Hochschule für Angewandte Wissenschaften Hamburg
Hamburg University of Applied Sciences

Project

Department of Automotive and Aeronautical Engineering

The Effect of Variations of the Height to Span Ratio of Box Wing Aircraft on Induced Drag and the Spanwise Lift Distribution

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Abstract

This project covers the effect of variations of the height to span (*h/b*) ratio of box wing aircraft on induced drag and the spanwise lift distribution. A box wing aircraft is an unconventional non planar configuration. It is comparable with a joined wing aircraft where the tips of the wings are connected. But for a box wing aircraft the tips are connected with winglets so it forms a rectangular box in the front view. This non planar configuration is known for its low induced drag and hence fuel reduction potential. The investigation is done using IDRAG, a program written by Joel Grasmeyer of Virginia Tech that calculates the distribution of the normal force coefficient, induced drag and span efficiency for non planar wings. The lift distribution of a box wing aircraft consists of a constant and an elliptical part. The magnitude and the variation in the shape of the lift distribution is investigated. Simple equations are derived. Equations from literature are compared with IDRAG results and a proposed new equation to calculate the span efficiency of box wings depending on the *h/b* ratio. Also according to this new equation span efficiency of box wings increases with increasing *h/b* ratio. The limit value for the span efficiency for *h/b* towards infinity e = 3.7.

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DEPARTMENT OF AUTOMOTIVE AND AERONAUTICAL ENGINEERING

The effect of variations of the height to span ratio of box wing aircraft on induced drag and the spanwise lift distribution

Task for a *project*

Background

A short to medium range box wing aircraft based on the Airbus A320 is examined within the framework of the research project Airport 2030, focusing on improved performance and a more efficient ground handling. For sizing the aircraft it is essential to estimate its span efficiency. The span efficiency factor depends on the height to span ratio (*h/b*) of the aircraft and the spanwise lift distribution. For minimum induced drag the lift distribution should consist of a constant and an elliptical part. The ratio between these two parts seems to depend on the *h/b* ratio. Knowing the exact ratio is important for assessing the wing loads needed for estimating the wing mass and also for determining the required spanwise distribution of the lift coefficient. This is why further aerodynamic analyses are necessary.

Task

The minimum induced drag and the spanwise lift distribution of box wing configurations is to be determined for different *h*/*b* ratios with the help of the aerodynamics software *IDRAG*. In detail the following subtasks are supposed to be treated:

- Get familiar with IDRAG and shortly describe its function
- Model box wing configurations with different *h*/*b* ratios ($0 < h/b < 0{,}5$), shortly describe the way of setting up the model
- Analyze their minimum induced drag and their lift distribution with regard to the ratio of the constant to the elliptical part
- Compare the results with predictions from theory.

The report has to be written in English based on German or international standards on report writing.

Declaration

Herewith I affirm that this project is entirely my own work. Where use has been made of the work of others, it has been fully acknowledged and referenced.

2012-07-13

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Datum Signature

Content

A **CD-ROM** is attached to this project report

List of Figures

List of Tables

List of Symbols

A	aspect ratio
b	wing span
e	span efficiency factor also called: Oswald (efficiency) factor
e_{wl}	span efficiency factor of a wing with winglet
c	chord
C	coefficient
$C_{D,0}$	zero lift drag coefficient
$C_{D,i}$	induced drag coefficient
C_D	drag coefficient
C_L	lift coefficient
$C_{L,md}$	lift coefficient at minimum drag condition
D	drag
$D_{i,box}$	drag of a box wing
$D_{i,wl}$	drag of a wing with winglet
h	height, vertical stagger
k	induced drag factor $k = 1/e$
k_1, k_2, k_3, k_4,	parameters in equations to calculate the induced drag factor (or span efficiency factor) of a box wing
k_{wl}	parameters in an equation to calculate the induced drag factor of a wing with winglet
L	lift
R	ratio
S_{ref}	wing reference area
$q_{l,r}$	wing loading (lift distribution) at the wing root
$q_{l,r,ellip}$	wing loading (lift distribution) at the wing root, elliptical part
$q_{l,t}$	wing loading (lift distribution) at the wing tip
$q_{l,const}$	wing loading (lift distribution) constant part
$q_{l,ellip}$	wing loading (lift distribution) elliptical part
V_{md}	speed at minimum drag condition

List of Abbreviations

CFD	Computational Fluid Dynamics
IDRAG	Program by Joel Grasmeyer used to calculate induced drag
DOC	Direct Operating Costs

1 Introduction

1.1 Motivation

Current civil aviation transport is facing challenges like increase of fuel prices, globalisation, climate change and many more. For the aviation industry to handle with these challenges, the current aircraft will have to be made less fuel consuming. The saving potentials of conventional configurations are almost exhausted. This states that there is a need for different configurations with better fuel efficiency. One of these non planar configurations is the box wing aircraft. It is most important benefits are its low induced drag.

This project serves to investigate the induced drag of the box wing. More specific, it covers the effect of variations of the height to span ratio of box wing aircraft on induced drag and the spanwise lift distribution

The results of this project are supposed to be integrated into the research project Airport2030 (**Scholz 2012**).

1.2 Objectives

The aim of this project is to investigate the effect of variations of the height to span ratio of box wing aircraft on induced drag and the spanwise lift distribution. This will be done by using IDRAG (**Grasmeyer 1997**) to calculate the span efficiency factor and wing loading.

The Airbus A320 is used as reference aircraft for this project. This means that the wing parameters of this aircraft will be used for the design of the box wing configurations with different height to span ratios.

These designs will be analysed by IDRAG and compared with results from theory. A new an d improved equation for the estimation of the induced of box wing should be established.

1.3 Structure of this Project

Chapters 2 and 3 give a theoretical background of the box wing. While Chapters 4 to 7 handle the design and analyse of the box wing configuration. In detail the distribution of the chapters is as follows:

Chapter 2 Gives a brief introduction on the box wing aircraft and non planar configurations in general.

Chapter 3 Gives more theoretical background about induced drag and it's relation to the box wing design.

Chapter 4 Describes IDRAG and its functions used for this project.

Chapter 5 Explains the modelling of the box wing configurations with different height to span ratios.

Chapter 6 Analyzes the minimum induced drag and the lift distribution with regard to the ratio of the constant to the elliptical part for the different designs.

Chapter 7 Results from IDRAG for the span efficiency factor will be compared with data and equations from literature.

Appendix A Contains the Matlab scripts.

Appendix B Contains the input files for IDRAG.

Appendix C Contains the tables with the wing loadings.

2 The Box Wing Aircraft

2.1 Overview of the Box Wing Aircraft

A box wing aircraft can be seen as a conventional aircraft with the combination of two wings that have winglets attached at their tips. The upper wing is attached to the vertical tail and has winglets that point downward. While the lower winglet points upwards such that both winglets are connected. In front view this wing configuration looks like a rectangular box. This unconventional non planar configuration is called a box wing. In Figure 2.1 a box wing design from Lockheed is shown. Lockheed did some limited research on this aircraft.

Figure 2.1 Example box wing aircraft (**Unwanted Blog 2012**)

The concept of box wing was for the first time presented by **Prandtl 1924**. Mainly he made three statements:

- A biplane has less drag than an equivalent monoplane.
- Minimum drag of a biplane is obtained when the two wings of the biplane are of the same span.
- When winglets are placed on the wingtips (making it a closed system), even a lower minimum drag can be achieved.

Figure 2.2 from **Khan 2010** gives a good overview of the wing geometry parameters of the box wing configuration.

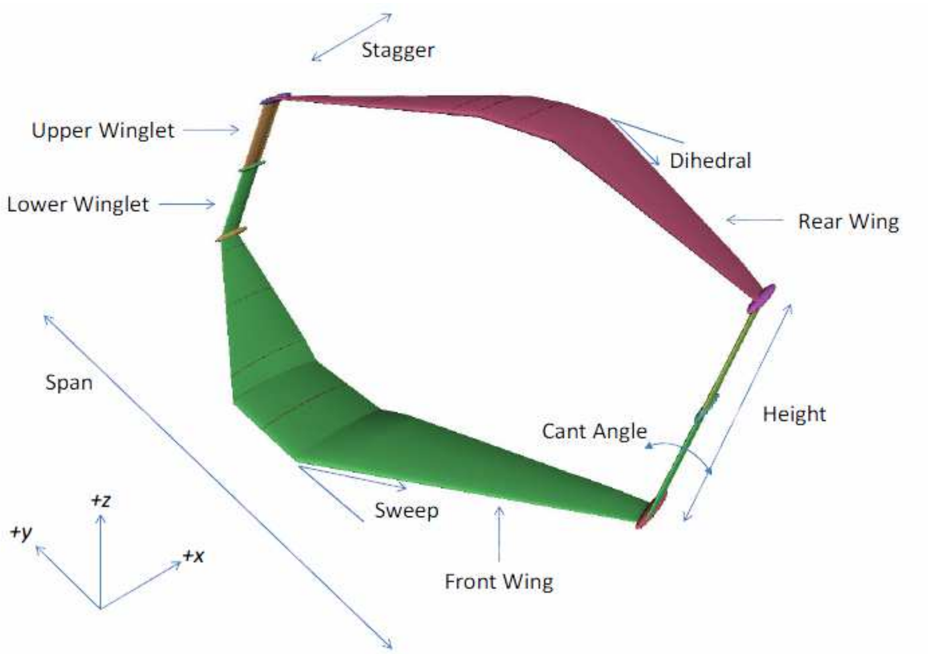

Figure 2.2 Box wing geometry outlining main design variables (**Khan 2010**).

Most of the parameters used for the box wing are the same as for the conventional configuration. Some additional parameters are added due the presence of the two wings.

Horizontal Stagger: This parameter describes the horizontal positioning of the wing relative to each other. There are two different horizontal staggers: positive and negative. It is called a positive horizontal stagger (Figure 2.3) when the upper wing is situated in front of the lower wing. In most of the box wing aircraft studies and designs the negative horizontal stagger is used. Higher *h/b* ratios can be gained and the integration of the two wings goes much easier.
Vertical stagger: This is the height of the wing configuration.

Decalage: This parameter describes the difference of the incidence angles between the two wings.

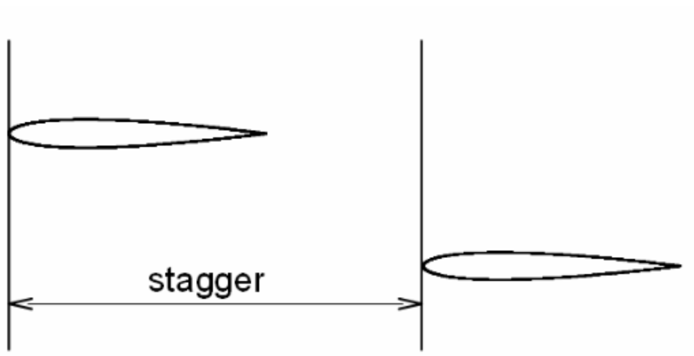

Figure 2.3 Positive stagger

Vertical Stagger: Is the vertical distance between the two wings.The more the wings are apart, the less interference between them; this results in a higher saving of induced drag and so a higher saving in fuel.

The effect of this interference can be described with the help of an interference factor. This factor is called σ. It was introduced by Prandtl in the early years of aviation. σ depends on a very important parameter. This is the *h/b*-ratio of the wing configuration. *h/b* is the vertical stagger of both wings divided by their average wing span.

Since we have two wings, the total wing area will be the sum of the wing areas from the two wings.

$$S = S_1 + S_2 \tag{2.1}$$

While the individual aspect ratios can be defined by

$$A_i = \frac{b_i^{\,2}}{S_i} \quad . \tag{2.2}$$

If both of the wings have the same span then the global aspect ratio can be calculated by

$$A = \frac{b^2}{S_1 + S_2} = \frac{b^2}{S} \quad . \tag{2.3}$$

2.2 Non Planar Concept

A box wing aircraft is a non planar aircraft. This means that the wing of the aircraft is not situated in one (x-y- plane). In Figure 2.4 different non planar configurations are shown. What you see are the front views of different wing layouts. The concept on the top left has the worst span efficiency while the box wing concept shown down on the right has the best span efficiency out of all the configurations.

Some of the layouts given in Figure 2.4 take advantage of reducing induced drag by using endplates or winglets. Other layouts take advantage by distributing one wing into multiple wings that have the same span but higher the individual aspect ratios (2.2). A layout that takes the advantage using both methods is the box wing concept.

A very important advantage of non planar configurations is that they increase the span efficiency factor without extending the wingspan. This means that the aircraft easily fits in the maximum space allowed for aircraft parking at terminals.

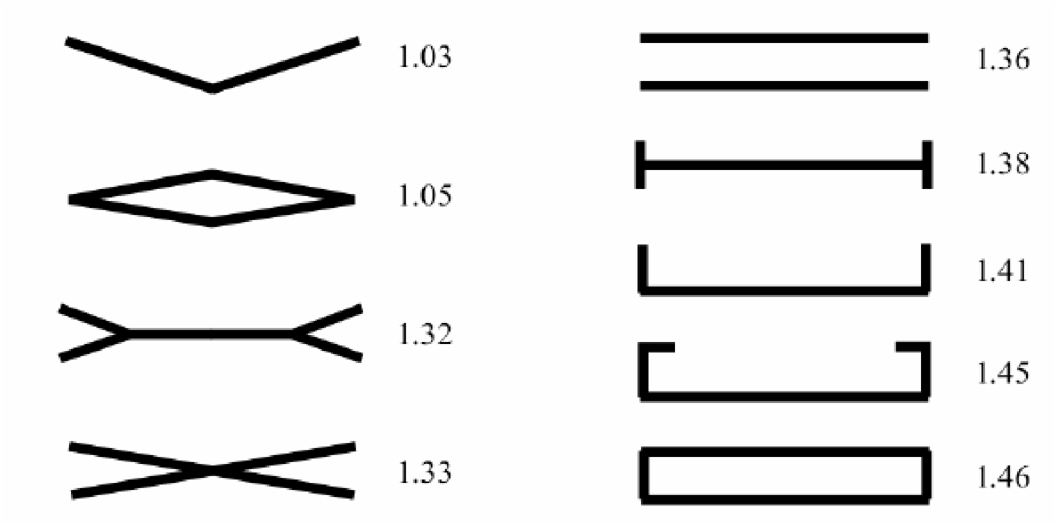

Figure 2.4 Span efficiencies for various optimally loaded non planar systems with a *h/b* ratio of 0.2 (**Kroo 2005**)

From the following equation it can be seen that an increase of the span efficiency factor causes a decrease of the induced drag.

$$C_{D,i} = \frac{C_L^{\ 2}}{\pi \ A \, e} \tag{2.4}$$

The span efficiency factor is also called Oswald's efficiency factor and the symbol used is *e*. In Figure 2.4 the span efficiency factor was obtained with a height to span ratio of 0.2. The span efficiency factor can even go higher by increasing this *h/b* ratio because the mutual interference drag reduces as the gap between the lifting surfaces increases.

2.3 Box Wing Aerodynamics

In order to have a maximum benefit, the box wing must have an equal lift distribution and the horizontal wings need to have the same total lift. Figure 2.5 gives a view of the lift distribution of a box wing aircraft. The lift on the horizontal wings consists of a constant and an elliptical part. While there is a butterfly shaped lift distribution on the vertical wings.

Demasi 2007 and **Kroo 2005** suggest adding a constant circulation loop to the overall distribution. This leads to an unequal lift distribution of both wings without the increase of induced drag. The elliptical parts will remain equal while there will be a difference in the constant part of both wings (Figure 2.6). This unequal lift distribution leads to a shift of the zero crossing of the winglet loading and a heavy loading of the wing. This heavier loading of the wing will generate more lift.

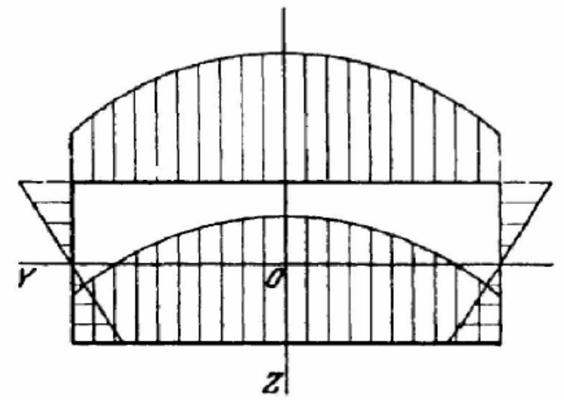

Figure 2.5 Spanwise lift distribution of a box wing aircraft (**Durand 1935**)

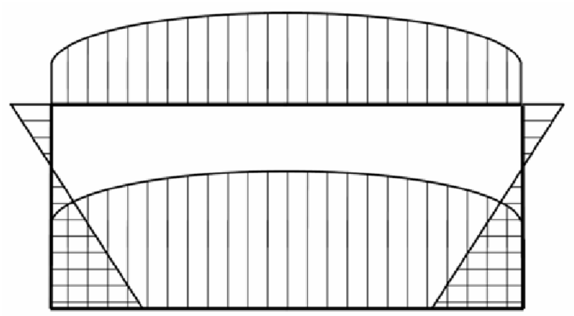

Figure 2.6 Unequal spanwise lift distribution of a box wing aircraft (**Schiktanz 2011b**)

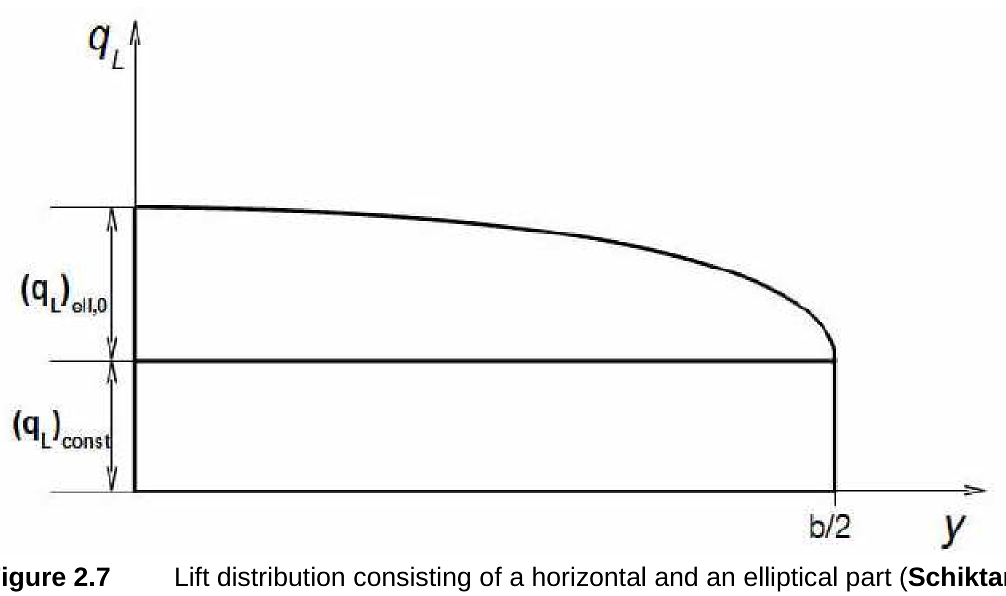

Figure 2.7 Lift distribution consisting of a horizontal and an elliptical part (**Schiktanz 2011a**).

Figure 2.7 gives a good representation of the constant and the elliptical part of the lift distribution. The elliptical part is the highest at the wing root and goes to zero when we move to the wing tip.

Schiktanz 2011a introduced an equation for the ratio between the constant part and the elliptical part of the lift distribution.

$$R_l = \frac{q_{l,const}}{q_{l,ellip}} = \frac{q_{l,t}}{q_{l,r} - q_{l,t}} \tag{2.5}$$

This ratio is important for knowing the exact wing loading and seems to depend on the *h/b* ratio. This is one topic of this project and shall be further investigated using IDRAG in Chapters 4, 5, 6 and 7.

2.4 Main Advantages of the Box Wing Aircraft

The greatest advantage of a box wing aircraft is that there is a reduction of induced drag. This leads to less fuel consumption which has a positive influence on the Direct Operating Costs (DOC). An other desirable effect is the reduced vortex wake hazard. Less noise production will be achieved by mounting engines on the back. Additional use of modern high bypass or fan turbines will decrease noise production even more. The box wing design has a potential to be used for smaller as well as for larger aircraft in the future.

The disadvantages are mainly heavier wing structure, stability and control issues more critical and the fact that not enough research has been done jet on this concept. Hence the development costs and development risk is higher. As one example there is a lack of current knowledge about the exact induced drag reduction. Although there has been research ongoing on non planar configuration since 1924, we are far from knowing everything about it jet. In order to build this aircraft, new design procedures will be required for this type of configuration. Another disadvantage is that the flutter speeds are expected to be lower than for conventional configurations.

Although a lot of research is still necessary on this unconventional non planar aircraft concept, box wing configurations do have a potential to lead the aviation industry in the future.

3 Drag

3.1 Drag Fundamentals

When an aircraft moves through the air, there is an aerodynamic force that opposes the aircrafts motion trough the air. This force is called drag. Since this force is in the opposite direction of the thrust it is obvious that the drag should be as low as possible. It has a direct influence on the fuel usage and so on the operating costs and overall weight of the aircraft. It’s obvious that drag should be reduced as much as possible. The four main forces of flight are shown in Figure 3.1.

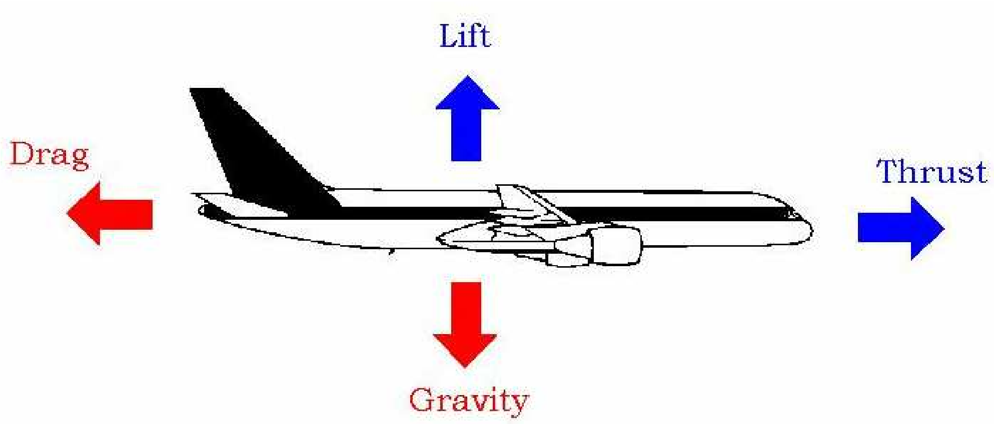

Figure 3.1 The four main forces of flight (**ThinkQuest 2012**)

Drag can be written in a coefficient form

$$C_D = C_{D,0} + C_{D,i} \quad . \tag{3.1}$$

It can be seen that there are two types of drag: Parasite drag and induced drag. One of the causes for parasite drag is the friction from the air on the aircraft skin. Parasite drag originates especially from the landing gear when this is extended during landing and take-off. Induced drag occurs due the development of lift. It can be seen that parasite and induced drag depend on airspeed (Figure 3.2). This means that there is a speed at which the lowest drag occurs. This speed is called the minimum drag speed V_{md} . The corresponding lift coefficient is the lift coefficient at minimum drag $C_{L,md}$.

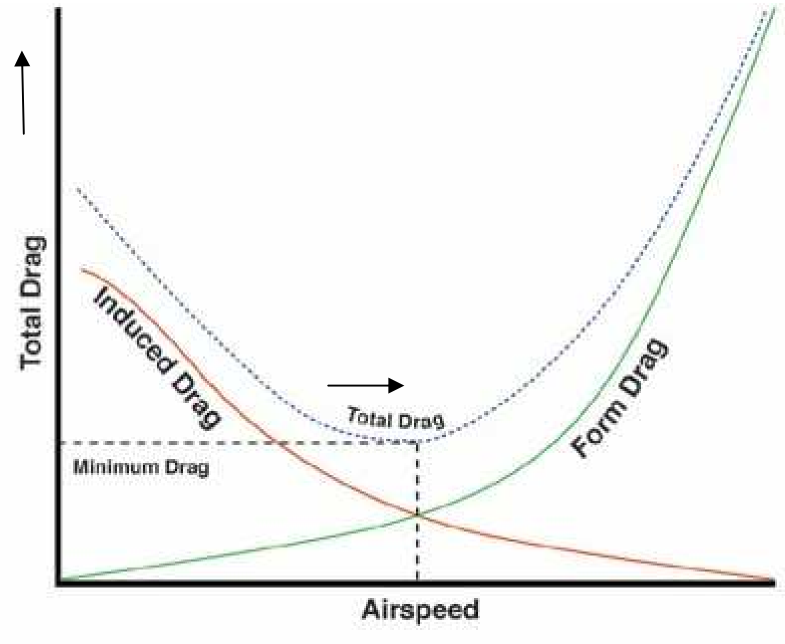

Figure 3.2 Drag in relation to airspeed (**Science Fair 2012**)

3.2 Induced Drag

The pressure differential created by the airfoil produces lift. Due this pressure difference the flow at the wing tip will curl around and causes a circular flow around the wingtip. Those circular flows result in wingtip vortices. Those vortices have a downward component called downwash. This downwash reduces the effective angle of attack which results in a local inclination of the lift vector relative to the incoming velocity vector. This produces induced drag. The vortices are one of the causes for induced drag.

Repeating (2.4), the induced drag coefficient can be written as

$$C_{D,i} = \frac{{C_L}^2}{\pi \, A \, e} \quad . \tag{3.2}$$

Here the span efficiency factor e is the crucial parameter; it should be as high as possible. It is also important to understand that the induced drag will increase rapidly as the lift coefficient increases. Think about the flight phases where the lift coefficient is increased, landing and takeoff, while the lift coefficient is optimum for low drag in cruise with $C_{L,md}$. With a reduction of induced drag an aircraft will benefit in all flight phases and not only during cruise.

From the (3.2) it can be seen that induced drag can be reduced either by increasing the span efficiency factor or the aspect ratio. In a conventional layout of an aircraft a certain span efficiency factor can be reached. The aspect ration can be increased causing an increase in wing span with the reference area kept constant. Due this increase in span the vortex strength along the wing tips will be decreased and so the induced drag will be decreased.

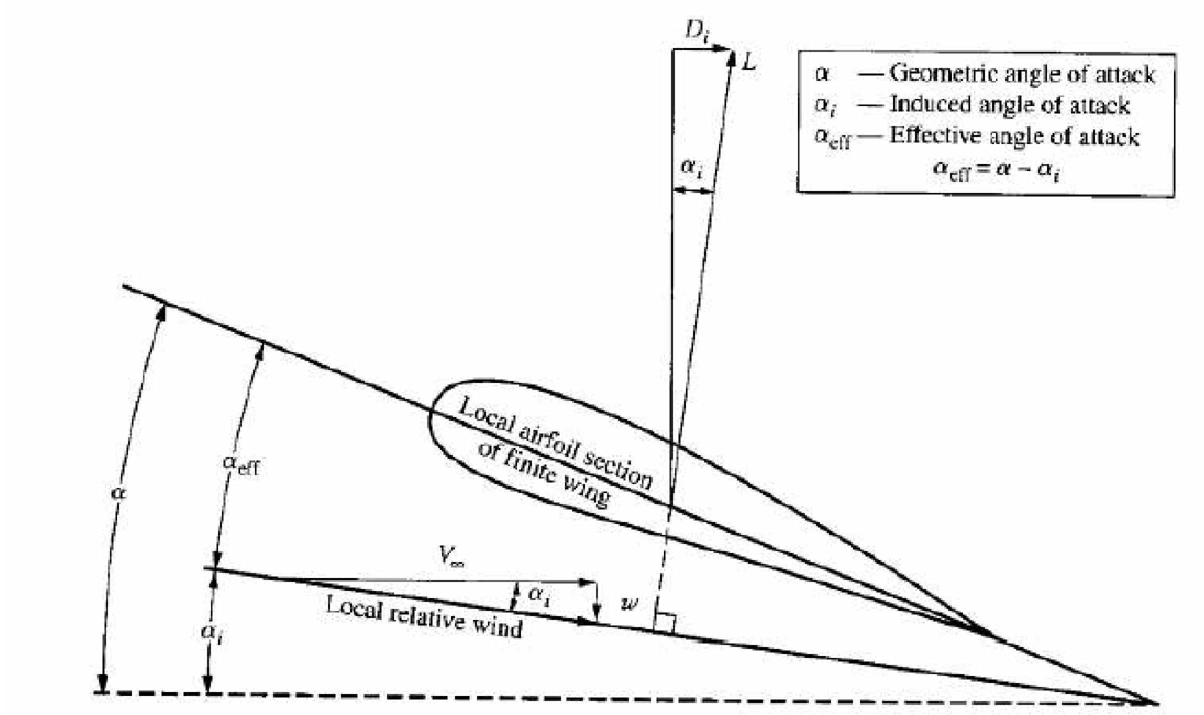

Figure 3.3 Effect of downwash. Flow over a local airflow section of a finite wing (**Anderson 2007**)

However an increase of the wing span will bring a few complications along with it. Wings with large span ...

- ... can cause aeroelastic problems.
- ... are structurally not the best option. The centre of lift will move more towards the tip. This results in the need of stronger and heavier construction.
- ... may face airport terminal restrictions. The aircraft has to fit in a box of a certain size depending on aircraft category. The largest of these categories limits aircraft size to the famous box 80 m x 80 m x 80 ft (b x l x h).

One possible solution for those complications could be the box wing aircraft because the span efficiency factor is increased without increasing wing span. With the same span the aircraft will have the same nominal aspect ration but a higher aspect ratio of the individual wings.

An additional advantage of the box wing aircraft is that the vortices caused by the horizontal wings and the vortices caused by the vertical wings counteract each other. This becomes clear considering Figure 3.4.

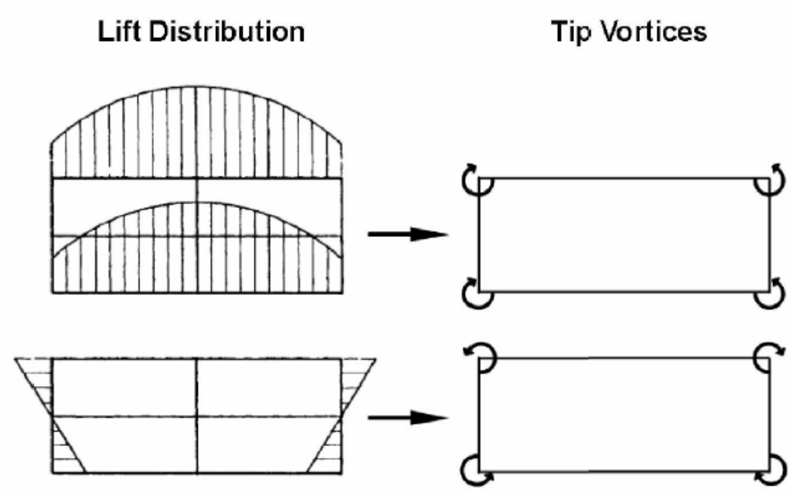

Figure 3.4 Counteracting wing tip vortices (**Schiktanz 2011a**)

3.3 Span Efficiency Factor

There are several methods to calculate the span efficiency factor of a box wing aircraft. In this Subchapter three of them will be discussed and parameters of a fourth method presented. The symbol *k* will be used in the following equations. It stands for the ratio between the induced drag of the box wing and the induced drag of the reference wing. It is also refered to as the induced drag factor.

$$k = \frac{D_{i,box}}{D_{i,ref}} \tag{3.3}$$

The span efficiency factor *e* and *k* are related to each other.

$$e = \frac{1}{k} \tag{3.4}$$

A first indication of the magnitude of k can be found in **Schiktanz 2012**. He shows that splitting a reference wing in two equal wings with same span and placing both wings far from each other yields a value of $k = 0.5$ or $e = 2$.

Prandtl proposed a method to calculate *k* for a box wing in **Prandtl 1924**. His equation is the oldest but seems to be a little optimistic. Following his theory the span efficiency factor will be determined from

$$k = \frac{1 + 0.45 \cdot h/b}{1.04 + 2.81 \cdot h/b} \quad . \tag{3.5}$$

Using this method there is a limit of 0.16 for *k* with *h/b* going to infinity. This limit is far below the value we obtained splitting a reference wing in two equal wings yielding $k = 0.5$. However Prandtl limits the useful range of his equation to 1/15 < *h/b* < 1/2 or 0.0667 < *h/b* < 0.5. This then leads to a minimum value of $k = 0.501$.

For $h/b = 0$ $k = 0.9615$ and not 1.0 as would be expected. However considering the limits Prandtl gave to his equation the inconsistency is removed.

Rizzo 2007 uses CFD analysis to derive the equation for the span efficiency factor. The equation he uses is

$$k = \frac{0.44 + 0.9594 \cdot h/b}{0.44 + 2.219 \cdot h/b} \quad . \tag{3.6}$$

Using this method there is a limit of 0.4324 for *k* with *h/b* going to infinity. This limit is of similar magnitude compared to the value we obtained splitting a reference wing in two equal wings with $k = 0.5$. For $h/b = 0$ $k = 1.0$ as would be expected.

Prandtl 1924 uses the following equation for <u>biplanes</u>

$$k = 0.5 + \frac{1 - 0.66 \cdot h/b}{2.1 + 7.4 \cdot h/b} \quad . \tag{3.7}$$

Using this method there is a limit of 0.4108 for *k* with *h/b* going to infinity. For $h/b = 0$ $k = 0.9762$ and not 1.0 as would be expected.

Figure 3.5 and Table 3.1 summarise the results of the presented methods. **Rizzo 2007** and **Prandtl 1924** (biplane) are giving the best results. Note that all the equations (3.5), (3.6) and (3.7) assume an optimal span loading for minimum induced drag. Only (3.6) shows $k = 1$ for $h/b = 0$ as it is expected.

Further information on induced drag calculation can also be obtained from **DeYoung 1980**. Tabulated data for rectangular box wing (Table 3.1) are very close to results from (3.5). Data from Table 3.1 is plotted in Figure 7.1.

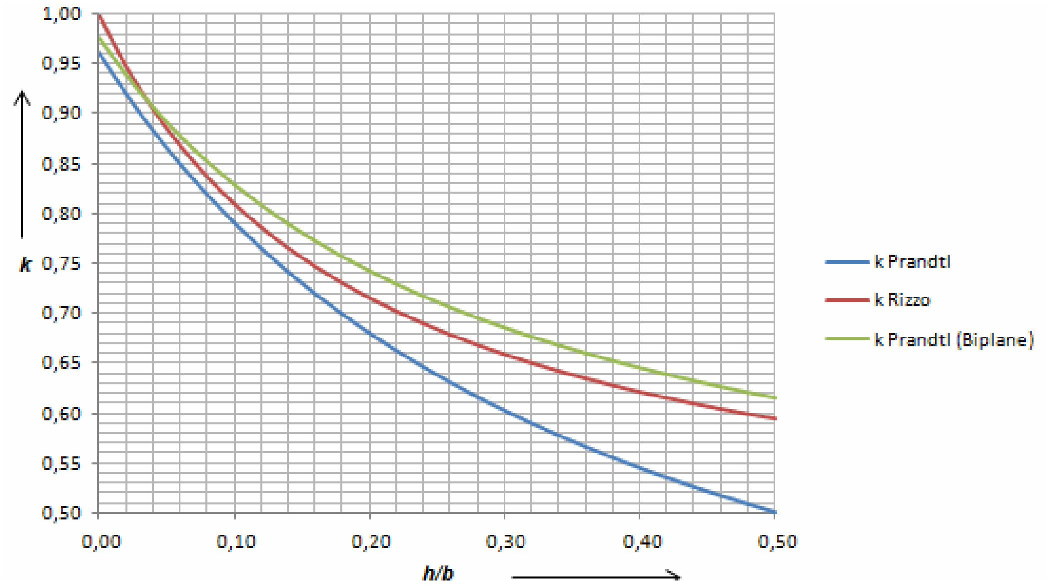

Figure 3.5 Induced drag factor *k* of a box wing configuration

Table 3.1 Induced drag factor *k* of a Box Wing (**DeYoung 1980**)

h/b	***k* DeYoung**
0,05	0,8682
0,15	0,7282
0,25	0,6385
0,35	0,5747
0,45	0,5229
0,70	0,4327
1,00	0,3589

4 IDRAG

4.1 Introduction

IDRAG (Figure 4.1) is a program written at the Virginia Polytechnic Institute and State University by **Grasmeyer 1997**. The program was written to calculate the induced drag of nonplanar wings composed of multiple panels. This program is ideal to do some basic induced drag calculations on the box wing. It has design and analysis capabilities, which means that it can either calculate the span loading required to obtain the minimum induced drag or the minimum induced drag when the span loading is given. IDRAG also calculates the span efficiency factor *e*. A typical IDRAG input file contains different reference values along with information regarding the geometry of the surfaces to be analysed.

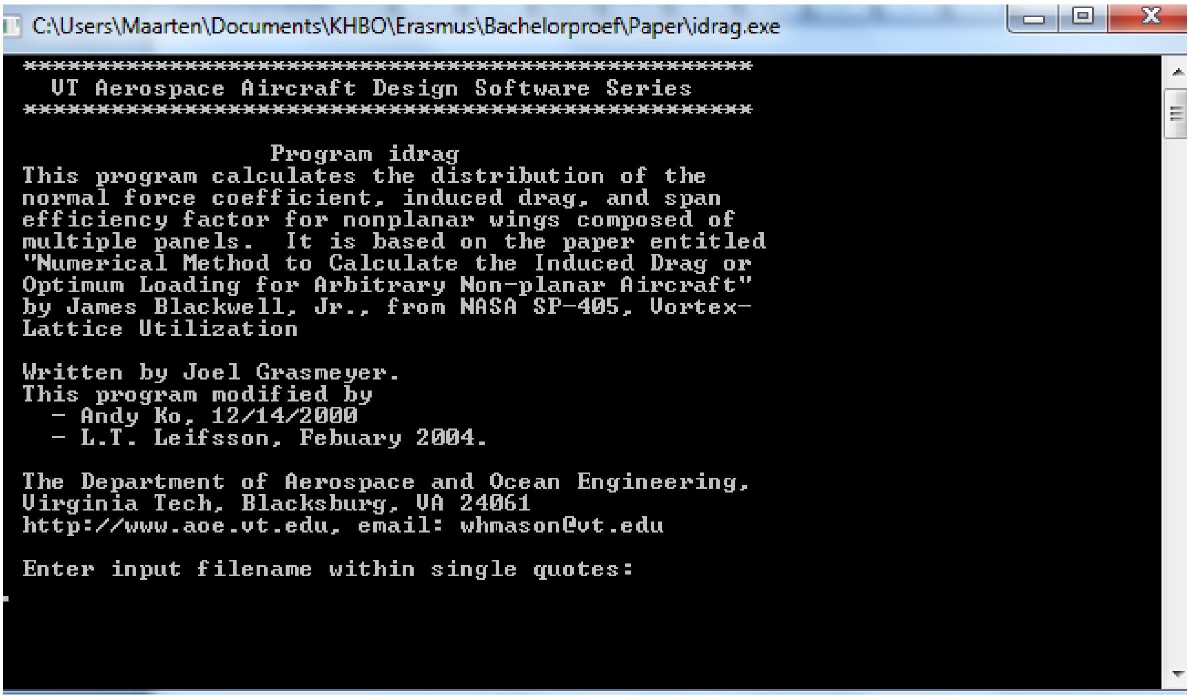

Figure 4.1 IDRAG. Initial screen

4.2 Discrete Vortex Algorithm

In the discrete vortex method is a two coordinate system used. This can be seen on Figure 4.2. The first coordinate system is the traditional aerodynamic reference frame. The x-axis points to the rear of the airplane, the y-axis points starboard and the z-axis is pointing up. Each vortex element has its own local reference frame. This is the second coordinate system. The discrete vortex method is based on the Kutta-Joukowsi theorem. For more information about the theoretical development refer to the IDRAG manual written by **Grasmeyer 1997**.

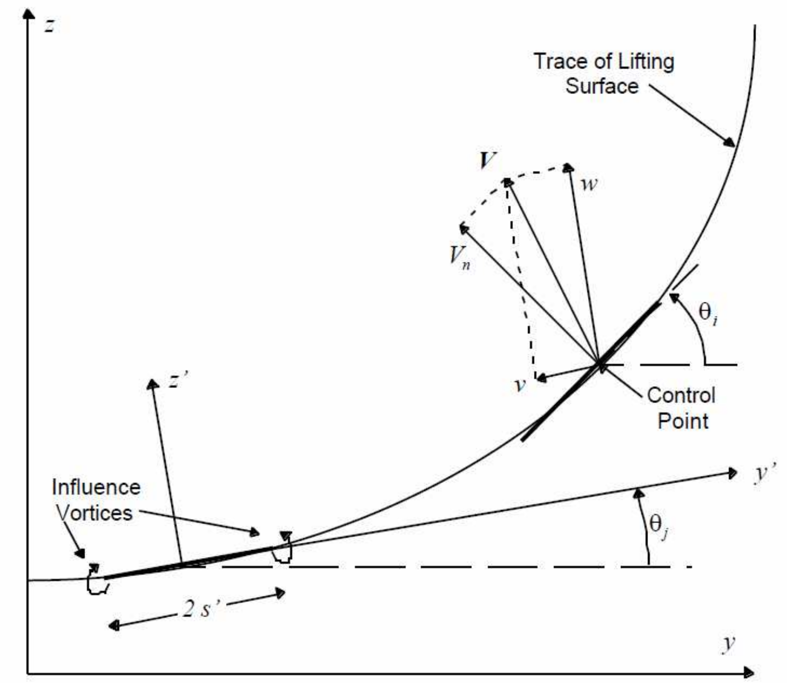

Figure 4.2 Coordinate systems for the discrete vortex method (**Blackwell 1976**)

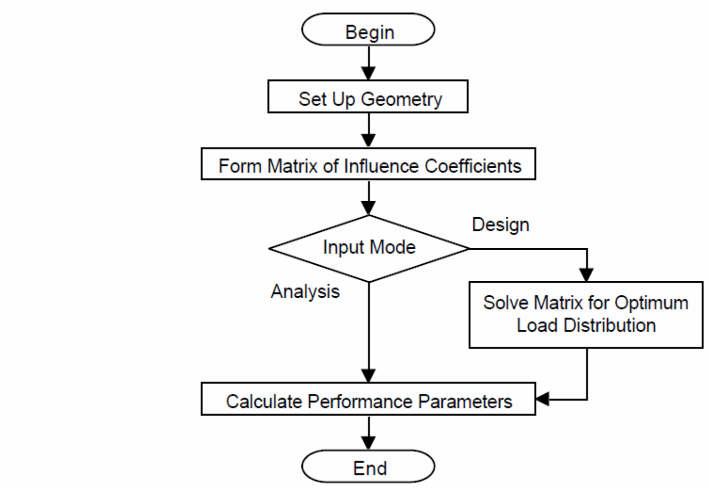

Figure 4.3 Flowchart of the IDRAG code

Figure 4.3 shows a flowchart of the IDRAG code. It begins with setting up the geometry. This phase includes the calculations of the dihedral angles, local chords, semi-widths of the lifting elements and the coordinates of the vortex control points. Two execution points are available: Analysis and design.

For the **analyse mode** the code will take the geometry and load distribution as inputs. Out of those data it calculates the performance parameters.

For the **design mode** the geometry and design conditions are taken as inputs. Out of those data it calculates the load distribution for the minimum induced drag. As output it gives the performance parameters.

4.3 Design Mode

A sample design input file is given in Figure 4.4.

```
idrag input file
winglet
0               input mode
1               write flag
1               symmetry flag
1.0             cl_design
1               cm_flag
0               cm_design
0.03            x cg position
0.25            center of pressure for airfoil sections
0.2             reference area
0.2             reference chord
3               number of panels
0   0   0       x,y,z for 4 corners of panel 1
0   0.5 0
0.2 0.5 0
0.2 0   0
10              number of vortices for panel 1
0               vortex spacing for panel 1
0   0.5 0       x,y,z for 4 corners of panel 2
0   0.5 0.1
0.2 0.5 0.1
0.2 0.5 0
5               number of vortices for panel 2
0               vortex spacing for panel 2
1   0   0.1     x,y,z for 4 corners of panel 3
1   0.2 0.1
1.1 0.2 0.1
1.1 0   0.1
6               number of vortices for panel 3
0               vortex spacing for panel 3
```

Figure 4.4 Sample design input file

The code is identified by the **first line** of the input file. In other words, it tells to which code the input file belongs to.

The **second line** is for the user to assign a title to the configuration.

The **input mode flag** tells the code whether it is about a design or analysis format.

The creation of the output file is controlled by the **write flag**. 1 means that an output file will be written while a 0 means that no output file will be written.

The **symmetry flag** has the function is to define if it is about a symmetric configuration or not: If the value is 1 than the code will mirror all the surfaces with respect to the xy-plane.

The **design lift coefficient** is the lift coefficient for the entire configuration.

The location of the centre of gravity along the x-axis is defined by the parameter **x cg position**.

The **center of pressure** parameter specifies the chordwise location of the centre of pressure for all the sections of the airfoil.

The possibility to input **reference area** and **reference chord** values give the user complete control over the values used to normalise the lift and moment into lift and moment coefficients.

Number of panels are specified next.

Than the **coordinates for the four corners for each panel** must be defined by using x, y and z coordinates.

As last point the **number of vortices** and the **vortex spacing** must be defined. For this project an even spacing shall be used.

4.4 Analysis Mode

A sample analysis output file is given in Figure 4.5. For the analyse input, the user must specify either the load distribution or the normal force coefficient distribution. This distribution is specified by points along the panel. The code will linearly interpolate between those points to determine the distribution. The accuracy of this distribution is defined by the number of vortices specified in the design mode.

The utility codes created can be imported to Matlab. It can create plots of the aircraft configuration. Matlab shall be used in this project to analyse the output files from IDRAG.

```
idrag output file
asample2
    1    = input mode
    1    = write flag
    1    = symmetry flag
    0.00 = design lift coefficient
    0    = moment coefficient flag
    0.00 = design moment coefficient
    0.00 = x cg position
    0.25 = center of pressure for airfoil sections
    0.15 = reference area
    0.15 = reference chord
    1    = number of panels

    x        y        z    for panel 1
    0.00     0.00     0.00
    0.00     0.50     0.00
    0.20     0.50     0.00
    0.20     0.00     0.00
   10    = number of vortices
    0    = vortex spacing flag

    i   x        y        z        load     cn
    1   0.0500   0.0250   0.0000   0.9975   0.7481
    2   0.0500   0.0750   0.0000   0.9874   0.7405
    3   0.0500   0.1250   0.0000   0.9668   0.7251
    4   0.0500   0.1750   0.0000   0.9352   0.7014
    5   0.0500   0.2250   0.0000   0.8913   0.6684
    6   0.0500   0.2750   0.0000   0.8330   0.6248
    7   0.0500   0.3250   0.0000   0.7571   0.5678
    8   0.0500   0.3750   0.0000   0.6571   0.4928
    9   0.0500   0.4250   0.0000   0.5179   0.3885
   10   0.0500   0.4750   0.0000   0.2180   0.1635

    0.77612 = actual lift coefficient
    -.25871 = actual moment coefficient
    0.02847 = induced drag coefficient
    1.01005 = span efficiency factor
```

Figure 4.5 Sample analysis output file

5 Modelling the Box Wing Configuration

5.1 Input

The model can be designed using a simple input file. In this project we are only looking for the effect of variations in the *h/b* ratio. So we will design a box wing without sweep and stagger. In addition, the wings will be designed with a constant chord length. This is done to be able to focus only on the different *h/b* ratios.

As stated in the beginning of this project, the A320 is used as the reference aircraft. So we will use the same wing reference area and wingspan:

S_{ref} = 122.4 m^2
b = 34 m

The chord can easily be calculated. First divide the wing area by two to obtain the wing area of one pair of wings from the box wing. Then you divide this number by the wingspan to find the chord

c = 1.8 m

Those dimensions shall be used for the design file. Looking at the input file (Figure 4.4) is apparent that more parameters have to be defined. For this project the parameters will stay constant except for the coordinates of the surfaces. This is obvious since we only want to change the *h/b* ratio of the box wing.

After defining the parameters, the amount of panels is asked and then the corners of those panels have to be defined. Since we use the symmetry flag in the '1' position we only have to define one side of the box wing. This means that we have to define three panels.

One last thing that has to be defined is the amount of vortices on each panel. The more vortices are used for the calculations the more accurate the results will be. The vortices will be spaced even. This is defined with a 1.

Figure 5.1 is the design file given for a box wing with a *h/b* ratio of 0.25.

```
idrag input file
BoxWingValidation
0                   input mode
1                   write flag
1                   symmetry flag
0.67                   cl_design
0                   cm_flag
0                   cm_design
6.71                   x cg position
0.25                   center of pressure for airfoil sections
122.4                   reference area
3.6                   reference chord
3                   number of panels
0    0    0     x,y,z for 4 corners of panel 1
0    17    0
1.8    17    0
1.8    0    0
30                   number of vortices for panel 1
0                   vortex spacing for panel 1
0    0    4.25    x,y,z for 4 corners of panel 2
0    17    4.25
1.8    17    4.25
1.8    0    4.25
30                   number of vortices for panel 2
0                   vortex spacing for panel 2
0    17    0     x,y,z for 4 corners of panel 3
0    17    4.25
1.8    17    4.25
1.8    17    0
15                   number of vortices for panel 3
0                   vortex spacing for panel 3
```

Figure 5.1 Input file for a box wing with a *h/b* ratio of 0.25

5.2 Output

In the next Chapter this input file will be imported in IDRAG and the results will be analysed using Matlab and Excel. Plotting the box wing geometry can be done by using the IDRAG output file and two Matlab scripts. One is able to read IDRAG (Appendix A.1). The second one is able to plot the wing (Appendix A.2).

When using Matlab it is important to set the current folder to the folder with the script and the output files from IDRAG. Type the command geom('test.idrag') in the command window and press enter. Now Matlab will generate a geometric view of the design. This is given in Figure 4.7.

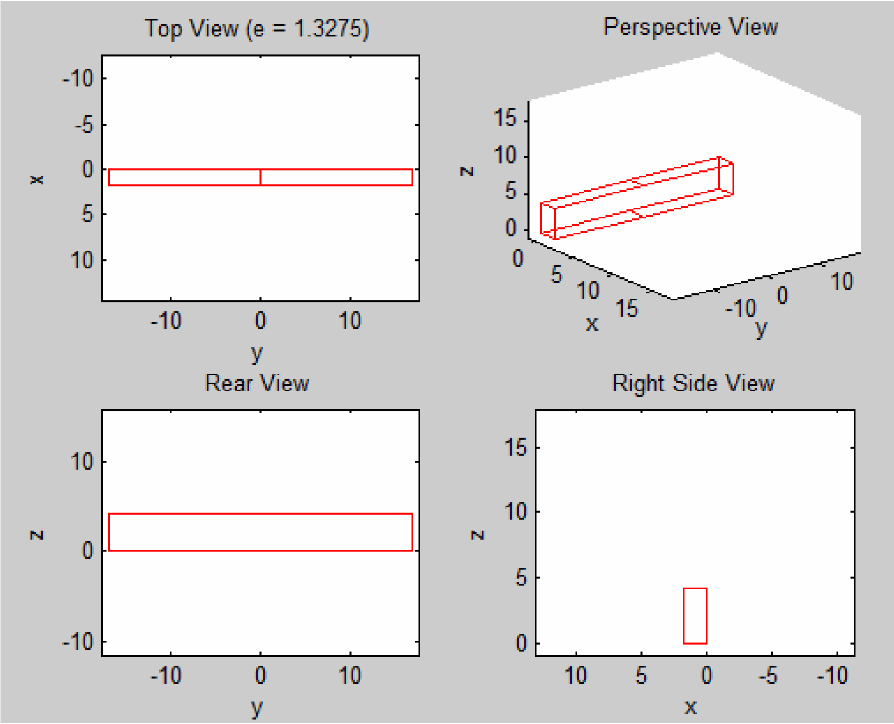

Figure 5.2 Geometric plot of the box wing design

To investigate the effect for different *h/b* ratios, configurations with ratios between 0 and 1.0 are designed. The following *h/b* ratios will are used:

h/b = 0.05
h/b = 0.15
h/b = 0.25
h/b = 0.35
h/b = 0.45
h/b = 0.70
h/b = 1.00

Their IDRAG input files can be found in Appendix B.

6 Analysis

In this chapter the designs will be analysed using IDRAG.

First you enter the input filename. In this example for the configuration with a *h/b* of 0,25 the input is hb25.in. After this, IDRAG will ask for an output filename. For this configuration the name hb25out.idrag is used. After pressing enter, the IDRAG window will close and a new text file is generated. The output files are too long to be shown here or in the Appendix. They can be found on the CD.

The load distribution can be plotted with Matlab. For this you need the IDRAG output file and the script for Matlab to plot the loads (Appendix A.3). The command that has to be typed into the command window is loads('hb25out.idrag'). Matlab will generate the load distribution graph. This graph is given in Figure 6.1.

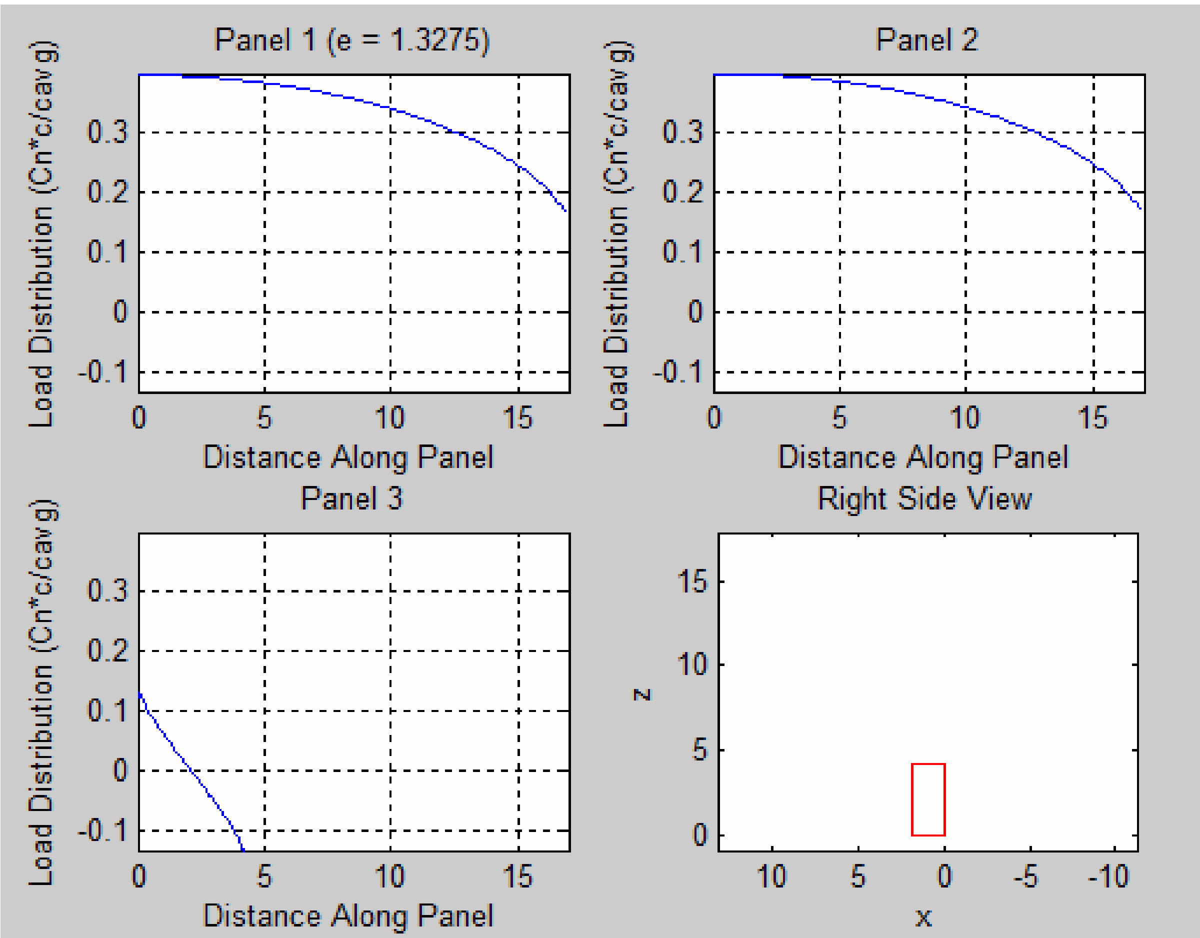

Figure 6.1 Load distribution plotted with Matlab

The graph is not precise enough for a good perception. This is why Excel graphs will be used further in this project. Note that the load distribution is given for each panel. Panel 1 and panel 2 are the main wings and have exactly the same load distribution. Panel 3 is the winglet.

6.1 Accuracy of the Span Efficiency Factor

In this subchapter the span efficiency factor *e* will be calculated by IDRAG for different amount of vortices. The example with a *h/b* ratio of 0,25 shall be used again. The span efficiency will be plotted for input files with 25, 50, 75, 100, 125, 150, 200, 250, 300, 350 and 400 vortices. The results are found in table 6.1 and plotted in figure 6.2.

Table 6.1 Span efficiency factor *e* for different number of vortices

Number of Vortices	*e*
25	1,35574
50	1,34144
75	1,33651
100	1,33394
125	1,33236
150	1,33127
200	1,32984
250	1,32896
300	1,32835
350	1,32784
400	1,32751

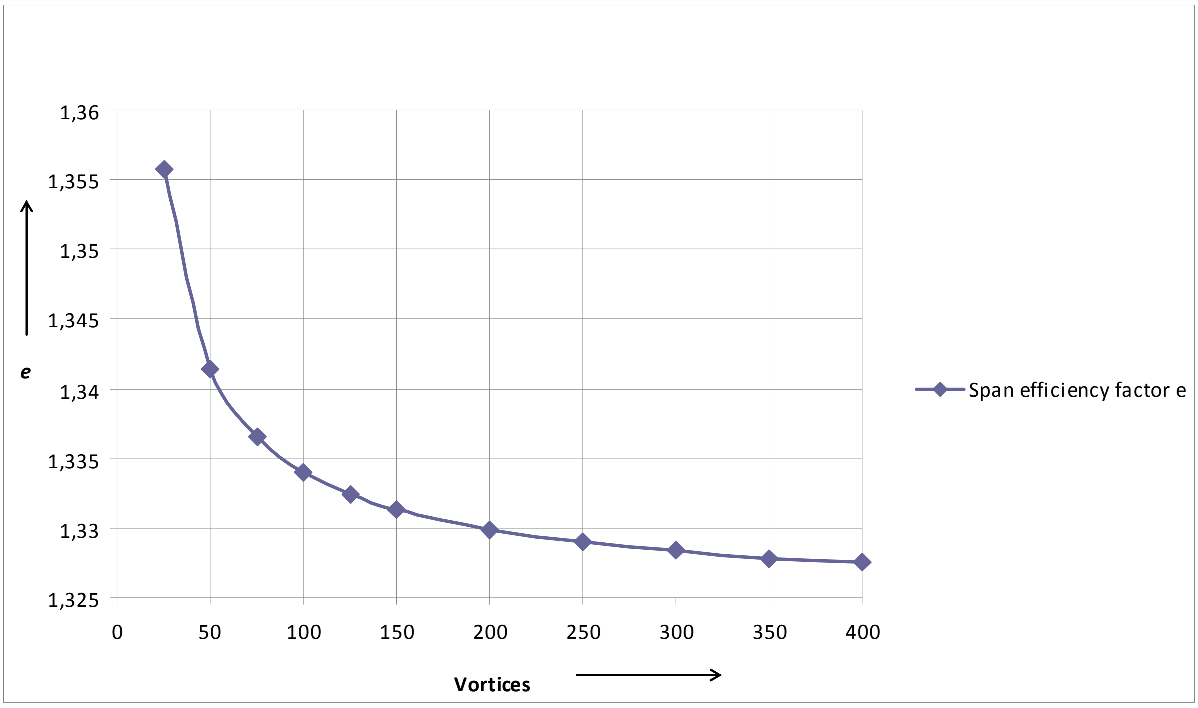

Figure 6.2 Span efficiency factor *e* for different number of vortices

Table 6.1 and Figure 6.2 make clear that the accuracy of the calculations done by IDRAG increases with a higher amount vortices. We can conclude that the span efficiency factor for the box wing configuration with a *h/b* of 0.25 is 1.327. This accuracy is reached by using 400 vortices. This amount of vortices shall be used for all the calculations.

6.2 Span Efficiency Factor for Different *h/b* Ratios

In this subchapter a graph is plotted with the span efficiency factor of the different *h/b* ratios. After plotting it out conclusions will be made about the *h/b* ratio with the best span efficiency factor.

Table 6.2 Span efficiency factor *e* for different *h/b* ratios

h/b	***e***
0.05	1.094
0.15	1.220
0.25	1.328
0.35	1.427
0.45	1.521
0.70	1.744
1.00	2.004

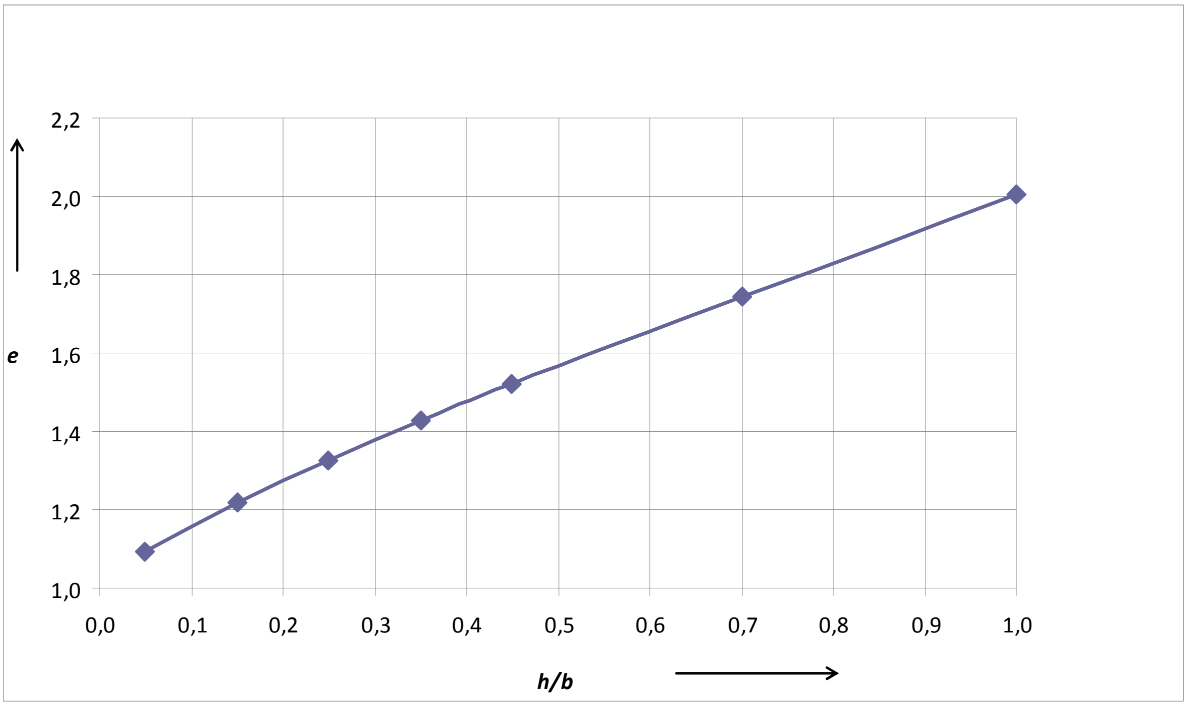

Figure 6.3 Span efficiency factor *e* for different *h/b* ratios

Clearly there is an increase of the span efficiency factor when the *h/b* ratio is chosen higher. We find a maximum *e* of about 2.0 for a *h/b* ratio of 1.0.

6.3 Wing Loading for Different *h/b* Ratios

In this subchapter the wing loadings for the different *h/b* ratios will be analysed. Two figures and two tables are shown. Figure 6.4 shows the wing loading of one wing of the box wing. It consists of a constant part and an elliptical part. The elliptical part of that wing loading is given in Figure 6.5. The constant part is given in Table 6.3. Numerical values for Figure 6.4 and Figure 6.5 are too long to be shown in this subchapter and can be found in Appendix C. A summary of the findings are Table 6.3 and Figure 6.6.

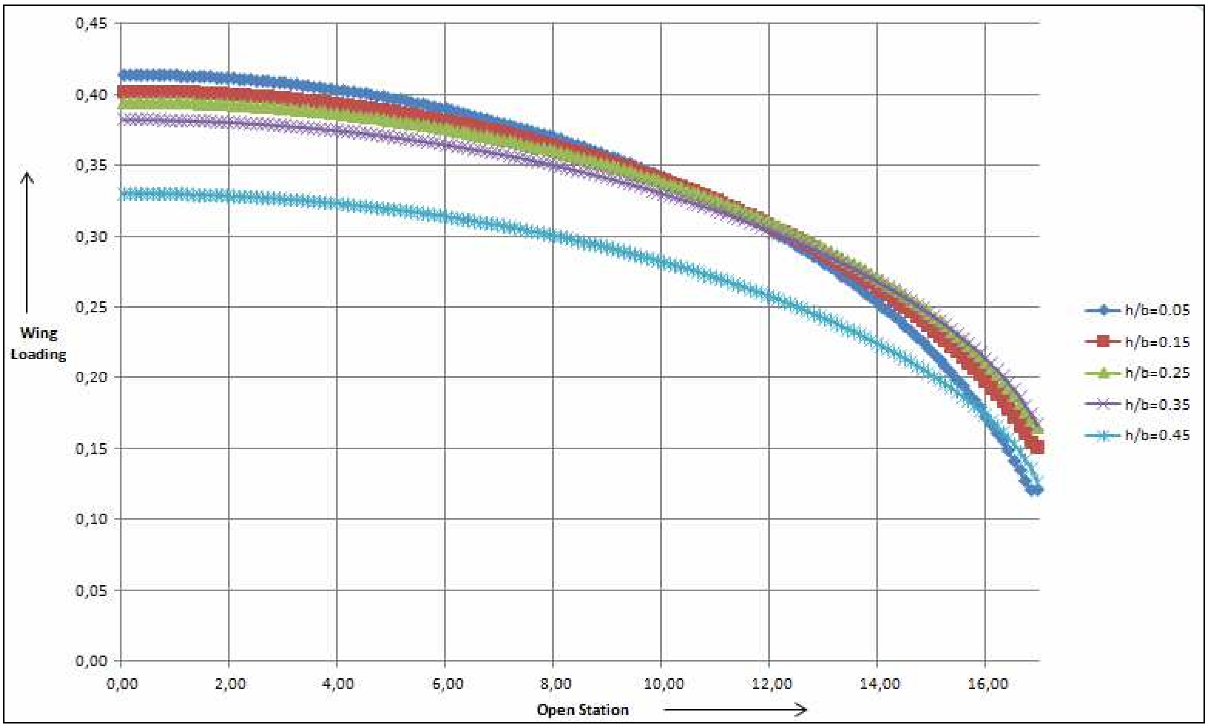

Figure 6.4 Wing loading of a reference wing with half span of 17 m

Now in more detail: Figure 6.4 shows the wing loading for just one side of the wing as a function of the wing station. The wing station is given in units of meters. The reference aircraft is the Airbus A320 with a span of 34 m and a half span of 17 m. The maximum load occurs at the wing root while the minimum load occurs at the wingtip. The total wing loading consists of a constant part and an elliptical part (Figure 2.7). A better view of just the elliptical part is given in Figure 6.5.

Table 6.3 show the summary of the wing loading results. Listed are

- Wing loading at wing root, $q_{l,r}$
- Wing loading at wing root, elliptical part, $q_{l,r,ellip}$ with $q_{l,r,ellip} = q_{l,r} - q_{l,t}$
- Wing loading at wing tip, $q_{l,t}$

Figure 6.6 is a graphical representation of Table 6.3.

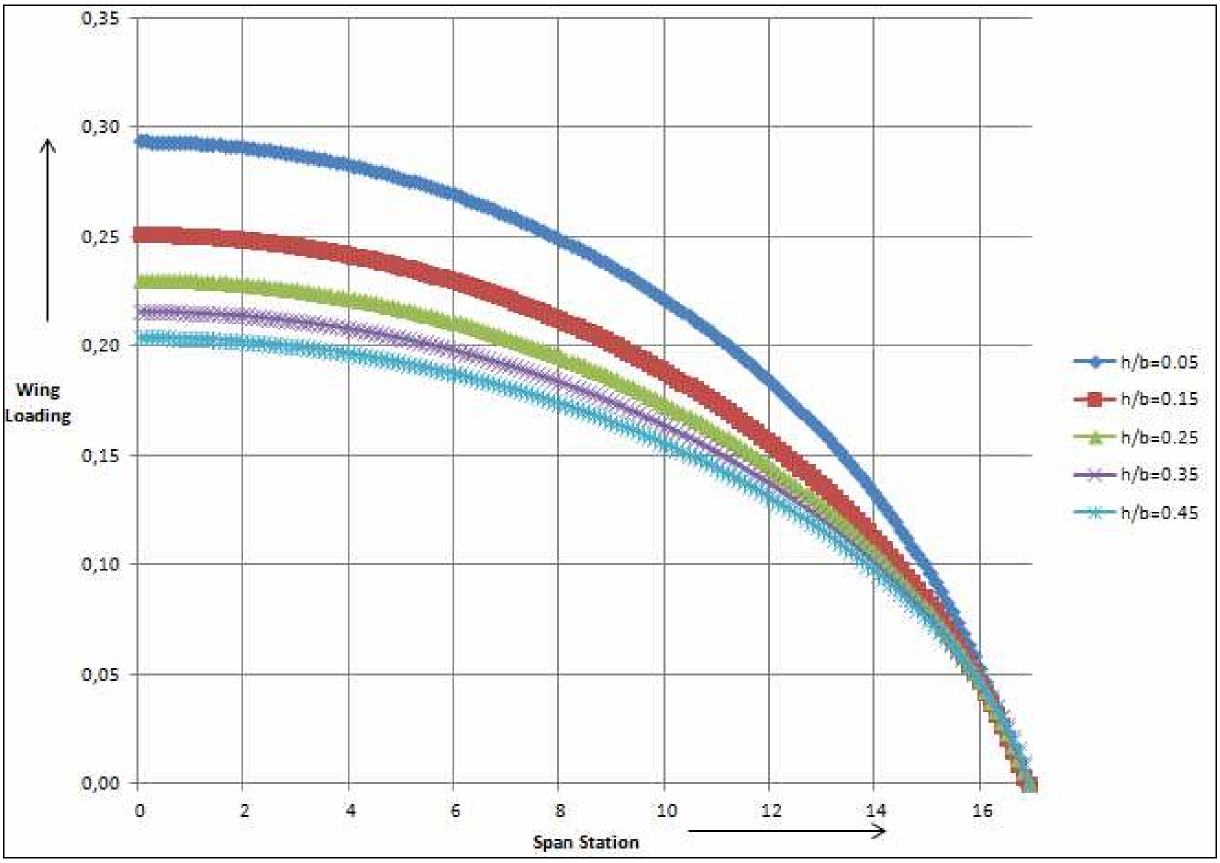

Figure 6.5 Elliptical part of the wing loading from Figure 6.4

Table 6.3 Wing loading at wing root and wing tip for different *h/b* ratios

h/b	**Wing Loading at Wing Root** $q_{l,r}$	**Wing Loading, Elliptical Part at Wing Root** $q_{l,r,ellip}$	**Wing Loading, at Wing Tip (Constant Part)** $q_{l,t}$
0,05	0,4142	0,2938	0,1204
0,15	0,4026	0,2512	0,1514
0,25	0,3950	0,2304	0,1646
0,35	0,3825	0,2158	0,1667
0,45	0,3301	0,2038	0,1263

It can be concluded (Figure 6.6) that the **wing loading at the wing root** decreases with increasing *h/b* ratio. The winglet (end plate) at the wing tip takes up wing loading with increasing *h/b* ratio, so that there is less wing loading left at the root.

It can be further concluded (Figure 6.6) that the **wing loading at the wing tip** (constant part) increases with *h/b* ratio up to *h/b* = 0.26. For larger *h/b* ratio than 0.26 the wing loading at the tip decreases. This follows from calculating the maximum of the polynomial describing the wing loading at the wing tip. The increase of wing loading with increasing *h/b* ratio can easily be interpreted with the end-plate-effect of the vertical wing. Less clear is why the constant part finally decreases.

The **elliptical part of the wing loading at the wing root** just follows $q_{l,r,ellip} = q_{l,r} - q_{l,t}$. $q_{l,r,ellip}$ is also the difference between wing loading between tip and root. In case we would be interested in a "as constant as possible" lift distribution we would find this (from Figure 6.6) at a h/b = 0.47 with a difference between wing loading between tip and root of 0.205. This follows from calculating the minimum of the polynomial describing the elliptical part of the wing loading.

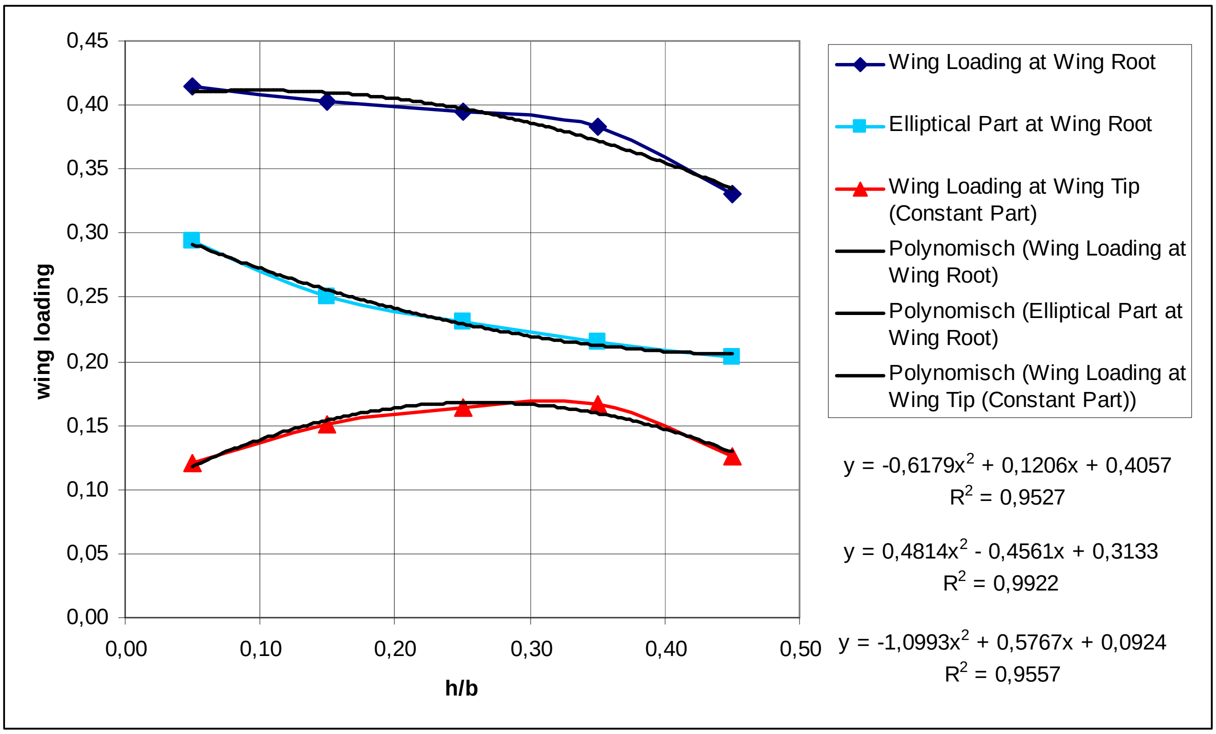

Figure 6.6 Results from Table 6.3 plotted and analysed further. Wing loading at wing root and wing tip for different h/b ratios

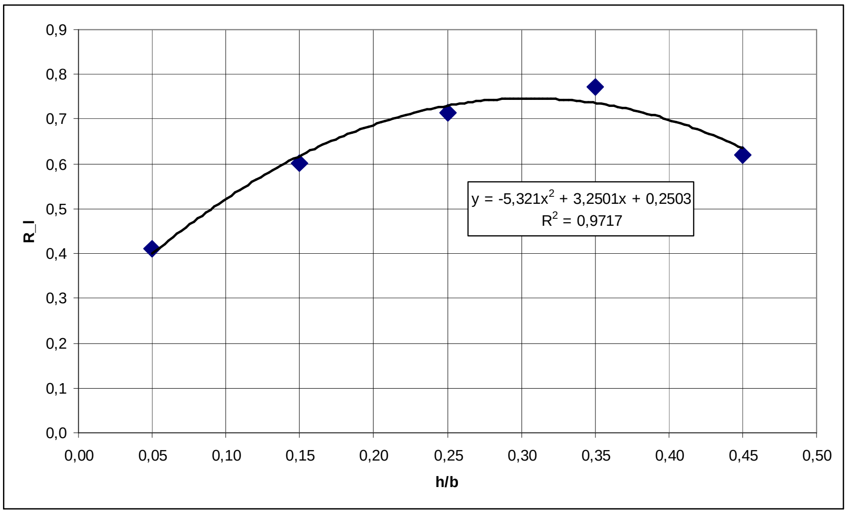

Figure 6.7 The ratio $R_l = q_{l,const} / q_{l,ellip}$ (2.5)

The ratio R_l from (2.5) is plotted in Figure 6.7. The constant part of the wing loading seems to have most influence at a h/b ratio of about 0.35. Here it is still less than the elliptical part.

7 IDRAG Results for Span Efficiency Compared with Literature

In this final chapter the results from IDRAG are compared with equations from literature. These were presented in Chapter 3.3. Graphical results are presented in Figure 7.1. In order to see the effect of high *h/b* ratios calculations were added for *h/b* = 0.7 and *h/b* = 1.0. These new *h/b* ratios where not considered when investigating the wing loading.

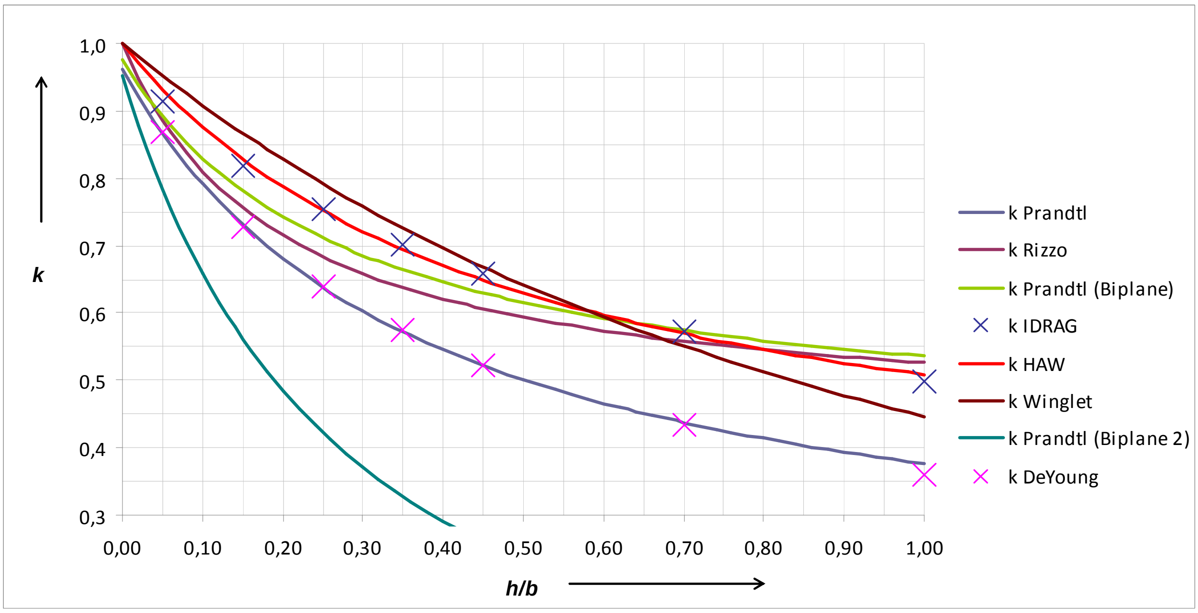

Figure 7.1 Induced drag factor *k* for a box wings with varying *h/b* ratios in comparison

It can be seen that the induced drag factor *k* calculated with IDRAG (X) follows the same general trend as the results from **Prandtl 1924** (biplane) Equation (3.7) and **Rizzo 2007** (3.6) though the *k* for each *h/b* ratio is a bit higher in IDRAG.

Now the intension is to find an own equation fitting even better to the calculated IDRAG data than those given in literature. The general form of the equations from literature is basically

$$\frac{D_{i,box}}{D_i} = k = \frac{k_1 + k_2 \cdot h/b}{k_3 + k_4 \cdot h/b} \quad . \tag{7.1}$$

k_2 / k_4 is the **limit value for very high *h/b* ratios**. A limit value could be forced on the equation. A possible limit value could be 0.5 which seems to be the limit value for two independent wings flying at a large distance from another. **Rizzo 2007** (3.6) gives a limit value of 0.4324 considerably lower than 0.5. So it seems wise to let the IDARG results tell what the limit value should be instead of forcing a value on the new equation that is itself not proven.

k_1 / k_3 is the **value for *h/b* = 0**. k_1 / k_3 = 1.0 is included in **Rizzo 2007** (3.6). This makes more sense than having a k_1 / $k_3 \neq 1.0$ as in the case of Prandtl's proposals (3.5) and (3.7). Hence also an own equation should have k_1 / k_3 = 1.0.

Table 7.1 Parameters for Equation (7.1) for best curve fit to IDRAG data

	1.) all k_i set free	**2.)** proposed parameters $k_3 = k_1$	***3.)*** $k_4 = 2 * k_2$	***4.)*** $k_3 = k_1$ and $k_4 = 2 * k_2$
k_1	1,30403	1,03687	0,89251	0,82508
k_2	0,37243	0,57082	2,53021	1,99730
k_3	1,35258	1,03687	0,84014	0,82508
k_4	1,98759	2,12587	5,06043	3,99460
h/b = 0	0,96410	1,00000	1,06234	1,00000
h/b => infinity	0,18738	0,26851	0,50000	0,50000
sum of error^2	5,57E-05	6,26E-04	9,94E-03	1,11E-02

Table 7.1 lists four possibilities forming an equation to the IDRAG data from Table 6.3 in the form of Equation (7.1). Curve fitting was done with the Excel solver minimizing the sum of the errors squared on the 7 IDRAG points from Table 6.3. The error on each point was taken as the difference between the IDRAG results and Equation (7.1) with the respective k-parameters from Table 7.1.

Interpretation of Table 7.1:

1. If all parameters are freely selected by the curve fitting algorithm the sum of the errors squared is the smallest. However this equation would show k_1 / $k_3 \neq 1.0$ at $h/b = 0$ which is not logical.
2. Requiring k_1 / k_3 = 1.0 yields another set of k-parameters and a somewhat larger sum of the errors squared which may still be tolerated. This set of k-parameters is finally selected for an own equation for the box wing.
3. k-parameters for a forced limit value of 0.5 for *h/b* going towards infinity are calculated. k_1 / k_3 = 1.062 is the value for h/b = 0 in this case.
4. k-parameters requiring k_1 / k_3 = 1.0 <u>and</u> a forced limit values of 0.5 for *h/b* going towards infinity are also given. The sum of the errors squared is now considerably larger. This does not seem to be advantageous.

Selecting k-parameters from Table 7.1 column marked 3. is

$$\frac{D_{i,box}}{D_i} = k = \frac{1.04 + 0.57 \cdot h/b}{1.04 + 2.13 \cdot h/b} \quad . \tag{7.2}$$

This equation and its k-parameters shows some similarity with the original box wing equation from Prandtl but seems to give better results.

The span efficiency factor for the box wing according to this own proposal is $e = 1/k$

$$\frac{D_i}{D_{i,box}} = e = \frac{1.04 + 2.13 \cdot h/b}{1.04 + 0.57 \cdot h/b} \quad . \tag{7.3}$$

For *h/b* going towards infinity the limit value for span efficiency is $e = 3.72$. Note that this span efficiency factor is a theoretical one. It assumes the span efficiency factor for the reference monoplane as 1.0.

One other approach for curve fitting was undertaken and presented in Figure 7.1 marked as "k Winglet". Here an equation was selected in the form used to estimate the induced drag parameter *k* for winglets

$$\frac{D_i}{D_{i,wl}} = e_{wl} = \left(1 + \frac{2}{k_{wl}} h/b\right)^2 \quad . \tag{7.4}$$

Similar to the approach from above Excel fitted a parameter $k_{wl} = 4.0291$ to the IDRAG data. The comparable value of the sum of error squared is $9.094 \cdot 10^{-3}$. Hence the curve fit is not as successful as for (7.2) respectively (7.3). For *h/b* going towards infinity the limit value for span efficiency would be $e \to$ infinity. This result is in contradiction to the box wing results of other authors who claim that there is a definite limit value (in the order of 2 to 4).

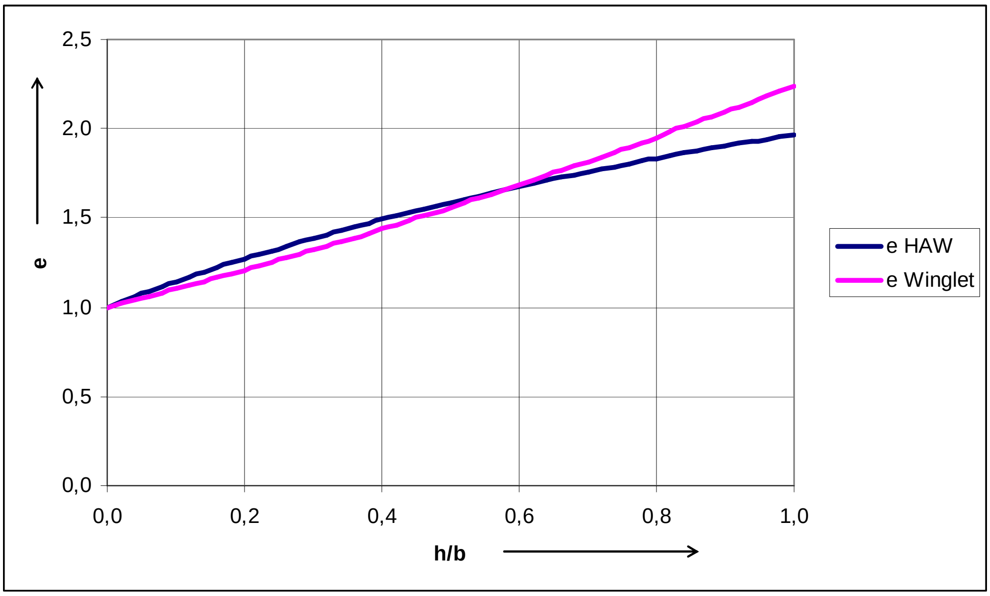

Figure 7.2 Span efficiency factor *e* for a box wing with varying *h/b* ratio in comparison. "e HAW" (proposed equation) is the result from (7.3) and "e Winglet" is the result from (7.4)

Summary

As early as 1924 the topic of this project report as been addressed by the "father" of aerodynamics Ludwig Prandtl. Biplanes are history and the box wing also known as the "best wing system" has not found much interest from industry until today. For this reason there is still much scope for exploration.

The report showed (as was expected) that the lift distribution flattens out to the wing tips if long (high) vertical wings are attached to the tips. The tendency to shift lift to the tips increases wing bending and requires more mass in wing construction to withstand these increased loads. This is detrimental to the idea of the box wing because the box wing approach is to reduce induced drag. Heavier wings cause more induced drag and consume some of the original advantage of the box wing.

So far it was not totally clear what the box wing potential is on induced drag reduction. Authors made different proposals for calculating these saving. The IDRAG result confirm the general layout of the form of equation usually used and made it possible to fit parameters to this equation. The result is this equation

$$\frac{D_i}{D_{i,box}} = e = \frac{1.04 + 2.13 \cdot h/b}{1.04 + 0.57 \cdot h/b}$$

Following this equation, for *h/b* going towards infinity the limit value for span efficiency is $e = 3.72$. Note that this span efficiency factor is a theoretical one. It assumes the span efficiency factor for the reference monoplane as 1.0. In other words the span efficiency calculated above would need to be multiplied with a usual monoplane span efficiency (like 0.82) to yield a useful and practical span efficiency of Oswald factor for the box wing.

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Appendix A - Matlab Scripts

A.1 Matlab Script to Read IDRAG

```
function [header,case_title,input_mode,write_flag,sym_flag,cl_design, ...
 cm_flag,cm_design,xcg,cp,sref,cavg,npanels,xc,yc,zc,nvortices,spacing, ...
 x,y,z,load,cn,cl_actual,cm_actual,cd_induced,e] = readidrag(filename)
%READIDRAG Reads the output file created by the code 'idrag'.
%          READIDRAG(FILENAME)
%              FILENAME - name of idrag output file (entered within single
quotes)
%
% Created by Joel Grasmeyer
% Last modified on 02/03/97
fid = fopen(filename,'r');
header = fscanf(fid,'%s %s %s',3);
case_title = fscanf(fid,'%s',1);
input_mode = fscanf(fid,'%f %s %s %s',4); input_mode = input_mode(1);
write_flag = fscanf(fid,'%f %s %s %s',4); write_flag = write_flag(1);
sym_flag = fscanf(fid,'%f %s %s %s',4); sym_flag = sym_flag(1);
cl_design = fscanf(fid,'%f %s %s %s %s',5); cl_design = cl_design(1);
cm_flag = fscanf(fid,'%f %s %s %s %s %s',5); cm_flag = cm_flag(1);
cm_design = fscanf(fid,'%f %s %s %s %s %s',5); cm_design = cm_design(1);
xcg = fscanf(fid,'%f %s %s %s %s %s',5); xcg = xcg(1);
cp = fscanf(fid,'%f %s %s %s %s %s %s %s %s',8); cp = cp(1);
sref = fscanf(fid,'%f %s %s %s %s',4); sref = sref(1);
cavg = fscanf(fid,'%f %s %s %s %s',4); cavg = cavg(1);
npanels = fscanf(fid,'%f %s %s %s %s %s',5); npanels = npanels(1);
for i=1:npanels,
  junk = fscanf(fid,'%s %s %s %s %s %f',6);
  for j=1:4,
    xc(j,i) = fscanf(fid,'%f',1);
    yc(j,i) = fscanf(fid,'%f',1);
    zc(j,i) = fscanf(fid,'%f',1);
  end
  nvor = fscanf(fid,'%f %s %s %s %s',5); nvortices(i) = nvor(1);
  spac = fscanf(fid,'%f %s %s %s %s',5); spacing(i) = spac(1);
end
junk = fscanf(fid,'%s %s %s %s %s %s',6);
k = 1;
for i=1:npanels,
  for j=1:nvortices(i),
    num = fscanf(fid,'%f',1);
    x(j,i) = fscanf(fid,'%f',1);
    y(j,i) = fscanf(fid,'%f',1);
    z(j,i) = fscanf(fid,'%f',1);
    load(k) = fscanf(fid,'%f',1);
    cn(k) = fscanf(fid,'%f',1);
    k = k + 1;
```

```
  end
end
cl_actual = fscanf(fid,'%f %s %s %s %s',5); cl_actual = cl_actual(1);
cm_actual = fscanf(fid,'%f %s %s %s %s',5); cm_actual = cm_actual(1);
cd_induced = fscanf(fid,'%f %s %s %s %s',5); cd_induced = cd_induced(1);
e = fscanf(fid,'%f %s %s %s %s',5); e = e(1);
fclose(fid);
```

A.2 Matlab Script to Plot the Geometry

```
function geom(filename,view_flag,vortex_flag)
%GEOM Creates plots of the configuration geometry.
%    GEOM(FILENAME,VIEW_FLAG,VORTEX_FLAG)
%    FILENAME  - name of idrag output file (entered within single quotes)
%    VIEW_FLAG - flag to denote the content of the plots:
%    0 = 1 perspective plot, 1 = 1 perspective and 3-views
%    VORTEX_FLAG - flag to determine whether vortex locations are shown:
%    0 = don't plot vortex locations, 1 = show vortex locations
%
%    If GEOM is called with just the FILENAME argument, VIEW_FLAG and
%    VORTEX_FLAG are set to their default values of 1 and 0, respectively.
%
% Created by Joel Grasmeyer
% Last modified on 12/2/96
% Set default plot parameters for 1 input argument
if nargin == 1
  view_flag = 1;
  vortex_flag = 0;
end
% Read idrag output file
[header,case_title,input_mode,write_flag,sym_flag,cl_design, ...
 cm_flag,cm_design,xcg,cp,sref,cavg,npanels,xc,yc,zc,nvortices,spacing, ...
 x,y,z,loading,cn,cl_actual,cm_actual,cd_induced,e] = readidrag(filename);
% Calculate ranges for plots
minx = min(min(xc));
miny = min(min(yc));
minz = min(min(zc));
maxx = max(max(xc));
maxy = max(max(yc));
maxz = max(max(zc));
[minmin,imin] = min([minx miny minz]);
[maxmax,imax] = max([maxx maxy maxz]);
del = maxmax - minmin;
margin = 0.05;
amin = minmin - margin*del;
amax = maxmax + margin*del;
% Close panels
xc(5,:) = xc(1,:);
```

```
yc(5,:) = yc(1,:);
zc(5,:) = zc(1,:);
% Create plots, depending on view_flag
if view_flag == 0
  for i=1:npanels
    if sym_flag == 1
      plot3(yc(:,i),xc(:,i),zc(:,i),'r-',-yc(:,i),xc(:,i),zc(:,i),'r-')
      axis([-amax amax amin amax amin amax])
    else
      plot3(yc(:,i),xc(:,i),zc(:,i),'r-')
      axis([amin amax amin amax amin amax])
    end
    title(['Perspective View (e = ',num2str(e),')'])
    xlabel('y')
    ylabel('x')
    zlabel('z')
    set(gca,'YDir','rev')
    hold on
    if vortex_flag == 1

plot3(y(1:nvortices(i),i),x(1:nvortices(i),i),z(1:nvortices(i),i),'b+')
    end
  end
  hold off
else
  for i=1:npanels
    if sym_flag == 1
      subplot(2,2,1), plot(yc(:,i),xc(:,i),'r-',-yc(:,i),xc(:,i),'r-')
      axis([-amax amax amin amax])
      axis('equal')
    else
      subplot(2,2,1), plot(yc(:,i),xc(:,i),'r-')
      axis([amin amax amin amax])
    end
    title(['Top View (e = ',num2str(e),')'])
    xlabel('y')
    ylabel('x')
    axis('equal')
    set(gca,'YDir','rev')
    hold on
    if vortex_flag == 1
      subplot(2,2,1), plot(y(1:nvortices(i),i),x(1:nvortices(i),i),'b+')
    end
    if sym_flag == 1
      subplot(2,2,2), plot3(yc(:,i),xc(:,i),zc(:,i),'r-', ...
       -yc(:,i),xc(:,i),zc(:,i),'r-')
      axis([-amax amax amin amax amin amax])
    else
      subplot(2,2,2), plot3(yc(:,i),xc(:,i),zc(:,i),'r-')
      axis([amin amax amin amax amin amax])
    end
```

```
    title('Perspective View')
    xlabel('y')
    ylabel('x')
    zlabel('z')
    set(gca,'YDir','rev')
    hold on
    if vortex_flag == 1
      subplot(2,2,2), plot3(y(1:nvortices(i),i),x(1:nvortices(i),i), ...
       z(1:nvortices(i),i),'b+')
    end
    if sym_flag == 1
      subplot(2,2,3), plot(yc(:,i),zc(:,i),'r-',-yc(:,i),zc(:,i),'r-')
      axis([-amax amax amin amax])
    else
      subplot(2,2,3), plot(yc(:,i),zc(:,i),'r-')
      axis([amin amax amin amax])
    end
    title('Rear View')
    xlabel('y')
    ylabel('z')
    axis('equal')
    hold on
    if vortex_flag == 1
      subplot(2,2,3), plot(y(1:nvortices(i),i),z(1:nvortices(i),i),'b+')
    end
    subplot(2,2,4), plot(xc(:,i),zc(:,i),'r-')
    title('Right Side View')
    xlabel('x')
    ylabel('z')
    axis([amin amax amin amax])
    axis('equal')
    set(gca,'XDir','rev')
    hold on
    if vortex_flag == 1
      subplot(2,2,4), plot(x(1:nvortices(i),i),z(1:nvortices(i),i),'b+')
    end
  end
  subplot(2,2,1),hold off
  subplot(2,2,2),hold off
  subplot(2,2,3),hold off
  subplot(2,2,4),hold off
end
```

A.3 Matlab Script to Plot the Loads

```
function loads(filename,dist_flag)
%LOADS Creats plots of load or normal force coefficient distribution.
%      LOADS(FILENAME,DIST_FLAG)
%           FILENAME   - name of idrag output file (entered within single
quotes)
%        DIST_FLAG - flag to denote the content of the plots:
%         (0 = loading (cn*c/cavg), 1 = normal force coefficient (cn))
%
%     If LOADS is called with just the FILENAME argument, DIST_FLAG is
%       set to its default value of 0.
%
% Created by Joel Grasmeyer
% Last modified on 12/2/96
% Set default plot parameters for 1 input argument
if nargin == 1
  dist_flag = 0;
end
% Read idrag output file
[header,case_title,input_mode,write_flag,sym_flag,cl_design, ...
 cm_flag,cm_design,xcg,cp,sref,cavg,npanels,xc,yc,zc,nvortices,spacing, ...
 x,y,z,loading,cn,cl_actual,cm_actual,cd_induced,e] = readidrag(filename);
% Set distribution to cn or load, depending on dist_flag
if dist_flag == 0
  dist = loading;
  label = 'Load Distribution (Cn*c/cavg)';
else
  dist = cn;
  label = 'Normal Force Coefficient Distribution (Cn)';
end
xmin = 0;
xmax = max( sqrt( (yc(2,:)-yc(1,:)).^2 + (zc(2,:)-zc(1,:)).^2 ) );
ymin = min(dist);
ymax = max(dist);
% Determine number of subplots to create (maximum of 4)
if npanels == 1
  rows = 1;
  cols = 1;
elseif npanels == 2
  rows = 2;
  cols = 1;
elseif npanels == 3 | npanels == 4
  rows = 2;
  cols = 2;
end
% Plot cn or load distribution for each panel
if npanels >= 1
  station = sqrt( (y(1:nvortices(1),1) - yc(1,1)).^2 + ...
   (z(1:nvortices(1),1) - zc(1,1)).^2 );
```

```
  subplot(rows,cols,1), plot(station,dist(1:nvortices(1)),'-')
  title(['Panel 1 (e = ',num2str(e),')'])
  ylabel(label)
  xlabel('Distance Along Panel')
  axis([xmin xmax ymin ymax])
    grid on
end
if npanels >= 2
  station = sqrt( (y(1:nvortices(2),2) - yc(1,2)).^2 + ...
   (z(1:nvortices(2),2) - zc(1,2)).^2 );
  subplot(rows,cols,2),
plot(station,dist(nvortices(1)+1:sum(nvortices(1:2))))
  title('Panel 2')
  xlabel('Distance Along Panel')
  ylabel(label)
  axis([xmin xmax ymin ymax])
  grid on
end
if npanels >= 3
  station = sqrt( (y(1:nvortices(3),3) - yc(1,3)).^2 + ...
   (z(1:nvortices(3),3) - zc(1,3)).^2 );
  subplot(rows,cols,3), plot(station, ...
   dist(sum(nvortices(1:2))+1:sum(nvortices(1:3))),'-')
  title('Panel 3')
  xlabel('Distance Along Panel')
  ylabel(label)
  axis([xmin xmax ymin ymax])
  grid on
end
if npanels >= 4
  station = sqrt( (y(1:nvortices(4),4) - yc(1,4)).^2 + ...
   (z(1:nvortices(4),4) - zc(1,4)).^2 );
  subplot(rows,cols,4), plot(station, ...
   dist(sum(nvortices(1:3))+1:sum(nvortices(1:4))),'-')
  title('Panel 4')
  xlabel('Distance Along Panel')
  ylabel(label)
  axis([xmin xmax ymin ymax])
  grid on
end
```

Appendix B - Input Files for IDRAG

B.1 Input File for a *h/b* of 0.05

```
idrag input file
BoxWingValidation
0               input mode
1               write flag
1               symmetry flag
0.67               cl_design
0               cm_flag
0               cm_design
6.71               x cg position
0.25               center of pressure for airfoil sections
122.4               reference area
3.6               reference chord
3               number of panels
0   0   0     x,y,z for 4 corners of panel 1
0   17   0
1.8   17   0
1.8   0   0
160               number of vortices for panel 1
0               vortex spacing for panel 1
0   0   0.85     x,y,z for 4 corners of panel 2
0   17   0.85
1.8   17   0.85
1.8   0   0.85
160               number of vortices for panel 2
0               vortex spacing for panel 2
0   17   0     x,y,z for 4 corners of panel 3
0   17   0.85
1.8   17   0.85
1.8   17   0
80               number of vortices for panel 3
0               vortex spacing for panel 3
```

B.2 Input File for a *h/b* of 0.15

```
idrag input file
BoxWingValidation
0               input mode
1               write flag
1               symmetry flag
0.67               cl_design
```

```
0               cm_flag
0               cm_design
6.71               x cg position
0.25               center of pressure for airfoil sections
122.4               reference area
3.6               reference chord
3               number of panels
0   0   0     x,y,z for 4 corners of panel 1
0   17   0
1.8   17   0
1.8   0   0
160               number of vortices for panel 1
0               vortex spacing for panel 1
0   0   2.55     x,y,z for 4 corners of panel 2
0   17   2.55
1.8   17   2.55
1.8   0   2.55
160               number of vortices for panel 2
0               vortex spacing for panel 2
0   17   0     x,y,z for 4 corners of panel 3
0   17   2.55
1.8   17   2.55
1.8   17   0
80               number of vortices for panel 3
0     vortex spacing for panel 3
```

B.3 Input File for a *h/b* of 0.25

```
idrag input file
BoxWingValidation
0               input mode
1               write flag
1               symmetry flag
0.67               cl_design
0               cm_flag
0               cm_design
6.71               x cg position
0.25               center of pressure for airfoil sections
122.4               reference area
3.6               reference chord
3               number of panels
0   0   0     x,y,z for 4 corners of panel 1
0   17   0
1.8   17   0
1.8   0   0
160               number of vortices for panel 1
0               vortex spacing for panel 1
0   0   4.25     x,y,z for 4 corners of panel 2
```

```
0   17   4.25
1.8   17   4.25
1.8   0   4.25
160              number of vortices for panel 2
0              vortex spacing for panel 2
0   17   0     x,y,z for 4 corners of panel 3
0   17   4.25
1.8   17   4.25
1.8   17   0
80              number of vortices for panel 3
0              vortex spacing for panel 3
```

B.4 Input File for a *h/b* of 0.35

```
idrag input file
BoxWingValidation
0              input mode
1              write flag
1              symmetry flag
0.67              cl_design
0              cm_flag
0              cm_design
6.71              x cg position
0.25              center of pressure for airfoil sections
122.4              reference area
3.6              reference chord
3              number of panels
0   0   0     x,y,z for 4 corners of panel 1
0   17   0
1.8   17   0
1.8   0   0
30              number of vortices for panel 1
0              vortex spacing for panel 1
0   0   5.95     x,y,z for 4 corners of panel 2
0   17   5.95
1.8   17   5.95
1.8   0   5.95
30              number of vortices for panel 2
0              vortex spacing for panel 2
0   17   0     x,y,z for 4 corners of panel 3
0   17   5.95
1.8   17   5.95
1.8   17   0
15              number of vortices for panel 3
0              vortex spacing for panel 3
```

B.5 Input File for a *h/b* of 0.45

```
idrag input file
BoxWingValidation
0               input mode
1               write flag
1               symmetry flag
0.67               cl_design
0               cm_flag
0               cm_design
6.71               x cg position
0.25               center of pressure for airfoil sections
122.4               reference area
3.6               reference chord
3               number of panels
0   0   0     x,y,z for 4 corners of panel 1
0   17   0
1.8   17   0
1.8   0   0
30               number of vortices for panel 1
0               vortex spacing for panel 1
0   0   7.65     x,y,z for 4 corners of panel 2
0   17   7.65
1.8   17   7.65
1.8   0   7.65
30               number of vortices for panel 2
0               vortex spacing for panel 2
0   17   0     x,y,z for 4 corners of panel 3
0   17   7.65
1.8   17   7.65
1.8   17   0
15               number of vortices for panel 3
0               vortex spacing for panel 3
```

Appendix C - Wing Loading Tables

C.1 Total Wing Loading

Table C.1 Total wing loading for different *h/b* ratios

Wing Loading					
Span Stations [m]	***h/b*=0.05**	***h/b*=0.15**	***h/b*=0.25**	***h/b*=0.35**	***h/b*=0.45**
0,0531	0,4142	0,4026	0,3950	0,3825	0,3301
0,1594	0,4142	0,4026	0,3950	0,3825	0,3301
0,2656	0,4141	0,4026	0,3950	0,3825	0,3301
0,3719	0,4141	0,4025	0,3949	0,3825	0,3301
0,4781	0,4140	0,4025	0,3949	0,3824	0,3300
0,5844	0,4140	0,4024	0,3948	0,3824	0,3300
0,6906	0,4139	0,4023	0,3948	0,3823	0,3299
0,7969	0,4138	0,4022	0,3947	0,3822	0,3298
0,9031	0,4136	0,4021	0,3946	0,3821	0,3298
1,0094	0,4135	0,4020	0,3945	0,3820	0,3297
1,1156	0,4134	0,4019	0,3944	0,3819	0,3296
1,2219	0,4132	0,4017	0,3942	0,3818	0,3295
1,3281	0,4130	0,4016	0,3941	0,3817	0,3293
1,4344	0,4128	0,4014	0,3939	0,3815	0,3292
1,5406	0,4126	0,4012	0,3938	0,3814	0,3291
1,6469	0,4124	0,4010	0,3936	0,3812	0,3289
1,7531	0,4122	0,4008	0,3934	0,3811	0,3288
1,8594	0,4119	0,4006	0,3932	0,3809	0,3286
1,9656	0,4116	0,4004	0,3930	0,3807	0,3284
2,0719	0,4113	0,4001	0,3928	0,3805	0,3282
2,1781	0,4110	0,3999	0,3926	0,3803	0,3280
2,2844	0,4107	0,3996	0,3923	0,3800	0,3278
2,3906	0,4104	0,3993	0,3921	0,3798	0,3276
2,4969	0,4101	0,3990	0,3918	0,3796	0,3274
2,6031	0,4097	0,3987	0,3915	0,3793	0,3271
2,7094	0,4093	0,3984	0,3912	0,3790	0,3269
2,8156	0,4089	0,3980	0,3909	0,3787	0,3266
2,9219	0,4085	0,3977	0,3906	0,3784	0,3263
3,0281	0,4081	0,3973	0,3902	0,3781	0,3260
3,1344	0,4077	0,3969	0,3899	0,3778	0,3258
3,2406	0,4072	0,3965	0,3895	0,3775	0,3255
3,3469	0,4068	0,3961	0,3892	0,3772	0,3251
3,4531	0,4063	0,3957	0,3888	0,3768	0,3248
3,5594	0,4058	0,3953	0,3884	0,3765	0,3245
3,6656	0,4053	0,3948	0,3880	0,3761	0,3241
3,7719	0,4047	0,3944	0,3876	0,3757	0,3238
3,8781	0,4042	0,3939	0,3872	0,3753	0,3234
3,9844	0,4036	0,3934	0,3867	0,3749	0,3230

4,0906	0,4030	0,3929	0,3863	0,3745	0,3226
4,1969	0,4024	0,3924	0,3858	0,3740	0,3222
4,3031	0,4018	0,3919	0,3853	0,3736	0,3218
4,4094	0,4012	0,3913	0,3848	0,3732	0,3214
4,5156	0,4006	0,3908	0,3843	0,3727	0,3210
4,6219	0,3999	0,3902	0,3838	0,3722	0,3205
4,7281	0,3992	0,3896	0,3833	0,3717	0,3201
4,8344	0,3985	0,3890	0,3827	0,3712	0,3196
4,9406	0,3978	0,3884	0,3822	0,3707	0,3192
5,0469	0,3971	0,3877	0,3816	0,3702	0,3187
5,1531	0,3964	0,3871	0,3810	0,3697	0,3182
5,2594	0,3956	0,3864	0,3804	0,3691	0,3177
5,3656	0,3948	0,3857	0,3798	0,3685	0,3171
5,4719	0,3940	0,3851	0,3792	0,3680	0,3166
5,5781	0,3932	0,3843	0,3785	0,3674	0,3161
5,6844	0,3924	0,3836	0,3779	0,3668	0,3155
5,7906	0,3915	0,3829	0,3772	0,3662	0,3149
5,8969	0,3907	0,3821	0,3765	0,3656	0,3144
6,0031	0,3898	0,3814	0,3758	0,3649	0,3138
6,1094	0,3889	0,3806	0,3751	0,3643	0,3132
6,2156	0,3880	0,3798	0,3744	0,3636	0,3126
6,3219	0,3870	0,3790	0,3737	0,3629	0,3119
6,4281	0,3861	0,3781	0,3729	0,3622	0,3113
6,5344	0,3851	0,3773	0,3722	0,3616	0,3106
6,6406	0,3841	0,3764	0,3714	0,3608	0,3100
6,7469	0,3831	0,3756	0,3706	0,3601	0,3093
6,8531	0,3821	0,3747	0,3698	0,3594	0,3086
6,9594	0,3810	0,3737	0,3690	0,3586	0,3079
7,0656	0,3800	0,3728	0,3681	0,3578	0,3072
7,1719	0,3789	0,3719	0,3673	0,3571	0,3065
7,2781	0,3778	0,3709	0,3664	0,3563	0,3057
7,3844	0,3766	0,3699	0,3655	0,3555	0,3050
7,4906	0,3755	0,3689	0,3646	0,3546	0,3042
7,5969	0,3743	0,3679	0,3637	0,3538	0,3035
7,7031	0,3731	0,3669	0,3628	0,3529	0,3027
7,8094	0,3719	0,3659	0,3619	0,3521	0,3019
7,9156	0,3707	0,3648	0,3609	0,3512	0,3010
8,0219	0,3695	0,3637	0,3599	0,3503	0,3002
8,1281	0,3682	0,3626	0,3589	0,3494	0,2994
8,2344	0,3669	0,3615	0,3579	0,3485	0,2985
8,3406	0,3656	0,3603	0,3569	0,3475	0,2976
8,4469	0,3643	0,3592	0,3558	0,3466	0,2967
8,5531	0,3629	0,3580	0,3548	0,3456	0,2958
8,6594	0,3615	0,3568	0,3537	0,3446	0,2949
8,7656	0,3601	0,3556	0,3526	0,3436	0,2940
8,8719	0,3587	0,3544	0,3515	0,3426	0,2931

8,9781	0,3572	0,3531	0,3504	0,3416	0,2921
9,0844	0,3558	0,3518	0,3492	0,3405	0,2911
9,1906	0,3543	0,3505	0,3481	0,3394	0,2901
9,2969	0,3528	0,3492	0,3469	0,3383	0,2891
9,4031	0,3512	0,3479	0,3457	0,3372	0,2881
9,5094	0,3496	0,3465	0,3444	0,3361	0,2871
9,6156	0,3480	0,3451	0,3432	0,3350	0,2860
9,7219	0,3464	0,3437	0,3419	0,3338	0,2849
9,8281	0,3448	0,3423	0,3407	0,3326	0,2838
9,9344	0,3431	0,3409	0,3394	0,3315	0,2827
10,0406	0,3414	0,3394	0,3380	0,3302	0,2816
10,1469	0,3397	0,3379	0,3367	0,3290	0,2805
10,2531	0,3379	0,3364	0,3353	0,3278	0,2793
10,3594	0,3361	0,3348	0,3339	0,3265	0,2781
10,4656	0,3343	0,3333	0,3325	0,3252	0,2769
10,5719	0,3325	0,3317	0,3311	0,3239	0,2757
10,6781	0,3306	0,3301	0,3297	0,3226	0,2745
10,7844	0,3287	0,3284	0,3282	0,3212	0,2732
10,8906	0,3268	0,3268	0,3267	0,3199	0,2720
10,9969	0,3248	0,3251	0,3252	0,3185	0,2707
11,1031	0,3229	0,3234	0,3236	0,3171	0,2694
11,2094	0,3208	0,3216	0,3221	0,3156	0,2680
11,3156	0,3188	0,3198	0,3205	0,3142	0,2667
11,4219	0,3167	0,3180	0,3189	0,3127	0,2653
11,5281	0,3146	0,3162	0,3172	0,3112	0,2639
11,6344	0,3124	0,3144	0,3155	0,3096	0,2625
11,7406	0,3102	0,3125	0,3138	0,3081	0,2611
11,8469	0,3080	0,3105	0,3121	0,3065	0,2596
11,9531	0,3057	0,3086	0,3104	0,3049	0,2581
12,0594	0,3034	0,3066	0,3086	0,3033	0,2566
12,1656	0,3010	0,3046	0,3068	0,3016	0,2551
12,2719	0,2986	0,3025	0,3050	0,2999	0,2535
12,3781	0,2962	0,3004	0,3031	0,2982	0,2519
12,4844	0,2937	0,2983	0,3012	0,2965	0,2503
12,5906	0,2912	0,2962	0,2992	0,2947	0,2486
12,6969	0,2886	0,2940	0,2973	0,2929	0,2470
12,8031	0,2860	0,2917	0,2953	0,2911	0,2453
12,9094	0,2834	0,2894	0,2933	0,2892	0,2436
13,0156	0,2807	0,2871	0,2912	0,2873	0,2418
13,1219	0,2779	0,2848	0,2891	0,2854	0,2400
13,2281	0,2751	0,2824	0,2869	0,2834	0,2382
13,3344	0,2722	0,2799	0,2848	0,2814	0,2363
13,4406	0,2693	0,2774	0,2825	0,2794	0,2344
13,5469	0,2663	0,2749	0,2803	0,2773	0,2325
13,6531	0,2633	0,2723	0,2780	0,2752	0,2305
13,7594	0,2602	0,2697	0,2756	0,2731	0,2285

13,8656	0,2570	0,2670	0,2732	0,2709	0,2265
13,9719	0,2538	0,2642	0,2708	0,2686	0,2244
14,0781	0,2505	0,2614	0,2683	0,2663	0,2223
14,1844	0,2471	0,2586	0,2657	0,2640	0,2201
14,2906	0,2436	0,2557	0,2631	0,2616	0,2179
14,3969	0,2401	0,2527	0,2605	0,2592	0,2156
14,5031	0,2364	0,2497	0,2578	0,2567	0,2133
14,6094	0,2327	0,2465	0,2550	0,2541	0,2110
14,7156	0,2289	0,2434	0,2521	0,2515	0,2085
14,8219	0,2250	0,2401	0,2492	0,2488	0,2060
14,9281	0,2210	0,2368	0,2463	0,2461	0,2035
15,0344	0,2169	0,2333	0,2432	0,2433	0,2009
15,1406	0,2126	0,2298	0,2401	0,2404	0,1982
15,2469	0,2082	0,2262	0,2368	0,2374	0,1954
15,3531	0,2037	0,2225	0,2335	0,2344	0,1926
15,4594	0,1990	0,2187	0,2301	0,2312	0,1896
15,5656	0,1942	0,2148	0,2266	0,2280	0,1866
15,6719	0,1892	0,2108	0,2229	0,2246	0,1834
15,7781	0,1841	0,2066	0,2192	0,2211	0,1802
15,8844	0,1787	0,2023	0,2153	0,2175	0,1768
15,9906	0,1731	0,1978	0,2112	0,2137	0,1733
16,0969	0,1673	0,1932	0,2070	0,2098	0,1696
16,2031	0,1613	0,1883	0,2025	0,2057	0,1657
16,3094	0,1550	0,1833	0,1979	0,2013	0,1616
16,4156	0,1484	0,1780	0,1930	0,1967	0,1573
16,5219	0,1416	0,1724	0,1878	0,1918	0,1527
16,6281	0,1346	0,1665	0,1823	0,1865	0,1476
16,7344	0,1274	0,1604	0,1763	0,1807	0,1420
16,8406	0,1208	0,1542	0,1700	0,1744	0,1357
16,9469	0,1204	0,1514	0,1646	0,1667	0,1263

C.2 Elliptical Part of the Wing Loading

Table C.1 Elliptical part of the wing loading for different *h/b* ratios

Wing Loading Elliptical Part					
Span Stations	***h/b*=0.05**	***h/b*=0.15**	***h/b*=0.25**	***h/b*=0.35**	***h/b*=0.45**
0,0531	0,2938	0,2512	0,2304	0,2158	0,2038
0,1594	0,2938	0,2512	0,2304	0,2158	0,2038
0,2656	0,2937	0,2512	0,2304	0,2158	0,2038
0,3719	0,2937	0,2511	0,2303	0,2158	0,2038
0,4781	0,2936	0,2511	0,2303	0,2157	0,2037
0,5844	0,2936	0,2510	0,2302	0,2157	0,2037
0,6906	0,2935	0,2509	0,2302	0,2156	0,2036

0,7969	0,2934	0,2508	0,2301	0,2155	0,2035
0,9031	0,2932	0,2507	0,2300	0,2154	0,2035
1,0094	0,2931	0,2506	0,2299	0,2153	0,2034
1,1156	0,2930	0,2505	0,2298	0,2152	0,2033
1,2219	0,2928	0,2503	0,2296	0,2151	0,2032
1,3281	0,2926	0,2502	0,2295	0,2150	0,2030
1,4344	0,2924	0,2500	0,2293	0,2148	0,2029
1,5406	0,2922	0,2498	0,2292	0,2147	0,2028
1,6469	0,2920	0,2496	0,2290	0,2145	0,2026
1,7531	0,2918	0,2494	0,2288	0,2144	0,2025
1,8594	0,2915	0,2492	0,2286	0,2142	0,2023
1,9656	0,2912	0,2490	0,2284	0,2140	0,2021
2,0719	0,2909	0,2487	0,2282	0,2138	0,2019
2,1781	0,2906	0,2485	0,2280	0,2136	0,2017
2,2844	0,2903	0,2482	0,2277	0,2133	0,2015
2,3906	0,2900	0,2479	0,2275	0,2131	0,2013
2,4969	0,2897	0,2476	0,2272	0,2129	0,2011
2,6031	0,2893	0,2473	0,2269	0,2126	0,2008
2,7094	0,2889	0,2470	0,2266	0,2123	0,2006
2,8156	0,2885	0,2466	0,2263	0,2120	0,2003
2,9219	0,2881	0,2463	0,2260	0,2117	0,2000
3,0281	0,2877	0,2459	0,2256	0,2114	0,1997
3,1344	0,2873	0,2455	0,2253	0,2111	0,1995
3,2406	0,2868	0,2451	0,2249	0,2108	0,1992
3,3469	0,2864	0,2447	0,2246	0,2105	0,1988
3,4531	0,2859	0,2443	0,2242	0,2101	0,1985
3,5594	0,2854	0,2439	0,2238	0,2098	0,1982
3,6656	0,2849	0,2434	0,2234	0,2094	0,1978
3,7719	0,2843	0,2430	0,2230	0,2090	0,1975
3,8781	0,2838	0,2425	0,2226	0,2086	0,1971
3,9844	0,2832	0,2420	0,2221	0,2082	0,1967
4,0906	0,2826	0,2415	0,2217	0,2078	0,1963
4,1969	0,2820	0,2410	0,2212	0,2073	0,1959
4,3031	0,2814	0,2405	0,2207	0,2069	0,1955
4,4094	0,2808	0,2399	0,2202	0,2065	0,1951
4,5156	0,2802	0,2394	0,2197	0,2060	0,1947
4,6219	0,2795	0,2388	0,2192	0,2055	0,1942
4,7281	0,2788	0,2382	0,2187	0,2050	0,1938
4,8344	0,2781	0,2376	0,2181	0,2045	0,1933
4,9406	0,2774	0,2370	0,2176	0,2040	0,1929
5,0469	0,2767	0,2363	0,2170	0,2035	0,1924
5,1531	0,2760	0,2357	0,2164	0,2030	0,1919
5,2594	0,2752	0,2350	0,2158	0,2024	0,1914
5,3656	0,2744	0,2343	0,2152	0,2018	0,1908
5,4719	0,2736	0,2337	0,2146	0,2013	0,1903
5,5781	0,2728	0,2329	0,2139	0,2007	0,1898

5,6844	0,2720	0,2322	0,2133	0,2001	0,1892
5,7906	0,2711	0,2315	0,2126	0,1995	0,1886
5,8969	0,2703	0,2307	0,2119	0,1989	0,1881
6,0031	0,2694	0,2300	0,2112	0,1982	0,1875
6,1094	0,2685	0,2292	0,2105	0,1976	0,1869
6,2156	0,2676	0,2284	0,2098	0,1969	0,1863
6,3219	0,2666	0,2276	0,2091	0,1962	0,1856
6,4281	0,2657	0,2267	0,2083	0,1955	0,1850
6,5344	0,2647	0,2259	0,2076	0,1949	0,1843
6,6406	0,2637	0,2250	0,2068	0,1941	0,1837
6,7469	0,2627	0,2242	0,2060	0,1934	0,1830
6,8531	0,2617	0,2233	0,2052	0,1927	0,1823
6,9594	0,2606	0,2223	0,2044	0,1919	0,1816
7,0656	0,2596	0,2214	0,2035	0,1911	0,1809
7,1719	0,2585	0,2205	0,2027	0,1904	0,1802
7,2781	0,2574	0,2195	0,2018	0,1896	0,1794
7,3844	0,2562	0,2185	0,2009	0,1888	0,1787
7,4906	0,2551	0,2175	0,2000	0,1879	0,1779
7,5969	0,2539	0,2165	0,1991	0,1871	0,1772
7,7031	0,2527	0,2155	0,1982	0,1862	0,1764
7,8094	0,2515	0,2145	0,1973	0,1854	0,1756
7,9156	0,2503	0,2134	0,1963	0,1845	0,1747
8,0219	0,2491	0,2123	0,1953	0,1836	0,1739
8,1281	0,2478	0,2112	0,1943	0,1827	0,1731
8,2344	0,2465	0,2101	0,1933	0,1818	0,1722
8,3406	0,2452	0,2089	0,1923	0,1808	0,1713
8,4469	0,2439	0,2078	0,1912	0,1799	0,1704
8,5531	0,2425	0,2066	0,1902	0,1789	0,1695
8,6594	0,2411	0,2054	0,1891	0,1779	0,1686
8,7656	0,2397	0,2042	0,1880	0,1769	0,1677
8,8719	0,2383	0,2030	0,1869	0,1759	0,1668
8,9781	0,2368	0,2017	0,1858	0,1749	0,1658
9,0844	0,2354	0,2004	0,1846	0,1738	0,1648
9,1906	0,2339	0,1991	0,1835	0,1727	0,1638
9,2969	0,2324	0,1978	0,1823	0,1716	0,1628
9,4031	0,2308	0,1965	0,1811	0,1705	0,1618
9,5094	0,2292	0,1951	0,1798	0,1694	0,1608
9,6156	0,2276	0,1937	0,1786	0,1683	0,1597
9,7219	0,2260	0,1923	0,1773	0,1671	0,1586
9,8281	0,2244	0,1909	0,1761	0,1659	0,1575
9,9344	0,2227	0,1895	0,1748	0,1648	0,1564
10,0406	0,2210	0,1880	0,1734	0,1635	0,1553
10,1469	0,2193	0,1865	0,1721	0,1623	0,1542
10,2531	0,2175	0,1850	0,1707	0,1611	0,1530
10,3594	0,2157	0,1834	0,1693	0,1598	0,1518
10,4656	0,2139	0,1819	0,1679	0,1585	0,1506

10,5719	0,2121	0,1803	0,1665	0,1572	0,1494
10,6781	0,2102	0,1787	0,1651	0,1559	0,1482
10,7844	0,2083	0,1770	0,1636	0,1545	0,1469
10,8906	0,2064	0,1754	0,1621	0,1532	0,1457
10,9969	0,2044	0,1737	0,1606	0,1518	0,1444
11,1031	0,2025	0,1720	0,1590	0,1504	0,1431
11,2094	0,2004	0,1702	0,1575	0,1489	0,1417
11,3156	0,1984	0,1684	0,1559	0,1475	0,1404
11,4219	0,1963	0,1666	0,1543	0,1460	0,1390
11,5281	0,1942	0,1648	0,1526	0,1445	0,1376
11,6344	0,1920	0,1630	0,1509	0,1429	0,1362
11,7406	0,1898	0,1611	0,1492	0,1414	0,1348
11,8469	0,1876	0,1591	0,1475	0,1398	0,1333
11,9531	0,1853	0,1572	0,1458	0,1382	0,1318
12,0594	0,1830	0,1552	0,1440	0,1366	0,1303
12,1656	0,1806	0,1532	0,1422	0,1349	0,1288
12,2719	0,1782	0,1511	0,1404	0,1332	0,1272
12,3781	0,1758	0,1490	0,1385	0,1315	0,1256
12,4844	0,1733	0,1469	0,1366	0,1298	0,1240
12,5906	0,1708	0,1448	0,1346	0,1280	0,1223
12,6969	0,1682	0,1426	0,1327	0,1262	0,1207
12,8031	0,1656	0,1403	0,1307	0,1244	0,1190
12,9094	0,1630	0,1380	0,1287	0,1225	0,1173
13,0156	0,1603	0,1357	0,1266	0,1206	0,1155
13,1219	0,1575	0,1334	0,1245	0,1187	0,1137
13,2281	0,1547	0,1310	0,1223	0,1167	0,1119
13,3344	0,1518	0,1285	0,1202	0,1147	0,1100
13,4406	0,1489	0,1260	0,1179	0,1127	0,1081
13,5469	0,1459	0,1235	0,1157	0,1106	0,1062
13,6531	0,1429	0,1209	0,1134	0,1085	0,1042
13,7594	0,1398	0,1183	0,1110	0,1064	0,1022
13,8656	0,1366	0,1156	0,1086	0,1042	0,1002
13,9719	0,1334	0,1128	0,1062	0,1019	0,0981
14,0781	0,1301	0,1100	0,1037	0,0996	0,0960
14,1844	0,1267	0,1072	0,1011	0,0973	0,0938
14,2906	0,1232	0,1043	0,0985	0,0949	0,0916
14,3969	0,1197	0,1013	0,0959	0,0925	0,0893
14,5031	0,1160	0,0983	0,0932	0,0900	0,0870
14,6094	0,1123	0,0951	0,0904	0,0874	0,0847
14,7156	0,1085	0,0920	0,0875	0,0848	0,0822
14,8219	0,1046	0,0887	0,0846	0,0821	0,0797
14,9281	0,1006	0,0854	0,0817	0,0794	0,0772
15,0344	0,0965	0,0819	0,0786	0,0766	0,0746
15,1406	0,0922	0,0784	0,0755	0,0737	0,0719
15,2469	0,0878	0,0748	0,0722	0,0707	0,0691
15,3531	0,0833	0,0711	0,0689	0,0677	0,0663

15,4594	0,0786	0,0673	0,0655	0,0645	0,0633
15,5656	0,0738	0,0634	0,0620	0,0613	0,0603
15,6719	0,0688	0,0594	0,0583	0,0579	0,0571
15,7781	0,0637	0,0552	0,0546	0,0544	0,0539
15,8844	0,0583	0,0509	0,0507	0,0508	0,0505
15,9906	0,0527	0,0464	0,0466	0,0470	0,0470
16,0969	0,0469	0,0418	0,0424	0,0431	0,0433
16,2031	0,0409	0,0369	0,0379	0,0390	0,0394
16,3094	0,0346	0,0319	0,0333	0,0346	0,0353
16,4156	0,0280	0,0266	0,0284	0,0300	0,0310
16,5219	0,0212	0,0210	0,0232	0,0251	0,0264
16,6281	0,0142	0,0151	0,0177	0,0198	0,0213
16,7344	0,0070	0,0090	0,0117	0,0140	0,0157
16,8406	0,0004	0,0028	0,0054	0,0077	0,0094
16,9469	0,0000	0,0000	0,0000	0,0000	0,0000

The attached **CD-ROM** contains ...
1.) ... the text of this project as PDF-file and as World-file.

2.) ... all spread sheets IDRAG files and Matlab scripts:
- IDRAG program file,
- IDRAG input files,
- IDRAG output files,
- Matlab scripts.

3.) ... Literature according to the Chapter “References”