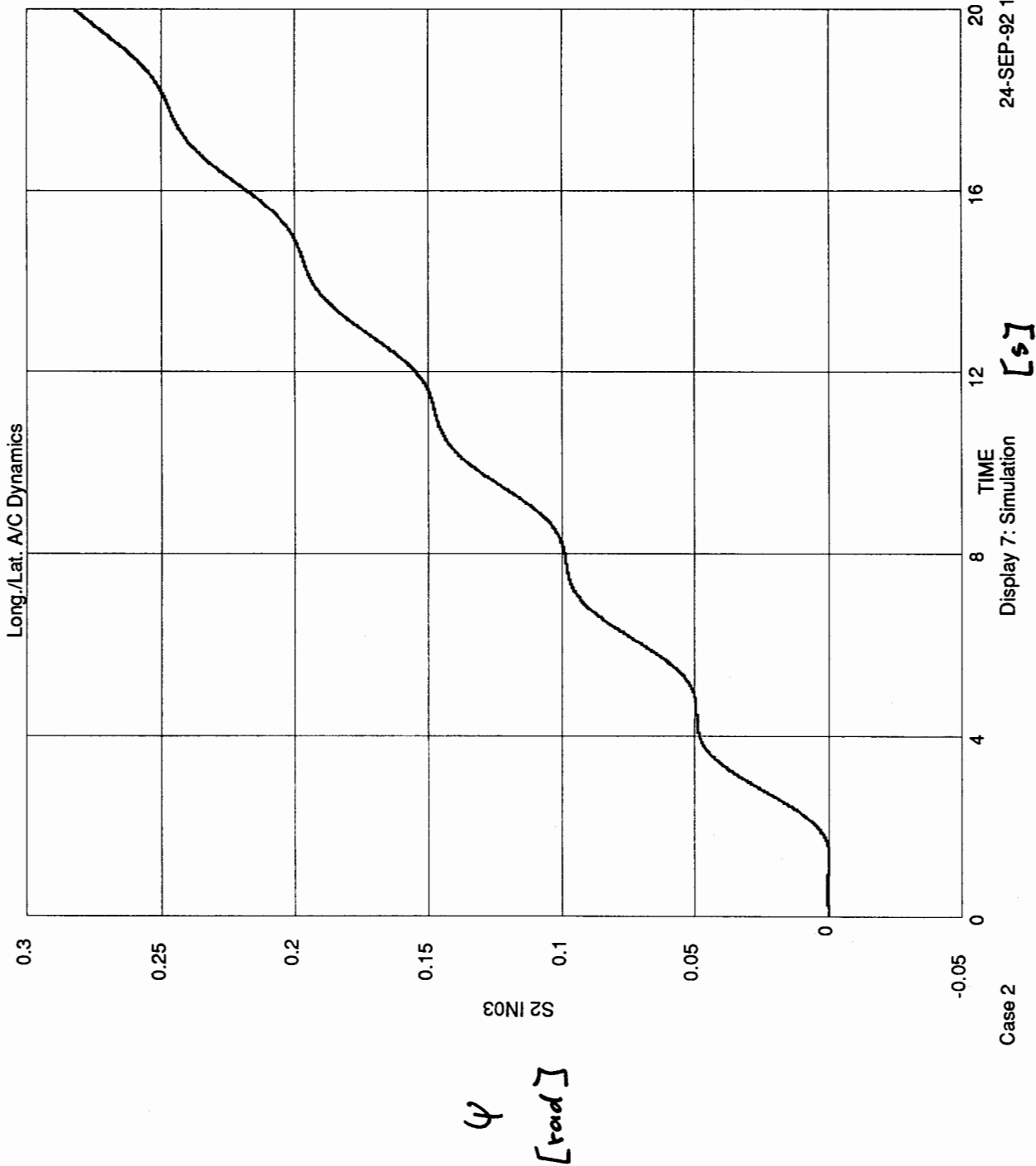
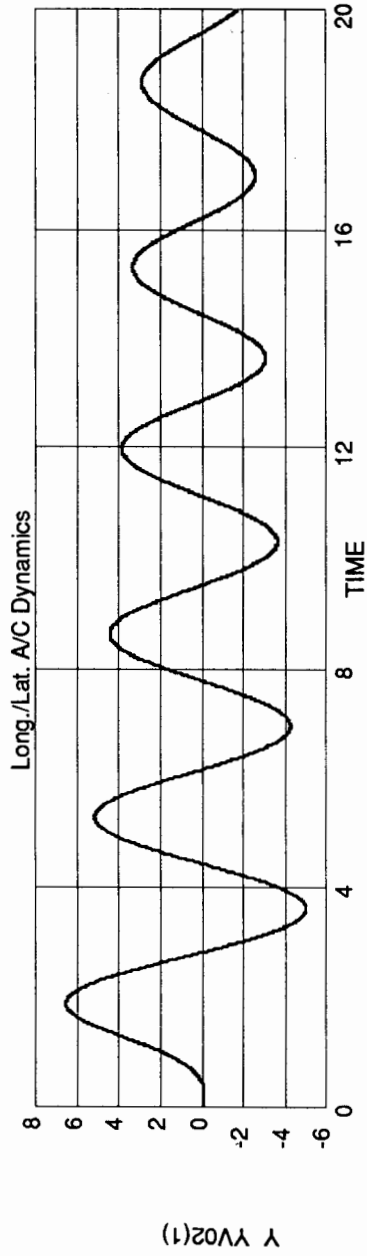


Fig. 15:

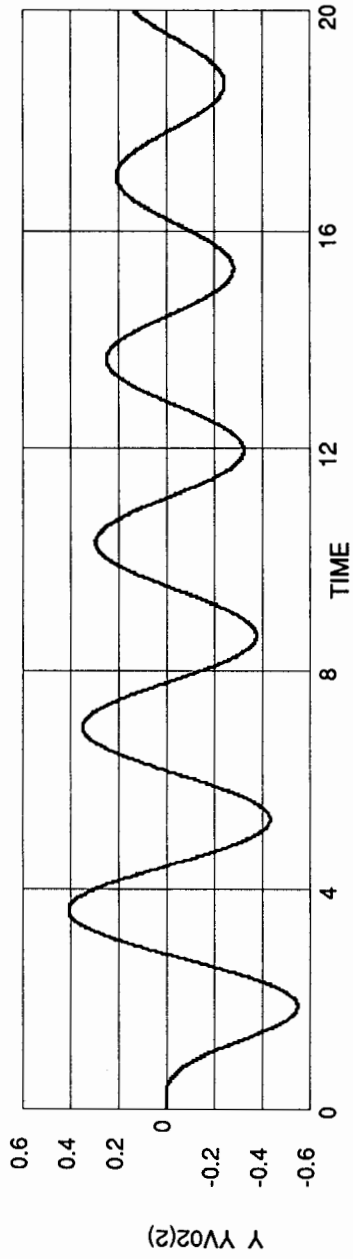
Response of 'test' aircraft to

1-second 0.02 rad aileron pulse input

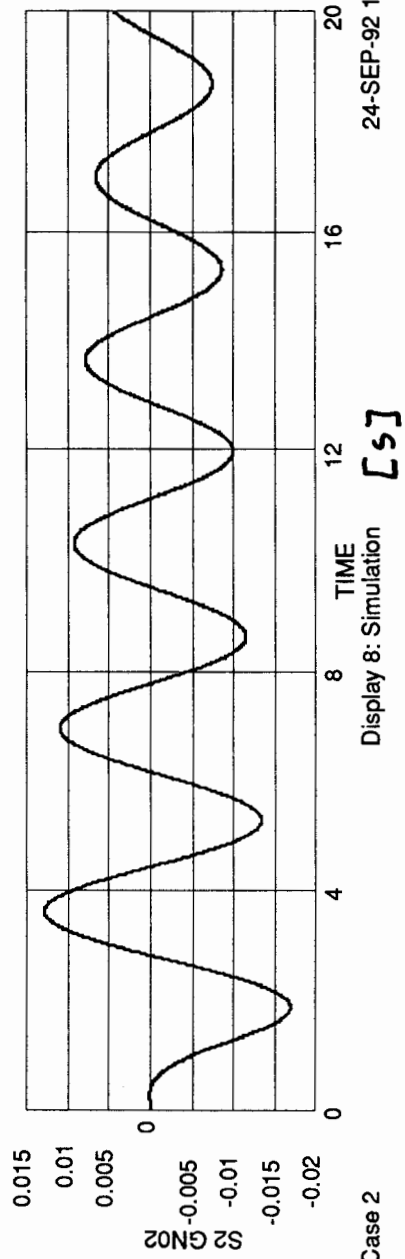
Compare to Fig. G-3 of Ref. 1.



V
[ft/s]



$\dot{\alpha}_{ycg}$
[ft/s²]



n_{ycg}
[-]

Fig. 16:

Response of 'test'
aircraft to
1-second 0.02 rad
aileron pulse input

Compare to Fig. 6-3
of Ref. 1 for $\dot{\alpha}_{ycg}$.

Long./Lat. A/C Dynamics

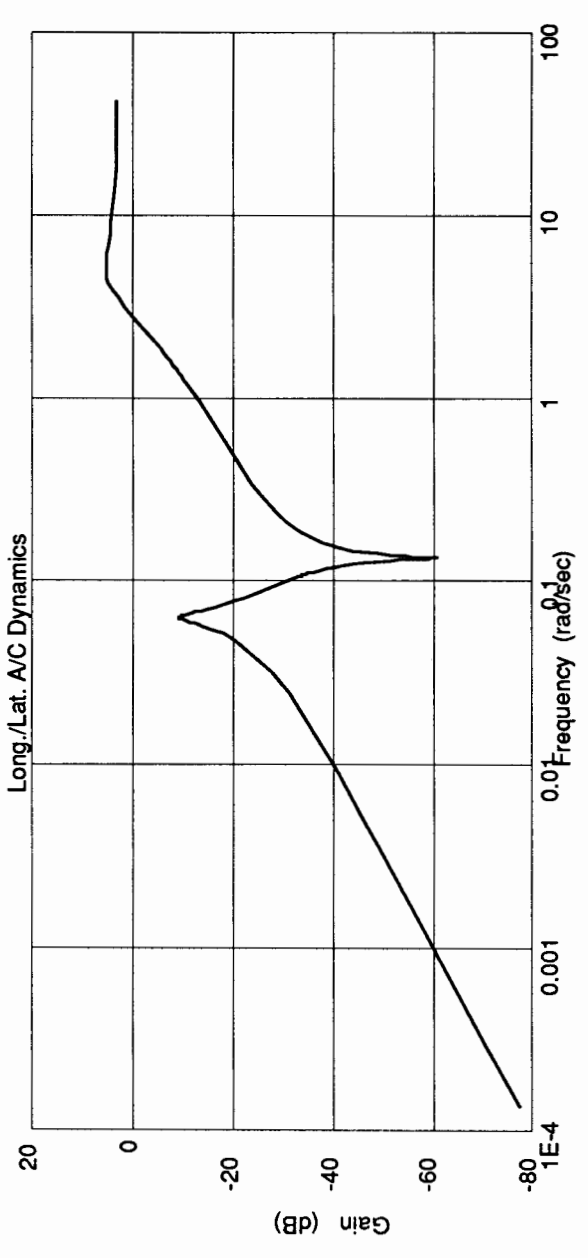
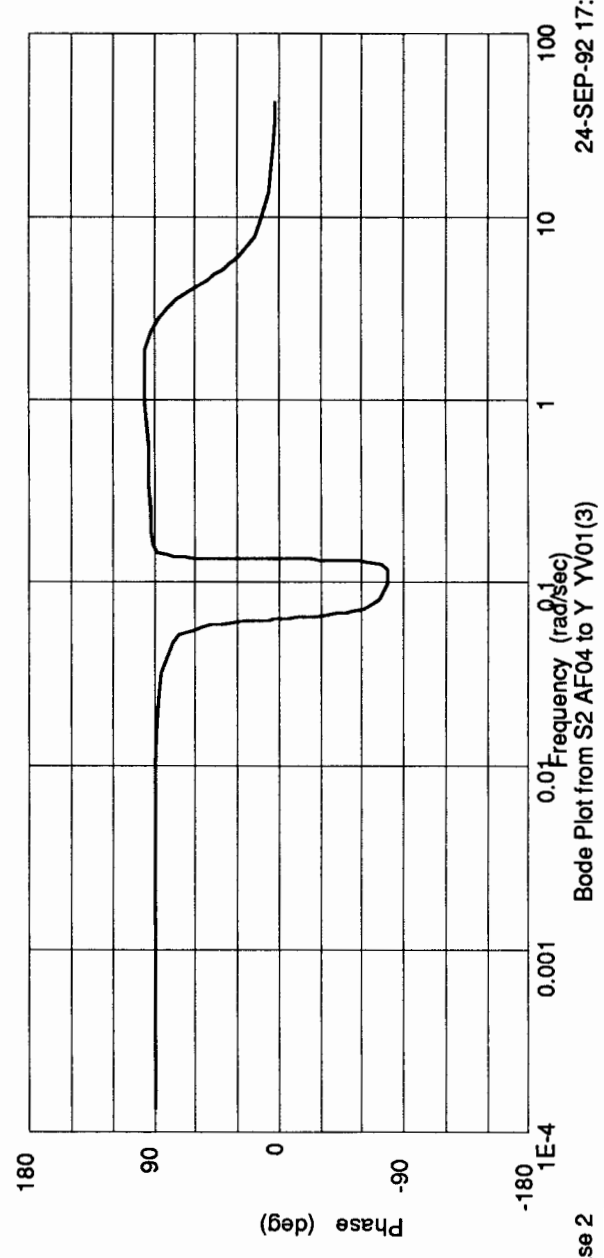


Fig. 17:

Bode plot

$$\frac{a_{zcg}(s)}{W_g(s)}$$

Compare with Fig. 5-11 d of Ref. 1.



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Bode Plot from S2 AF04 to Y YV01(3)

Case 2

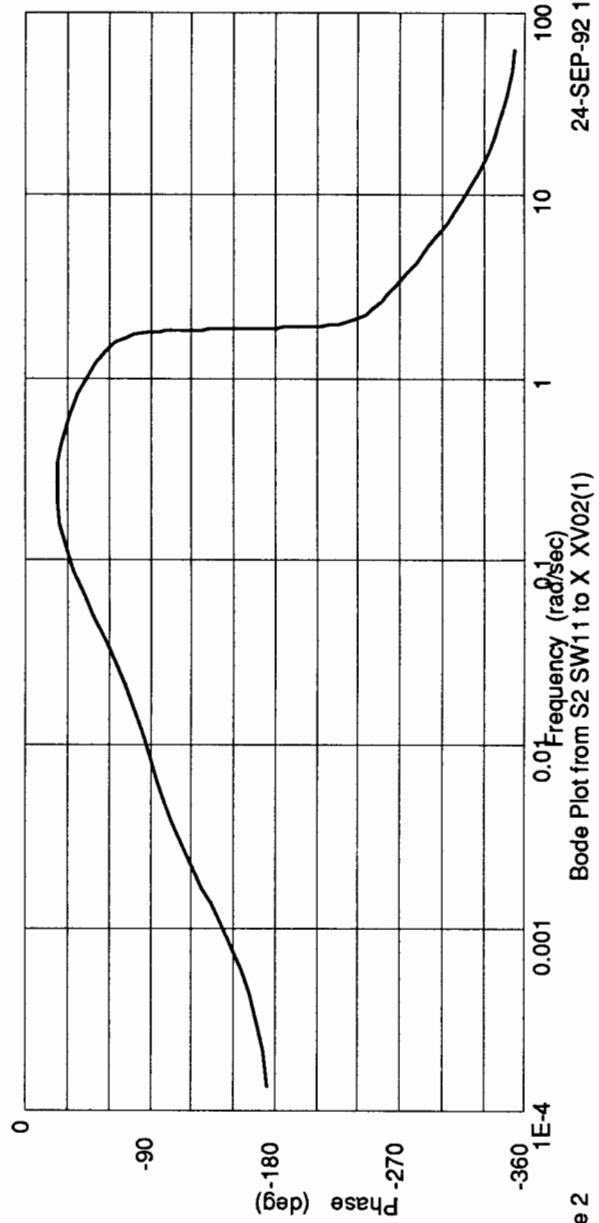
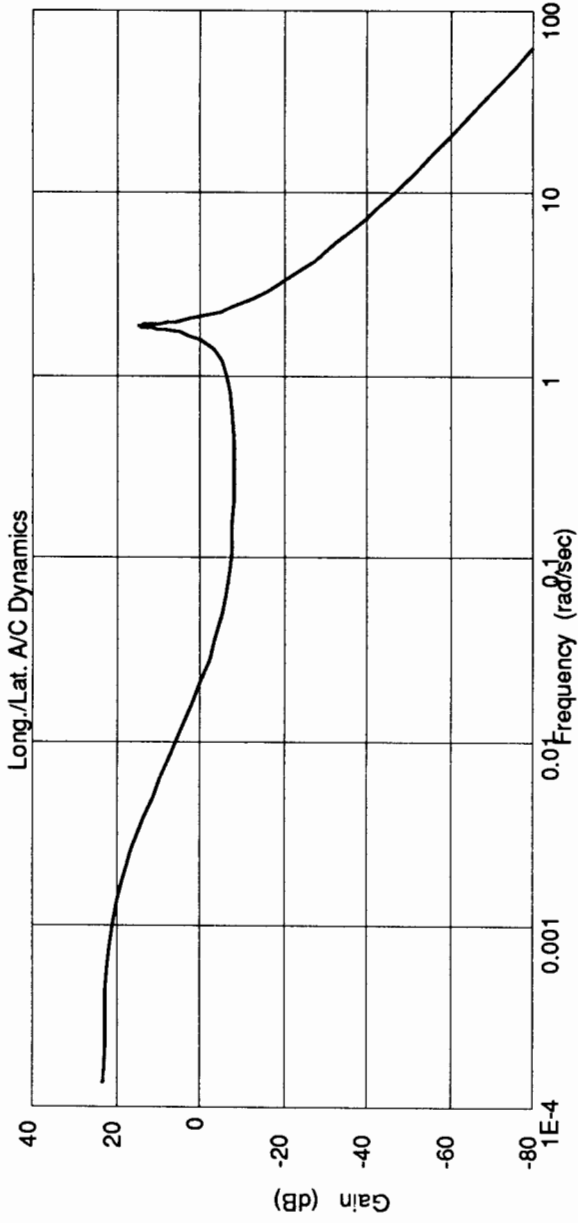
Fig. 18

Bode plot

$$\frac{\beta(s)}{S_A(s)}$$

Compare with

Fig. 6-2 a of Ref. 1.



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Case 2

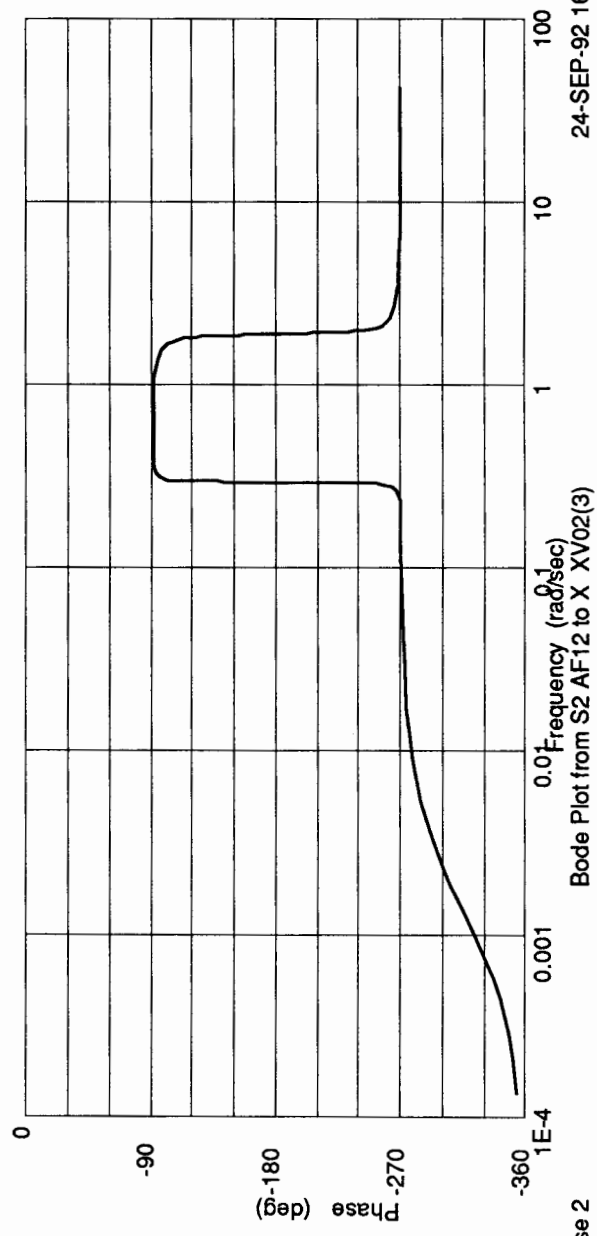
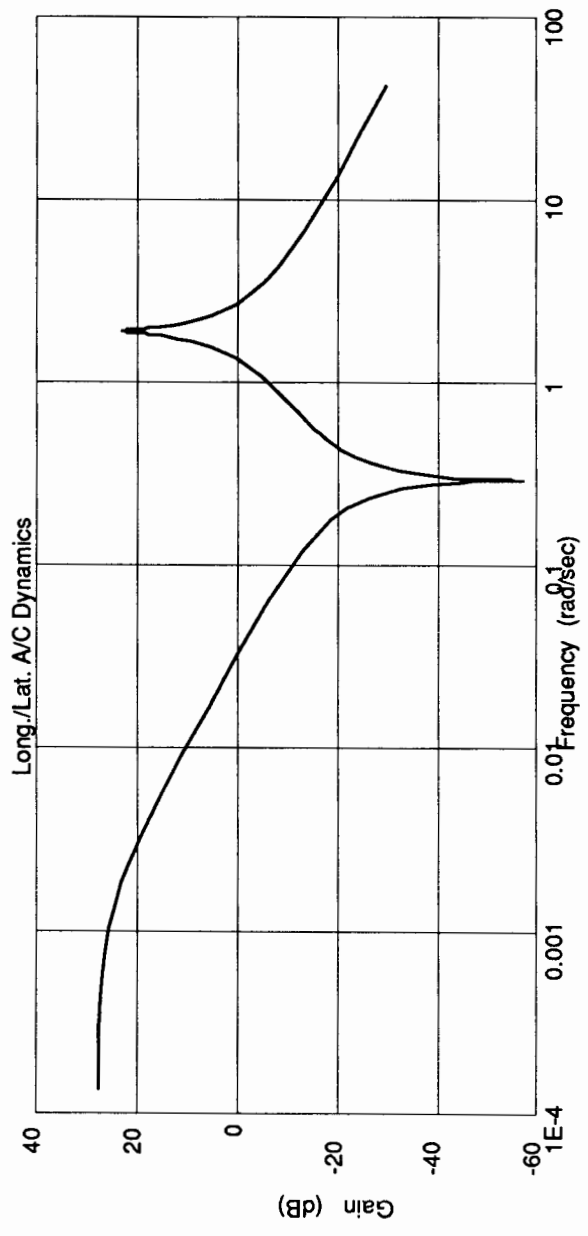
Fig. 19:

Bode plot

$$\frac{r(s)}{\delta_R(s)}$$

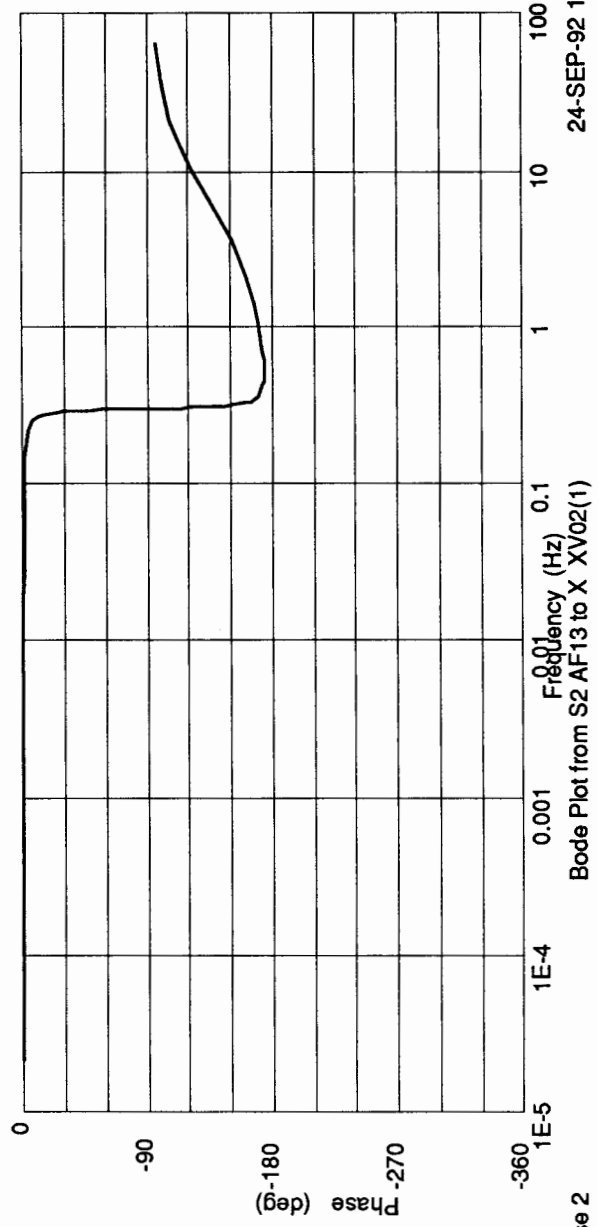
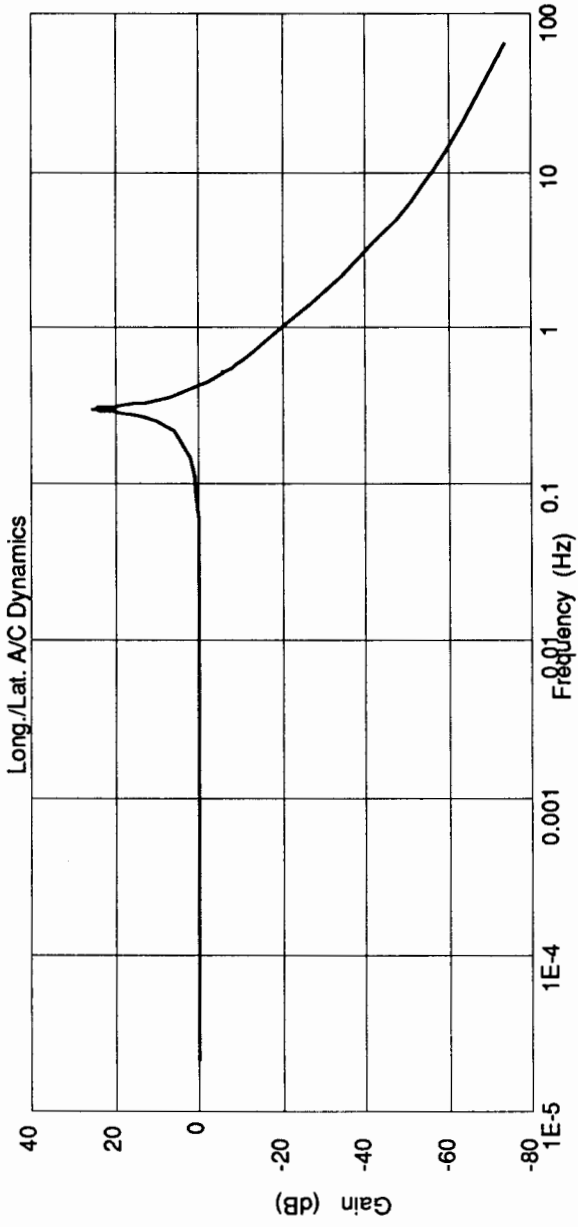
Compare with

Fig. 6-1 c of Ref. 1.



Case 2

24-SEP-92 16:50



Case 2

Bode Plot from S2 AF13 to X XV02(1)

24-SEP-92 16:53

Fig. 20:

Bode plot

$$\frac{\beta(s)}{\beta_g(s)}$$

Compare with

Fig. 6-11 a of Ref. 1.

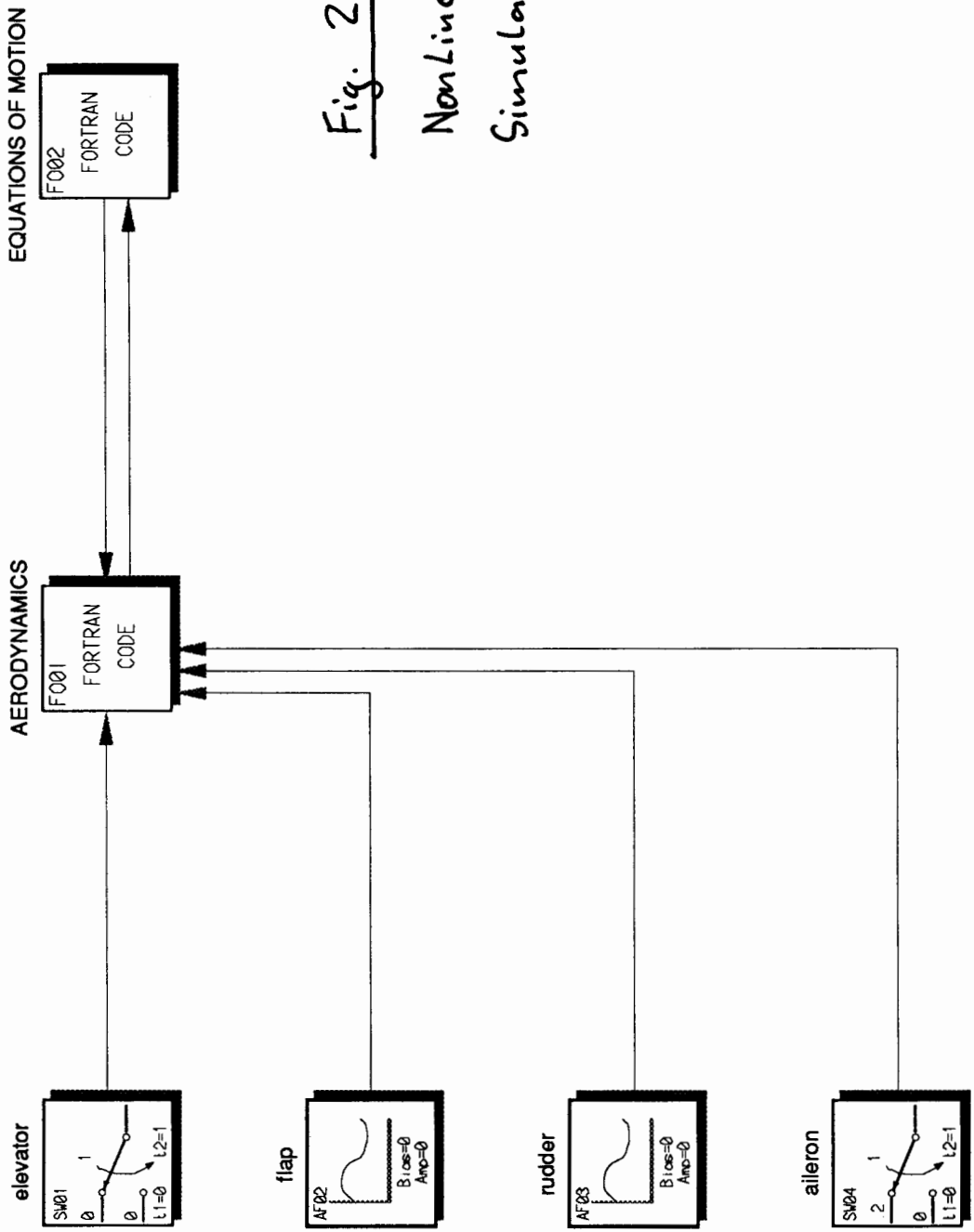


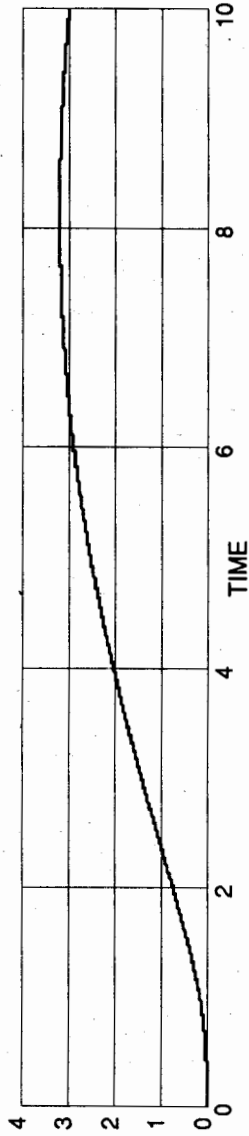
Fig. 21:

NonLinear Aircraft
Simulation (open loop)

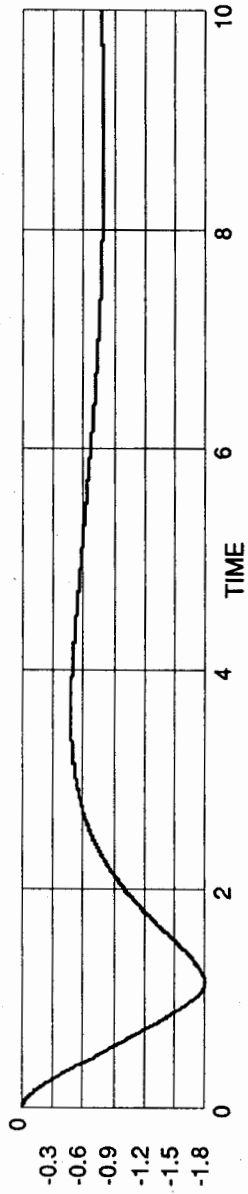
Fig. 22:

DC 3 response
to elevator input
from Fig. 2

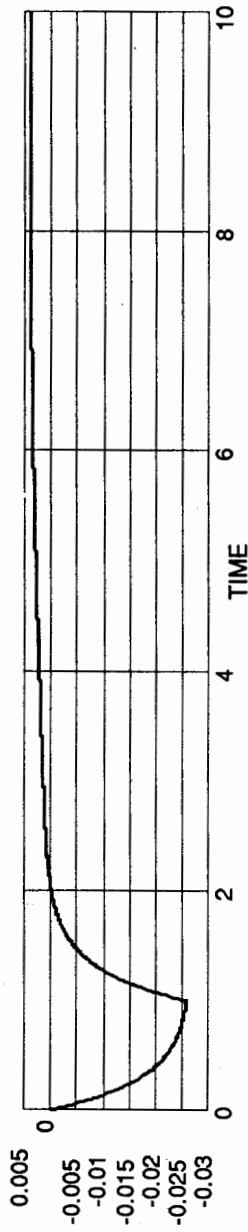
Compare with Fig. 3
to see differences
between 'test' aircraft
and DC 3.



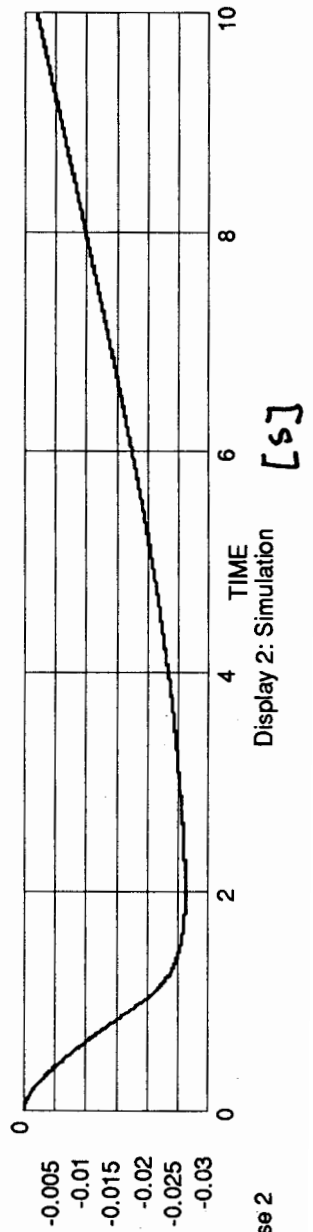
u
[ft/s]



w
[ft/s]



q
[rad/s]



θ
[rad]

Case 2

Display 2: Simulation

[s]

Fig. 24:

Combining aircraft dynamics with sensor, controller and actuator to a closed loop simulation

