ESTIMATING THE OSWALD FACTOR FROM BASIC AIRCRAFT GEOMETRICAL PARAMETERS

M. Niţă, D. Scholz
Hamburg University of Applied Sciences
Aero – Aircraft Design and Systems Group
Berliner Tor 9, 20099 Hamburg, Germany

Abstract

This paper provides a sufficient accurate estimation method for the span efficiency factor, e, during the preliminary design stage. First, several approaches are identified from an exhaustive literature study and the accuracy and logic of these approaches are assessed. Second, a simple and logical general form is proposed as evaluation method for the Oswald factor for conventional configurations. The method starts with the calculation of a theoretical Oswald factor that accounts for the two parameters taper ratio and sweep – one depending on the other for a maximum Oswald factor. It then corrects the theoretical Oswald factor for the fuselage influence, zero lift drag influence and Mach number influence, making use of statistical aircraft data. Third, the paper delivers a method to calculate the Oswald factor for non-planar configurations, hence covering both conventional and unconventional designs. Out of the non-planar configurations, more attention is given to the wing with winglets, wing with dihedral and the box wing. For the box wing configuration an additional equation is proposed. The method proposed for estimating the Oswald factor is simple and accurate, with deviations under 4 %. It requires only few basic aircraft geometrical parameters and is useful for an efficient implementation into code for aircraft design optimization.

1. INTRODUCTION

There are different sources of drag for an airplane. The summation of all these effects composes the total drag. Drag breakdowns may vary from one author to another, yet it is agreed that there is a:

– zero-lift drag term and a
– lift-dependent drag term.

Additional terms are compressibility drag, trim drag, interference drag or gap drag [1], [2], [3].

Estimating drag coefficients is required already during the preliminary design stage. The zero lift drag coefficient $C_{D,0}$ has gained much more attention in aircraft design literature compared to the induced drag coefficient $C_{D,i}$.

$C_{D,i}$ is expressed mostly with help of the parameter e or the term $(1 + \delta)$. e is called the span efficiency factor or Oswald efficiency factor "after Dr. W.B. Oswald who first used it" [2]. e is used most often as a parameter to account for induced drag. This is why this paper focuses on the Oswald factor e and has it in the title.

(1) $$C_D = C_{D,0} + C_{D,i} = C_{D,0} + \frac{C_L^{\,2}}{\pi A e} = C_{D,0} + \frac{C_L^{\,2}}{\pi A}(1+\delta)$$

In Equation (1) C_L is the lift coefficient and A is the aspect ratio.

For preliminary sizing in aircraft design usually standard fixed values are applied for e without further reference to the aircraft geometry. It seems that more can be done.

[4] estimates that drag due to lift amounts to about 40 % of the total drag of a typical transport aircraft in cruise conditions. During the second segment climb, the induced drag can be 70 % to 80 % of the total drag [4]. It is thus important to accurately estimate this amount already in the preliminary design phase.

Disregarding this importance, it seems also that efforts to reduce induced drag are given less attention than efforts to reduce zero lift drag from the side of aircraft manufacturers or academia. Hence, this paper tries to focus on the (forgotten) basics of induced drag calculation as a first step. Future publications will show the integration of meaningful induced drag reduction techniques into preliminary aircraft design optimization.

2. THEORETICAL BACKGROUND

The lift-dependent drag term has two components [2], [3]:

– an **inviscid** part, which is caused by induced velocities from the wake (also called **vortex** drag); it includes the effect of a zero-lift term due to wing twist;
– a **viscous** part, which is caused by increases in skin friction and pressure drag due to changes in angle of attack. It hence depends on wing parameters (such as leading edge geometry, camber, thickness ratio, sweep) and on the interference of other aircraft components with the wing flow (pylon interference, fuselage upsweep, tail induced drag, engine power effects, etc.).

Equation (1) is the common way to express the induced drag coefficient. The Oswald factor, e, accounts for any deviation from an ideal elliptical lift distribution, for which this factor is 1. After looking at different authors, (presented in Section 3), it was found suitable to express the lift-dependent drag (as a parabolic variation with C_L^2) in the following form:

(2) $$C_{D,i} = \left(\frac{Q}{\pi A} + P\right) \cdot C_L^{\ 2} .$$

From Equation (2) the Oswald factor e (for the whole aircraft) can be extracted in the form:

(3) $$e = \frac{1}{Q + P\pi A} .$$

The term Q covers the **inviscid** part of the induced drag coefficient. $e_{inviscid} = 1/Q$ is just the inviscid part of the Oswald factor without consideration of other effects on the aircraft.

The term P is used to express the **viscous** part of the induced drag coefficient (in addition to the viscous drag in the zero lift drag coefficient).

The next section presents an exhaustive literature study on lift-dependent drag estimation methods available for preliminary aircraft design. The equations were evaluated for a set of aircraft, for which the Oswald factor values were known from literature (see Appendix A).

3. LITERATURE STUDY

Most of the authors give expressions of the Oswald factor for aircraft design as in form (3), proposed before. Others give empirical solutions, following wind tunnel data, or by sampling a virtual design space and using regressions.

Obert [3] gives an empirical diagram, obtained from flight testing a number of modern and less modern aircraft. He proposes (3) to calculate e:

(4) $$e = \frac{1}{1.05 + 0.007\pi A} ,$$

and gives constant values for the factors Q and P: $Q = 1.05$ and $P = 0.007$. This yields a value $e_{inviscid} = 0.95$ and with a typical value for $C_{D,0} = 0.02$ a parameter K as in (7) of 0.35.

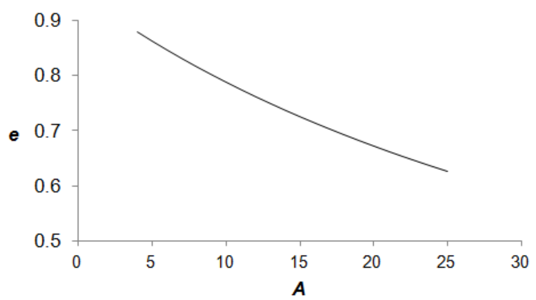


FIG. 1 Oswald variation with aspect ratio based on Obert

A more accurate approach is given by **Kroo** [2]. The **inviscid** part is:

(5) $$Q = \frac{1}{u \cdot s} .$$

The factor u can be taken, according to [2], as 0.99. u represents the theoretical Oswald factor and is defined as $u = e_{theo}$ that only considers the wing, without additional corrections, such as fuselage influence or Mach number influence. e_{theo} has been calculated from lifting line theory. Equations and diagrams are given in Section 3 and Section 5.

The factor s is a factor that accounts for the added lift-dependent drag caused by the modification of the span loading due to the presence of the fuselage. Kroo [2] expresses this factor as a function of the fuselage diameter, d_F and wing span, b:

(6) $$s = 1 - 2 \cdot \left(\frac{d_F}{b}\right)^2 .$$

For diameter-to-span ratios greater than 0.7 the s factor and, respectively the inviscid Oswald factor, $e_{inviscid} = u \cdot s$, is zero. The *theoretical* s factor, given by $s = 1 - (d_F / b)^2$, has the value of zero for $d_F / b = 1$ (see FIG. 2).

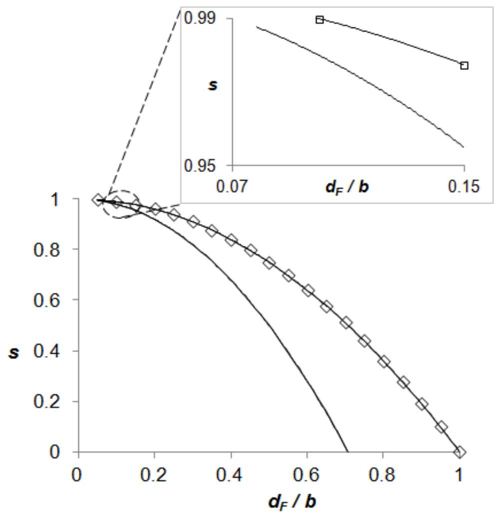


FIG. 2 s factor from Equation (6) (continuous line) and theoretical s factor (dashed line). Zoomed in: the domain of typical values.

The **viscous** part, P, is given by [2]:

(7) $$P = K \cdot C_{D,0} ,$$

where K was determined from flight test data for the DC-8-62 and 63 and for the DC-9-10, -20, and -30 airplanes to be approximately 0.38 [2]. This value was confirmed by own research, based on several other aircraft, including propeller and general aviation aircraft (see Appendix A).

The expression of the Oswald factor for the whole aircraft is, following [2]:

(8) $$e = \frac{1}{\frac{1}{u \cdot s} + K C_{D,0} \pi A} .$$

If the *wing twist, θ*, is accounted for, the inviscid drag coefficient has two additional terms, which may be generally expressed via the factors v and w as in relation (9) (this general form was derived based on Kroo [2] and Dubs [5]).

(9) $$C_{D,inviscid,\,twist} = C_{L_\alpha}\theta \cdot C_L \cdot v + (C_{L_\alpha}\theta)^2 \cdot w\,.$$

The Oswald factor based on [2] becomes, compared to Equation (8):

(10) $$e = \frac{1}{\frac{1}{u \cdot s} + \frac{\pi A \cdot C_{L_\alpha}\theta \cdot v}{C_L} + \frac{\pi A \cdot (C_{L_\alpha}\theta)^2 \cdot w}{C_L{}^2} + KC_{D,0}\pi A}\,,$$

i.e. the inviscid term is now:

(11) $$Q = \frac{1}{s \cdot u} + \frac{\pi A \cdot C_{L_\alpha}\theta \cdot v}{C_L} + \frac{\pi A \cdot (C_{L_\alpha}\theta)^2 \cdot w}{C_L{}^2}\,,$$

while P is the same as in Equation (7).

The lift curve slope can be approximated with [6]:

(12) $$C_{L_\alpha} = \frac{2\pi A}{2 + \sqrt{A^2 \cdot \left(1 + \tan^2 \varphi_{50} - M^2\right) + 4}}\,.$$

In Equation (12), φ_{50} is the 50 % chord line sweep, and M is the Mach number.

The lift curve slope drops with increased sweep, especially at higher aspect ratios.

The twist influence is, however, quite small and is neglected by most of the authors. [2] gives little information on how to compute the twist terms, respectively the v and w factors. An approach was found in [5], but it delivered realistic results only for aircraft with high taper ratios. Böhnke [7] includes the twist angle in his proposed equation for the Oswald factor estimation, but his equation is only valid for a limited interval.

Stinton [8], explained in view of Section 2 with Equation (2) and (3), is such that $Q = 1/e_{inviscid} = K$ with $e_{inviscid}$ having values of 0.83 for "normal" aircraft and 0.95 for gliders[1]. $P = m$ and $m\pi A = 0.25\ ...\ 0.45$:

(13) $$e = \frac{1}{\frac{1}{e_{inviscid}} + m\pi A} = \frac{1}{K'}\,.$$

Schaufele [9] gives no equation, but a plot based on which e can be determined, for given aspect ratio and profile drag. The plot (see FIG. 3) is very similar to the one that can be produced based on Kroo's from [2], in relation (8) (see the plot in [2]). The value of K extracted from [9] is almost the same as the value of Kroo, respectively $K = 0.379$. Q can be extracted form the diagram as 1.03.

Extracting the equations from FIG. 3 and using them for the aircraft in Appendix A, results in a deviation from reference Oswald factors of 12 % on average.

Another author that gives an Oswald factor equation in a similar form as in Kroo [2] and Stinton [8] is **Grosu** [10]. He expresses the drag coefficient as:

(14) $$C_D = C_{D,0} + 0.028 \cdot (t/c)_{\max} + 1.08\frac{C_L{}^2}{\pi A}\,.$$

The Oswald factor, written after expression (3), is then:

(15) $$e = \frac{1}{1.08 + \frac{0.028 \cdot (t/c)_{\max}}{C_L{}^2} \cdot \pi A}\,.$$

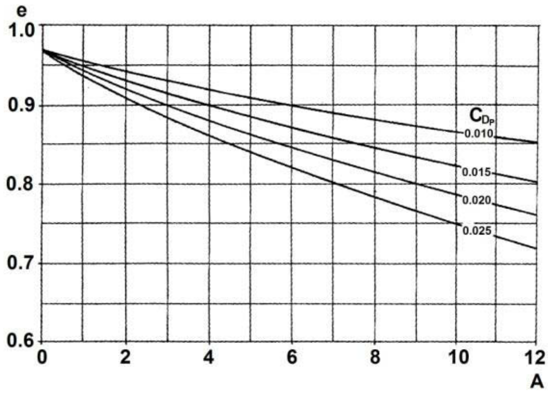


FIG. 3 Schaufele's diagram for calculating the Oswald factor [9]

The author gives a very useful estimation method in preliminary design for the lift coefficient:

(16) $$C_L = \frac{5.53 \cdot A}{A + 1.76} \cdot \alpha_{eff}\,,$$

where α_{eff} is the effective angle of attack. The effective angle of attack is the sum between the angle of attack, α, and the induced angle of attack, α_i, due to an induced velocity, w, which is the difference between the velocity of the airplane and the velocity perpendicular to the leading edge, V_∞. It is: $\alpha_{eff} = \alpha + \alpha_i$; $\alpha_i \cong \tan\alpha_i = \frac{w}{V_\infty}$.

Evaluating this equation for a number of aircraft from the available aircraft database (see Appendix A) showed a good accuracy towards literature values for the Oswald factor.

Since the dependency of the Oswald factor on thickness ratio was not found at other authors, it is important to underline this method as a useful approach. Thickness ratio is plotted in FIG. 4 for constant aspect ratio and angle of attack. Higher (maximum) thickness ratios are useful for reducing the wing bending moments. A wing with a higher thickness allows a higher aspect ratio, which overall reduces lift dependent drag. Yet the Oswald factor, as indicated by FIG. 4, reduces with increasing thickness ratio.

[1] Q is called K in [8], not to be confused with K in Equation (7). K' is the modified parameter K in [8]

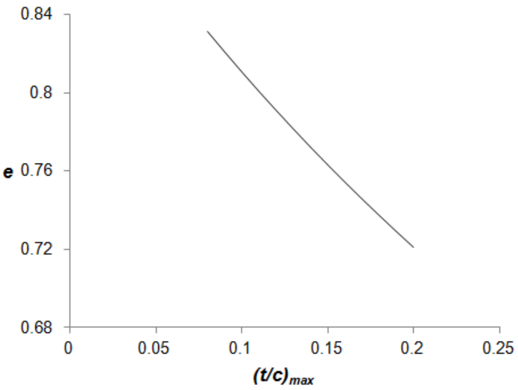


FIG. 4 Oswald factor e plotted versus thickness ratio for $A = 9.5$ and $C_L = 0.74$ based on [10]

Hörner (page 7-4) provides a function $f(\lambda) = \delta / A$, $\delta = k$ in [11]. This function is given in FIG. 5.

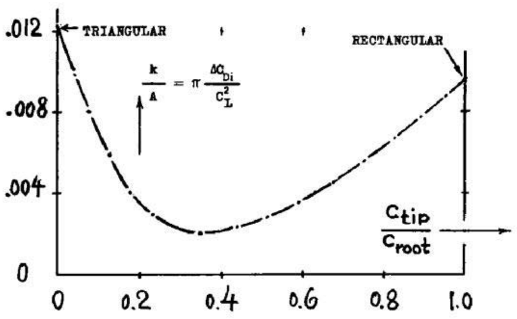


FIG. 5 $f(\lambda) = \delta / A$ from [11]

Hörner [11] intends to provide a theoretical Oswald factor (without further corrections) which can play the role of u in Equation (8). It is:

(17) $$e_{theo} = \frac{1}{1 + f(\lambda) \cdot A} .$$

Hörner [11] accounts for additional effects on span efficiency, other than taper and aspect ratio, by proposing a set of corrections to the theoretical Oswald factor. As such, the author gives corrected values, e' for sweep angle (negative and positive), wing tip tanks and end plates, flaps and dihedral. Examples are:

$e' = e \cdot \cos \varphi$ - sweep angle effect,

$e' = e \cdot \cos^2 \Gamma$ - dihedral angle effect.

Hörner [11] also considers the twist effect, but only quantifies its negative influence through the following relation (without specifying the benefits of twist):

(18) $$\Delta C_{D,i} = 4(\Delta \alpha)^2 / 10^5 ,$$

where $\Delta \alpha$, in degrees, is the wing twist i.e. the difference in angle of attack of the wing tips against the angle of attack at the wing center part.

McCormick [12] and **Dubs** [5] produced similar diagrams to that of Hörner [11]. The diagrams of both authors, [12] and [5], are presented in FIG. 6 and FIG. 7.

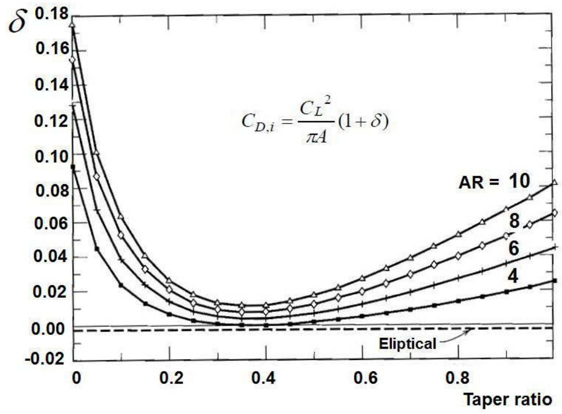


FIG. 6 Diagram for calculating the theoretical Oswald efficiency factor, $e_{theo} = 1 / (1+\delta)$ from [12]

In FIG. 7 the term δ_G is represented as a function of taper ratio (horizontal axis) and aspect ratio-to-profile efficiency, noted with Λ/η. No data is given on profile efficiency η and so it is set to 1.

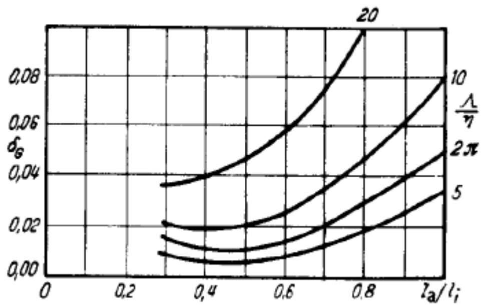


FIG. 7 Diagram for calculating the theoretical Oswald efficiency factor, $e_{theo} = 1 / (1+\delta_G)$ from [5]

Dubs's diagram is only valid for taper ratios higher than 0.3. A closer look reveals that otherwise FIG. 6 and FIG. 7 are identical for $\eta = 1$. Also Hörner with FIG. 5 leads to the same results.

The values of e obtained from FIG. 5, FIG. 6 or FIG. 7 are unrealistically high. Comparing with aircraft data leads to the conclusion that all these results are theoretical. McCormick [12] states that his results are based on lifting line theory. So they need further correction.

ESDU [13] proposes a series of diagrams (for a number of taper ratios) from which Oswald factor values can be deducted as:

(19) $$e = \frac{1}{1 + \delta} .$$

These diagrams are the result of a simplified theoretical approach. Their drawback is the difficulty in reading data, if one cannot refer directly to the electronic version of the calculation. The term $(1+\delta)$ is represented as a function of aspect ratio, A, compressibility factor, $\beta = \sqrt{1 - M^2}$ and sweep, φ_{50}, for a constant taper ratio (an example is given in FIG. 8).

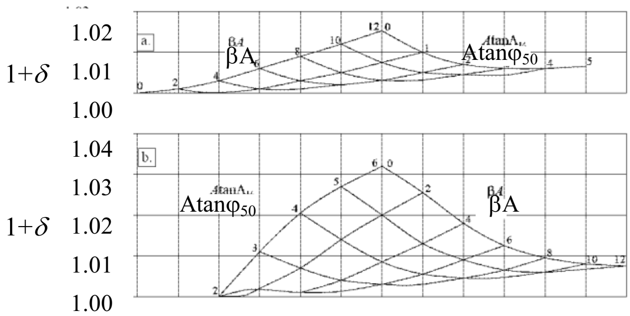


FIG. 8 ESDU diagram for estimating the Oswald factor, valid for taper ratio of 0.25 [13]

All the diagrams in [13] deliver unrealistically high values for the Oswald factor (with deviations of up to 28 % from literature values of e). Yet, from this theoretical approach, dependencies between the span efficiency factor and other factors can be confirmed. When the taper ratio is small, a better efficiency (i.e. a higher e) is obtained for a higher sweep angle. For high taper ratios, a smaller sweep is better. In other words, there is an optimal taper ratio for every sweep angle, that minimizes the induced drag and these two parameters should be accounted for together. This will be discussed further in Section 5.

Torenbeek [14] cites three methods in his book *Synthesis of Subsonic Airplane Design*. The author expresses the Oswald factor in terms of δ as indicated before in (1):

1.) Garner**,** who calculates δ as a function of the spanwise center of pressure, η_{cp}:

(20) $\delta = 46.264 \cdot \left(\eta_{cp} - \frac{4}{3\pi} \right)^2$, with:

$$\eta_{cp} = C_1 \cdot \frac{1+2\lambda}{3(1+\lambda)} + (C_2 + C_3) \cdot \frac{4}{3\pi} + 0.001 \cdot \varphi_{25,\beta} \cdot C_3 ,$$

where $\varphi_{25,\beta}$ is the corrected sweepback angle:

$$\tan(\varphi_{25,\beta}) = \tan(\varphi_{25}) / \beta$$

$$\beta = \sqrt{1 - M^2} .$$

The drawback of this method lies, again, in reading values from a diagram for the coefficients C_1, C_2 and C_3 (as given in FIG. 9).

2.) Anderson, who proposes an expression for δ, which is only valid for unswept wings:

(21) $\delta = \left[0.0015 + 0.016 (\lambda - 0.4)^2 \right] (\beta A - 4.5)$,
for $6 < \beta A < 30;\ 0.3 < \lambda < 1.0;\ \varphi_{25} = 0$

3.) Labrujere, who, with the aid of an NLR computer program, obtains charts (FIG. 10) from which, for different taper ratios (between 0.2 and 0.5), aspect ratios and sweeps, the δ factor can be read. This approach is similar to the ESDU method, so it will not be discussed in detail.

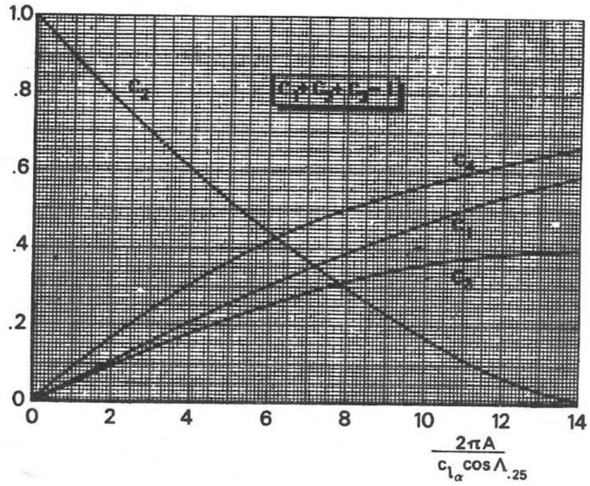


FIG. 9 Garner's diagram for the coefficients C_1, C_2 and C_3 (quoted from [14])

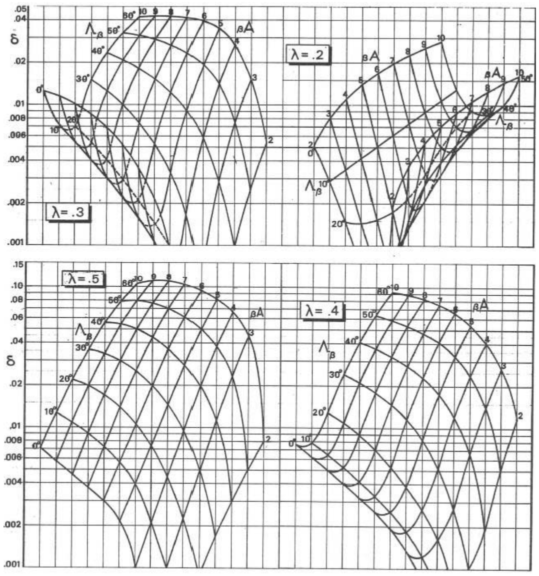


FIG. 10 δ factor according to Labrujere [14]

Niţă & Patraulea [15]. For trapezoidal wings, in incompressible regime, the authors give the following estimation method for δ, from which the Oswald factor can be calculated as before:

(22) $\delta \cong 3 \left(\frac{\beta_2 - \beta_4}{3\mu_0 + \beta_0} \right)^2 + 5 \left(\frac{\beta_2}{5\mu_0 + \beta_0} \cdot \frac{\beta_2 - \beta_4}{3\mu_0 + \beta_0} - \frac{\beta_4}{5\mu_0 + \beta_0} \right)^2$

with:

$$\mu_0 = \frac{\left(C_{L_\alpha} \right)_\infty}{2A} \cdot \frac{(1/\lambda)}{(1/\lambda)+1}$$

$$\beta_0 = 0.5 \left(\frac{0.383}{1 - 0.924q} + \frac{0.924}{1 - 0.383q} \right)$$

(23) $\beta_2 = \frac{0.707}{2} \left(\frac{0.383}{1 - 0.924q} - \frac{0.924}{1 - 0.383q} \right)$

$$\beta_4 = 0.25 \left(\frac{0.383}{1 - 0.924q} + \frac{0.924}{1 - 0.383q} \right) - \frac{1}{2} \cdot \frac{0.707}{1 - 0.707q}$$

$$q = 1 - \lambda;\ \lambda = c_t / c_r$$

The minimum value for δ is obtained for:

(24) $\beta_2 - \beta_4 \cong 0.0183 \dfrac{3\mu_0 + 1.07}{(5\mu_0 + 1.07)^3}$.

The lift curve slope can be approximated with Equation (12).

This method delivered high values for the Oswald factor, which lead to the conclusion that the result is a theoretical Oswald factor that needs to be further corrected.

Jenkinson [16] gives a 3D chart displaying a theoretical distribution of the induced drag as a function of the aspect and taper ratios.

The factor C_1 resulting from FIG. 11, is corrected with the factor C_2:

(25) $C_2 = 1.235 - 0.0245A$ for old wing designs,

(26) $C_2 = 1.113 - 0.0116A$ for new wing designs.

The Oswald factor is ultimately given by:

(27) $e = C_2 / C_1$.

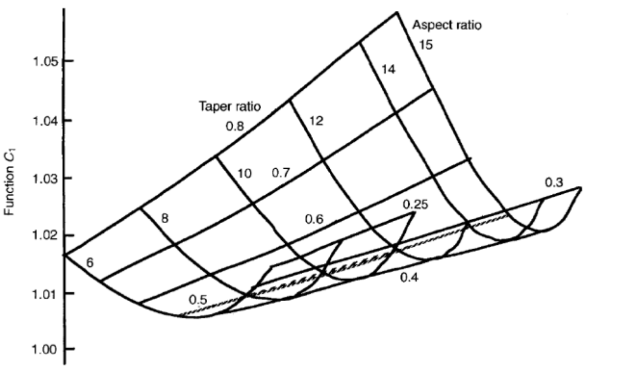


FIG. 11 Jenkinson's diagram for estimation the factor C_1 required to calculate the Oswald factor [16]

By splitting the 3D plot in 2D equations, and by interpolating (where necessary) for the taper ratios of the airplanes from the database in Appendix A, it was found that, compared to literature values, the deviations for the respective aircraft, were on average 26.6 %.

Samoylovitch [17] proposes an equation that accounts for the influence of the leading edge suction force, as well as the fuselage cross section, through the form factor k_F:

$$e = e_w k_F$$

(28) $$e_w = \frac{e_{w/S_e=1}\, e_{w/S_e=0}}{S_e\, e_{w/S_e=0} - (1 - S_e)\, e_{w/S_e=1}} .$$

In the above equation, S_e is the relative leading edge suction force. The terms $e_{w/S_e=1}$ and $e_{w/S_e=0}$ represent the wing Oswald factor calculated by considering and, respectively, by not considering the suction force. The author proposes in [17] a simplified model to calculate the required terms of Equation (28). The results present rather high inaccuracies, with deviations from (author's) experimental values of 12.7 %, on average [17].

Howe [18] proposes the following equation for preliminary aircraft design:

(29) $$e = \frac{1}{\left(1 + 0.12 \cdot M^2\right)\left(1 + \dfrac{0.142 + f(\lambda) \cdot A \cdot (10 \cdot t/c)^{0.33}}{(\cos\varphi_{25})^2} + \dfrac{0.1 \cdot (3N_e + 1)}{(4 + A)^{0.8}}\right)}$$

with:

$$f(\lambda) = 0.005 \cdot \left(1 + 1.5 \cdot (\lambda - 0.6)^2\right).$$

Among the factors he includes for estimating e, there is also the thickness ratio that was found in [10]. The Oswald factor increases with increased aspect ratio and drops with increased thickness ratio. Compared to the ESDU method, the influence of the taper ratio is smaller, and that of sweep angle stronger. As it can be noticed in FIG. 12, the two parameters seem not to be coupled, as it should be.

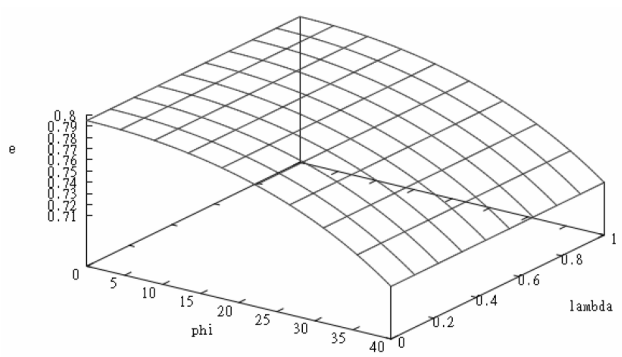


FIG. 12 Variation of Oswald factor with sweep and taper ratio, based on Howe [18], for $M = 0.82$, $A = 9.39$, $(t / c) = 0.117$

In FIG. 12 it can be noticed that the influence of sweep is stronger, compared to the influence of taper ratio (in contrast to the ESDU method, where an optimal taper ratio is to be found for each sweep, hence minimizing the induced drag).

When comparing with reference values of real aircraft, Howe's equation delivers realistic results, with deviations under 10 %.

Other authors have tried different approaches for estimating the Oswald factor. **Böhnke** [7] constructed an equation with the aid of Eureqa, a symbolic regression toolbox [19], which post-processed data gathered from a Latin Hypercube sampling of a virtual design space given by the following variables: aspect ratio, A, quarter chord sweep, φ_{25}, taper ratio, λ, twist angle, θ and kink ratio, η_k:

(30) $$e = 0.04 - 0.0007 \cdot A - 0.00019 \cdot \varphi_{25} \cdot \theta + \lambda^{0.03} \cdot \cos(0.16 - 0.0007 \cdot A \cdot \varphi_{25} - 0.0007 \cdot A \cdot \varphi_{25} \cdot \eta_k - 0.55 \cdot \lambda)$$

Yet, outside the given limits, the equation is invalid. Also, compared to aircraft data from literature (given in Appendix A), it produces rather high deviations (of about 20 %), giving higher values for the Oswald factor. This leads to the conclusion that the result represents a theoretical Oswald factor that would need additional corrections to match statistics.

Brandt [20] presents an equation fitting a curve of experimental wind tunnel data. The variables are aspect ratio and leading edge sweep angle:

(31) $e = 4.61 \cdot (1 - 0.045 A^{0.68}) \cdot (\cos \varphi_{LE})^{0.15} - 3.1$

For typical sweep angles, the equation is valid for aspect ratios between about 4 and 15. It yields, however, an Oswald factor e much too low for typical civil aircraft aspect ratios. Applying the equation to the set of aircraft (given in Appendix A), results in, on average, a deviation of about 24 %.

Raymer [21] proposes two sets of empirical equations, for straight and swept wings:

(32) $e = 1.78 \cdot (1 - 0.045 \cdot A^{0.68}) - 0.64$, for straight wings,

(33) $e = 4.61 \cdot (1 - 0.045 \cdot A^{0.68}) \cdot (\cos \varphi_{LE})^{0.15} - 3.1$, for swept wings with φ_{LE} > 30°.

Equation (32) is not valid for $A < 2.27$, where $e > 1$. Equation (33) leads to unrealistic low values for $A > 10$. The average deviation from the aircraft values listed in Appendix A is 15 %.

DATCOM [22] proposes the following empirical equation, accounting for the leading edge suction force and the planform geometry of the wing:

(34) $$e = \frac{1.1\left(C_{L_\alpha} / A\right)}{R\left(C_{L_\alpha} / A\right) + (1 - R)\pi}$$.

The equation is applicable to twisted, sweptback wings, with straight-tapered planforms. C_{L_α} is the lift curve slope and R is a leading edge suction parameter, defined as the ratio of leading edge suction actually attained, to that theoretically possible. R depends on leading edge radius Reynolds number, Re_{LER}, Mach number, M, aspect ratio, A and sweepback, φ_{LE} (see diagram in FIG. 13).

The DATCOM method delivers good results. The disadvantage is the difficulty to implement the method in an automatic calculation environment, as values need to be manually red from the diagram in FIG. 13.

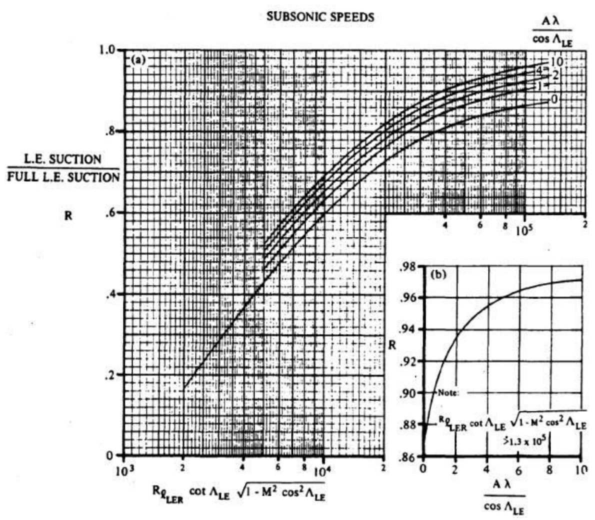


FIG. 13 Diagram for calculating the R factor [22]

4. SUMMARY OF EXISTING METHODS

In the previous section most of the available methods for estimating the Oswald factor in preliminary aircraft design were evaluated. Literature values were compared with calculated values and deviations were presented.

The general, theoretically grounded form to express the Oswald factor is: $e = \frac{1}{Q + P\pi A}$, which accounts for the inviscid part (Q) and the viscous part (P) of the induced drag. The inviscid part can usually be expressed as an inviscid (theoretical) Oswald factor, $e_{inviscid}$. Especially these authors for this approach should be named: Kroo [2], Stinton [8], Grosu [10] and Obert [3].

Other authors express the Oswald factor in terms of δ. It is

$$C_{D,i} = \left(\frac{Q}{\pi \mathrm{A}} + P\right) C_L^{\,2} = \frac{C_L^{\,2}}{\pi \mathrm{A}}(1 + \delta) = \left(\frac{1}{\pi \mathrm{A}} + \frac{\delta}{\pi \mathrm{A}}\right) C_L^{\,2} .$$

One way to interpret this is with $Q = 1$ and $P = \frac{\delta}{\pi A}$. However, if the respective author is just considering the theoretical induced drag, then $Q = 1 + \delta$ and $P = 0$. So, $e_{theo} = \frac{1}{1 + \delta}$.

In other cases authors use empirical data from wind tunnel tests or make use of mathematical regressions to match an equation to a virtual design space. This type of equations is valid only for a specific domain, and in some of the cases, it is difficult to read the diagrams which are given.

Twist influence is by most of the authors neglected, or only partially covered. Hörner [11], for instance, only covers the negative effect, while Dubs [5] and Kroo [2] give limited data.

TAB. 1 and TAB. 2 present an overview of the entire section of the literature study.

TAB. 1 Summary of literature study (1 of 2 tables)

Author	Ref.	Scope	Q	P	Remarks
Obert	[3]	whole aircraft	$Q=1.05$	$P=0.007$	Constant values based on experimental data.
Kroo	[2]	whole aircraft	$Q=1/(u\cdot s)$	$P=K\cdot C_{D,0}$	The product ($u{\cdot}s$) represents the inviscid part of the induced drag. $u=e_{theo}$ is the theoretical Oswald factor. s accounts for the fuselage interference. The viscous part of the induced drag is proportional to the (viscous) zero lift drag coefficient. $K=0.38$.
Stinton	[8]	whole aircraft	$Q=K$	$P=m$	Values are indicated for K (K from [8] may not be confused with K from [2] and [9]) and $m\pi A$
Schaufele	[9]	whole aircraft	$Q=1.03$	$P=K\cdot C_{D,0}$	Data given in the form of a diagram. The viscous part of the induced drag is proportional to the (viscous) zero lift drag coefficient. $K=0.38$.
Hörner	[11]	inviscid, theoretical	$Q=1+A\cdot f(\lambda)$	$P=0$	$\delta/A=f(\lambda)$ given in the form of a diagram.
McCormick	[12]	inviscid, theoretical	$Q=1+\delta$	$P=0$	$\delta=f(\lambda,A)$ given in the form of a diagram.
Dubs	[5]	inviscid, theoretical	$Q=1+\delta$	$P=0$	$\delta=f(\lambda,A/\eta)$ given in the form of a diagram.
ESDU	[13]	inviscid, theoretical	$Q=1+\delta$	$P=0$	Data given in the form of diagrams. Requires time consuming manual process. The result represents a theoretical Oswald factor
Garner	[14]	inviscid, theoretical	$Q=1+\delta$	$P=0$	Method taken from Torenbeek. See original for further details.
Anderson	[14]	inviscid, theoretical	$Q=1+\delta$	$P=0$	Method taken from Torenbeek. See original for further details.
Labrujere	[14]	inviscid, theoretical	$Q=1+\delta$	$P=0$	Method taken from Torenbeek. See original for further details.
Niţă & Patraulea	[15]	inviscid, theoretical	$Q=1+\delta$	$P=0$	Empirical, laborious method, which results in theoretical values, that need to be further corrected.
Jenkinson	[16]	whole aircraft	$Q=\frac{C_1}{C_2}$	$P=0$	$C_1=f(\lambda,A)$ given in the form of a diagram. C_2 given in form of equations.

TAB. 2 Summary of literature study (2 of 2 tables)

Author	Ref.	Scope	Type of model	Remarks
Samoylovitch	[17]	whole aircraft	Theoretical	The theoretical background seems to be incomplete.
Howe	[18]	whole aircraft	Unknown	It is a good aircraft design equation, yet it lacks the coupling between taper ratio and sweep angle.
Böhnke	[7]	inviscid, theoretical	Nonlinear regression	The equation is obtained based on multiple lifting line method, sampling with Latin Hypercube and forming an equation with the help of the tool Eureqa. The result represents a theoretical Oswald factor. Equation is applicable only in the limited investigated design space.
Brandt	[20]	whole aircraft	Empirical	Equation is applicable only in a limited design space. Results in small Oswald factors. Large error.
Raymer	[21]	whole aircraft	Empirical	Equation is applicable only in a limited design space. Depends only on aspect ratio. Error quite large.
DATCOM	[22]	whole aircraft	Empirical	Data given in the form of diagrams. Requires time consuming manual process.

5. PROPOSED METHOD FOR OSWALD FACTOR ESTIMATION

5.1. Optimum Combination of Taper Ratio and Sweep Angle

Authors analyzed in Section 3 (some more, such as ESDU [13], some less, such as Howe [18]) have accounted for the *coupling* between the *taper ratio* and the *sweep angle*. For each sweep, there is an optimal taper ratio that minimizes the induced drag, because it yields a close to elliptical lift distribution. Reference [23] delivers a curve that relates the taper ratio to sweep for an approximate elliptical loading (pp. 648, Figure 21). An equation that approximates this curve very well was found to be the following exponential equation (with *e* standing for Euler number):

(35) $\lambda_{opt} = 0.45 \cdot e^{-0.0375\varphi_{25}}$.

φ_{25} is the sweep angle of the 25 %-line in degrees. For unswept wings, the optimal taper ratio for achieving a near elliptical loading is 0.45.

5.2. Estimating a Theoretical Oswald Factor

In Section 3, $f(\lambda) = \delta / \mathrm{A}$ from Hörner [11] was introduced that delivered a theoretical Oswald factor (without corrections) e_{theo} form Equation (17). Taking values from FIG. 5 and curve fitting a fourth order polynomial, a function $f(\lambda)$ was found:

(36) $f(\lambda) = 0.0524\,\lambda^4 - 0.15\lambda^3 + 0.1659\lambda^2 - 0.0706\lambda + 0.0119$.

This equation produces the diagram of FIG. 14, which is practically identical to FIG. 5.

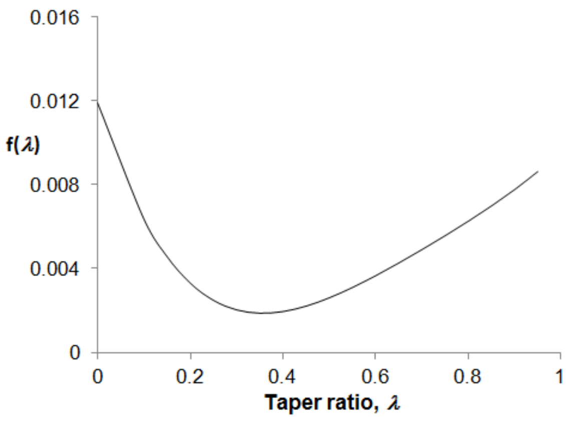


FIG. 14 Function f(λ) from (36)

Equation (17) and (36) as well as FIG. 5 and FIG. 14 are only valid for unswept wings. If wing sweep is present, the minimum from FIG. 14 has to be shifted to the respective optimum taper ratio. The problem is now that following (35) the optimum taper ratio for unswept wings is $\lambda_{opt} = 0.45$, whereas another optimum taper ratio is calculated from the derivative of (36): $\lambda_{opt} = 0.357$. We stick to the optimum from (35), which is a function of sweep. So the taper ratio, λ has to be shifted by an amount:

(37) $\Delta\lambda = -0.357 + 0.45 \cdot e^{0.0375\varphi_{25}}$.

So, Equation (17) is modified to:

(38) $e_{theo} = \dfrac{1}{1 + f(\lambda - \Delta\lambda) \cdot A}$.

Equation (38), with (37) and $f(\lambda - \Delta\lambda)$ from (36), represents the expression of a theoretical Oswald factor that accounts for the coupling between taper ratio and sweep. It can play the role of u in Kroo's [2] expression, or the role of an $e_{inviscid}$.

5.3. Corrections for Fuselage, Zero-Lift Drag and Mach Number Influence

The method proposed in this paper starts from the theoretical evaluation of the Oswald factor given in Equation (38) and then corrects it for the fuselage influence, zero lift drag influence and Mach number influence:

(39a) $e = e_{theo} \cdot k_{e,F} \cdot k_{e,D_0} \cdot k_{e,M}$.

As alternative method, based on Kroo [2], Equation (39b) is proposed:

(39b) $e = \dfrac{k_{e,M}}{Q + P\pi A}$,

with $Q = \dfrac{1}{e_{theo} \cdot k_{e,F}}$, $P = KC_{D,0}$ and $K = 0.38$.

Equation (39a) has the advantage that e can be calculated without knowledge of $C_{D,0}$. However, if the drag polar in form of Equation (1) has been established, also $C_{D,0}$ has to be known. This knowledge can be used in (39b) and may improve the accuracy of e further.

The factor $k_{e,F}$ depends on the ratio between fuselage diameter and span (is the same as s in [2]):

(40) $k_{e,F} = 1 - 2\left(\dfrac{d_F}{b}\right)^2$.

If data is available, the real ratio d_F / b can be used. Otherwise an average of 0.114 represents a realistic value for all aircraft types (see Appendix A).

The factor k_{e,D_0} is a statistical factor that accounts for the change of Oswald factor based on a change of zero lift drag roughly depending on the aircraft category (see TAB. 3). The factor was set by matching statistical data given in Appendix A to (39a).

TAB. 3 $k_{e,F}$ and k_{e,D_0} factors for each aircraft category

Aircraft category	d_F / b	$k_{e,F}$	k_{e,D_0}
All	0.114	0.974	-
Jet	0.116	0.973	0.873
Business Jet	0.120	0.971	0.864
Turboprop	0.102	0.979	0.804
General Aviation	0.119	0.971	0.804

The factor $k_{e,M}$ has the form:

(41) $$k_{e,M} = \begin{cases} a_e \left(\dfrac{M}{M_{comp}} - 1 \right)^{b_e} + 1, & M > M_{comp} \\ 1, & M \leq M_{comp} \end{cases},$$

$$a_e < 0; \quad b_e > 0$$

where M_{comp} stands for compressibility Mach number and has the constant value of 0.3. The constant terms a_e and b_e where determined statistically, based on a set of commercial transport aircraft for which aerodynamical data was available. The values are:

(42) $$\begin{aligned} a_e &= -0.001521 \\ b_e &= 10.82 \end{aligned}.$$

A Prandtl-Glauert correction was also considered, but it matched poorly with experimental data. Equation (41) allowed a better manipulation of the correction towards real aircraft data. FIG. 15 compares *e* and $k_{e,M}$ obtained with a Prandtl-Glauert correction, $k_{e,M} = \sqrt{1-M^2}$, and the correction in Equation (41), for the A320 aircraft. The form given in Equation (41) matches real data better.

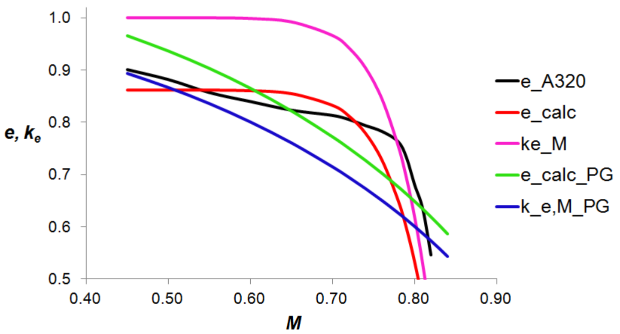


FIG. 15 Calculated Oswald factor (e_calc) and Mach number correction (ke_M) from Prandtl-Glauert (PG) and own estimation method (without further index), compared with Oswald factor calculated for A320 aircraft (e_A320)

The proposed method given as (39a) or (39b) is simple, logical and delivers quite accurate results for e and hence induced drag. It was observed that the influence of $C_{D,0}$ (alternatively expressed as $ke_{,D0}$) on e is of high importance together with the Mach influence already for $M > 0.3$. The average deviation from the literature values of Oswald factors e for the aircraft as given in TAB. A1 is under 4 %.

A drag polar (1) can be calculated even for subsonic cruise Mach numbers using the approach from this paper: 1.) An incompressible $C_{D,0}$ has to be calculated from one of many available methods. 2.) Wave drag influence on $C_{D,0}$ has to be estimated. [6] presents a simplified method for wave drag estimation. 3.) The Oswald factor has to be calculated from (39a) or (39b). The Mach influence is represented by $k_{e,M}$.

6. ESTIMATING THE OSWALD FACTOR FOR NON-PLANAR CONFIGURATIONS

Induced drag can be substantially reduced by increasing wing span. Kroo [24] calculates that a 10 % increase in span leads to a 17 % reduction in vortex drag at a fix speed and lift. The drawback of this strategy is, however, a substantially increased structural weight, and hence a higher cost. Non-planar configurations have the potential to reduce drag compared with planar wings of the same span and lift. But their assessment needs to account for both structural and aerodynamic characteristics.

The most known and currently used non-planar configuration is the wing with winglets.

6.1. Wing with Winglets

The simplest approach in understanding winglets is to consider the effect of the winglets equal to that of a wing that prolongs its span with the size of the winglets, as in FIG. 16.

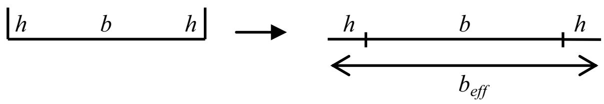


FIG. 16 Simple geometrical consideration for winglets evaluation

The following relations can be written:

(43) $$\begin{aligned} \frac{b_{eff}}{b} &= 1 + 2\frac{h}{b} \\ C_{D,i} &= \frac{{C_L}^2}{\pi A e} \\ C_{D,i,WL} &= \frac{{C_L}^2}{\pi A_{eff} e} = \frac{{C_L}^2}{\pi A e_{WL}} \\ e_{WL} &= \frac{A_{eff}}{A} \cdot e = \left(\frac{b_{eff}}{b} \right)^2 \cdot e \end{aligned}$$

Hence:

(44) $$e_{WL} = \left(1 + 2\frac{h}{b} \right)^2 \cdot e\,.$$

This simple geometrical consideration aids in understanding the phenomenon, but it is not accurate enough. Proposed is a penalization via a factor k_{WL}, which accounts for the effectiveness of winglets:

(45) $$e_{WL} = \left(1+\frac{2}{k_{WL}}\frac{h}{b}\right)^2 \cdot e = k_{e,WL} \cdot e = \\ = e_{theo} \cdot k_{e,F} \cdot k_{e,D_0} \cdot k_{e,M} \cdot k_{e,WL}$$

If the winglet with its height has the same effect as a span increase, then $k_{WL} = 1$. This is the geometrical equivalence of the winglet. I.e. the winglet sticking up is as good as folding it down. If, however, the winglet height needs to be divided by 2 and only this reduced height taken as a span increase gives the performance of the winglet, then $k_{WL} = 2$. This is the approach taken by Howe [18]. Data from Kroo [24] (see Section 6.3) can be used to calculate $k_{WL} = 2.13$. Dubs [5], and Zimmer (from Müller [25]) have studied winglet efficiency. They have plotted the ratio A/A_{eff} which is smaller than one. This also means that winglets sticking up are not as efficient as folding them down. Data points taken from [5] are plotted in FIG. 17. Equation (45) with a penalty $k_{WL} = 2.45$ represents their graph well.

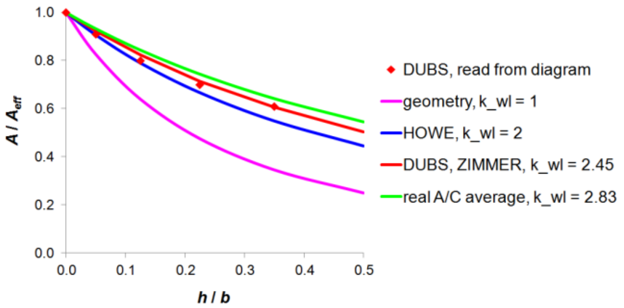


FIG. 17 A / A_{eff} as a function of h/b, from different authors

Whitcomb [28] from NASA is often quoted in support of winglet efficiency. Whitcomb shows a glide ratio increase due to winglets in wind tunnel measurements compared to a reference configuration without winglets. If however his winglet height would have been used for a span increase, performance gains would have been even greater. It can be estimated from [28] that his winglets can be represented by $k_{WL} = 2.22$. This puts the NASA measurements well in line with the other data in TAB. 4.

Real aircraft performance with winglets is published by Boeing [26] in a form shown in FIG. 18. Assuming that induced drag is 40 % of total drag in cruise [4] we can calculate penalties k_{WL} for each aircraft given in FIG. 18. All results are listed in TAB. 4. It can be seen that there is much scatter in data on winglet drag reduction. The average penalty of $k_{WL} = 2.83$ (obtained from a least square fit) indicates that winglets in real life are on average not performing any better than other sources have indicated.

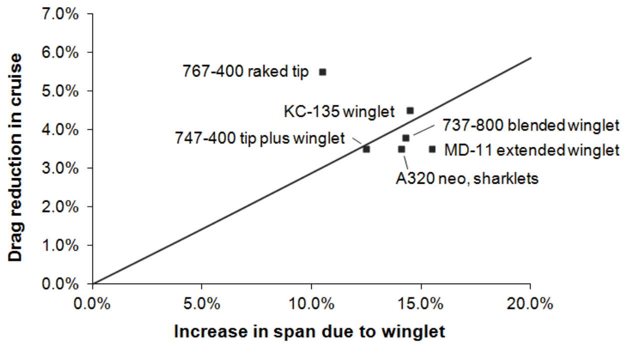


FIG. 18 Increase in span due to winglets as a function of drag reduction in cruise, with data from [26], [27]

TAB. 4 The k_{WL} penalty obtained from different sources

Approach / Source	Reference	k_{WL}
Geometry	-	1.00
Howe	[18]	2.00
Kroo	[24]	2.13
Whitcomb	[28]	2.20
Dubs, Zimmer	[5], [25]	2.45
Real aircraft average	[26], [27]	2.83
767-400 raked tip	[26]	1.58
747-400 tip plus winglet	[26]	2.92
737-800 blended winglet	[26]	3.08
KC-135 winglet	[26]	2.65
MD-11 extended winglet	[26]	3.62
A320 NEO	[27]	3.29

The main reason for fitting winglets on an aircraft is to reduce induced drag for a given span. Aircraft are span limited due to airport restrictions and hangar space. Winglets can be an improvement solution for existing aircraft. However, they shift the lift distribution further to the tip of the wing and increase wing bending. Their overall optimization is a multidisciplinary one and rather complicated. But for sure, people agree: winglets look good [29].

6.2. Wing with Dihedral

The wing with dihedral reduces to a similar geometrical representation as the wing with winglets (see FIG. 19).

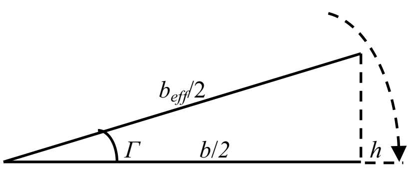


FIG. 19 Geometrical representation of the wing with dihedral

The following relations can be written:

$$\frac{b}{2} = \frac{b_{eff}}{2} \cdot \cos\Gamma \Rightarrow \frac{b_{eff}}{b} = \frac{1}{\cos\Gamma}$$

(46) $$h = \frac{1}{2}\left(b_{eff} - b\right) \Rightarrow \frac{b_{eff}}{b} = 1 + 2\frac{h}{b}$$

$$\frac{h}{b} = \frac{1}{2}\left(\frac{1}{\cos\Gamma} - 1\right)$$

Following the same derivation as for winglets, the Oswald factor for a V-shaped wing is:

(47) $$e_\Gamma = \frac{A_{eff}}{A} \cdot e \ .$$

In other words, the Oswald factor obtained with Equations (39a) or (39b) needs to be multiplied by a factor $k_{e,\Gamma}$:

(48) $$e = e_{theo} \cdot k_{e,F} \cdot k_{e,D_0} \cdot k_{e,M} \cdot k_{e,\Gamma} \ .$$

From the simple geometrical consideration, the factor would be:

(49) $$k_{e,\Gamma} = \left(1 + 2 \cdot \frac{h}{b}\right)^2 = \left(\frac{1}{\cos\Gamma}\right)^2 .$$

A more accurate evaluation is achieved by penalizing the relation with the factor k_{WL} (TAB. 4):

(50) $$k_{e,\Gamma} = \left(1 + \frac{2}{k_{WL}} \cdot \frac{h}{b}\right)^2 = \left[1 + \frac{1}{k_{WL}} \cdot \left(\frac{1}{\cos\Gamma} - 1\right)\right]^2 .$$

6.3. Non-Planar Configurations in General

Kroo [24] researched most of the non-planar configurations and delivered the chart in FIG. 20, where he gives the span efficiency for every type of configuration. He states:

"Among the well-known results we note that a ring wing has half of the vortex drag of a monoplane of the same span and lift. A biplane achieves this same drag savings in the limit of very large vertical gap and the box-plane achieves the lowest drag for a given span and height, although winglets are quite similar." [24]

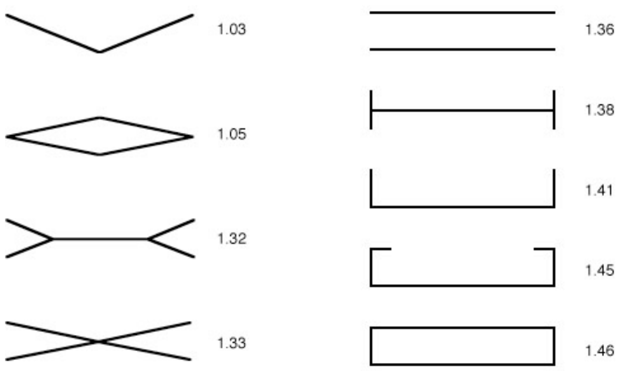


FIG. 20 Span efficiency for various optimally loaded non-planar systems ($h/b = 0.2$) [24]

The configurations listed in FIG. 20 have a vertical extent of 20 % of the wing span (i.e. $h/b = 0.2$). Each design has the same projected span and total lift. The results were generated by specifying the geometry of the trailing vortex wake and solving for the circulation distribution with minimum drag. So, each of the designs is assumed to be optimally twisted [24].

If we assume that the Oswald factor can be calculated in a similar fashion as before, via a penalty factor, this time called k_{NP}, the following relation can be written:

(51) $$e_{NP} = \left(1 + \frac{2}{k_{NP}}\frac{h}{b}\right)^2 \cdot e \Leftrightarrow e_{NP} = k_{e,NP} \cdot e \ .$$

The factor k_{WL} for wings with winglets and dihedral, investigated above, becomes now a particular case of the k_{NP} factor. Having the $k_{e,NP}$ from FIG. 20, k_{NP} can be calculated for each configuration, including for the particular case of wing with winglets, value which is already included in TAB. 4 (under the name k_{WL}) and wing with dihedral:

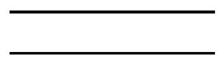
(52) $$k_{NP} = 2\frac{h}{b} \cdot \frac{1}{\sqrt{k_{e,NP}} - 1} \ .$$

Results are listed in TAB. 5.

For wings with winglets, the value is 2.13, while for wings with dihedral, the resulting value is 26.9. There is another literature source from where a k_{NP} can be calculated for wings with dihedral, namely DeYoung [30]. From his Table 2 (pp. 14), $k_{NP} = 12.1$ was calculated. The dihedral angle alone, is a bad solution for reducing induced drag. Yet, if required for other reasons (like roll stability), or in combination with other solutions, it brings a 3 % advantage (according to [24]) compared to the reference solution.

From TAB. 5 the most promising concept seems to be *the box wing configuration*. The box wing can be looked at as a particular case of a biplane, where the two wings are connected via winglets.

TAB. 5 k_{NP} calculated for each non-planar configuration from Kroo [24]

Non-planar configuration	$k_{e,NP}$ (h/b = 0.2)	k_{NP}
	1.03	26.9
	1.05	16.2
	1.32	2.69
	1.33	2.61
	1.36	2.41
	1.38	2.29
	1.41	**2.13**
	1.45	1.96
	1.46	1.92

6.4. The Box Wing Aircraft

Induced drag characteristics of a box wing aircraft compared to a reference conventional aircraft of same span, weight and dynamic pressure can be expressed as follows (see [31] for a complete description of the box wing fundamentals):

(53) $\frac{D_{i,box}}{D_{i,ref}} = \frac{e_{ref}}{e_{box}} = k$.

Relation (53) can be found in literature as a function of the ratio h/b, in the general form:

(54) $\frac{D_{i,box}}{D_{i,ref}} = k = \frac{k_1 + k_2 \cdot h/b}{k_3 + k_4 \cdot h/b}$.

In terms of Oswald factors, (54) can be rewritten as:

(55) $\frac{e_{box}}{e_{ref}} = \frac{e_{NP}}{e} = \frac{k_3 + k_4 \cdot h/b}{k_1 + k_2 \cdot h/b}$.

For $h/b = 0$, the value of the ratio in Equation (54) should be $k = 1$. This means that, in order to follow logic, k_1 should be equal to k_3. In this case, $k = k_2 / k_4$ should be the limit for very high h/b ratio, i.e. at $h/b = \infty$.

The factors k_1, k_2, k_3 and k_4 from different literature sources are given in TAB. 6. From Prandtl [34] two equations for biplanes are included for comparison purposes. An own equation for the box wing is created, with the help of a tool called iDrag. iDrag was developed at Virginia Polytechnic Institute and State University by Grasmeyer [32]. It calculates induced drag of non-planar configurations composed of multiple panels. Values of k were obtained from iDrag for seven points (h/b from 0 to 1) for a reference box wing aircraft. By using the Excel Solver, the factors k_1, k_2, k_3 and k_4 were obtained as such as to minimize the deviations between calculated values and iDrag values of k (read more in [33]). In TAB. 6, two possibilities for forming the equation are listed: first, by letting all parameters to be freely selected by the curve fitting algorithm. This gives a very small deviation, yet k_1 does not result equal to k_3. Second, by imposing $k_1 = k_3$. This returns a set of factors which gives slightly higher deviations, but it respects the logic. Parameters from case (f) in TAB. 6 are proposed here to be used for calculating the Oswald factor for a box wing configuration:

(56) $\frac{e_{box}}{e_{ref}} = \frac{e_{NP}}{e} = \frac{k_3 + k_4 \cdot h/b}{k_1 + k_2 \cdot h/b} = \frac{1.04 + 2.13 \cdot h/b}{1.04 + 0.57 \cdot h/b}$.

Data from TAB. 6 is plotted in FIG. 21. In this figure plotted is also a curve "(f) k from iDrag fit to (51)".This curve represents a k calculated from Equation (51). A new k_{WL} (or k_{NP}) resulted by fitting the equation to iDrag data with the aid of MS Solver. The value $k_{WL} = 4.03$ is higher than literature values given in TAB. 5. For $h/b = 0.2$, $k_{e,NP}$ is calculated to only 1.21. A value $k_{e,NP} = 1.46$ is achieved only for as much as $h/b = 0.42$. Compared to own iDrag results TAB. 5 seems to be over-predicting the benefit of non-planar configurations. e calculated with the equation based on winglets (51) and any value k_{WL} (or k_{NP}) leads always to a value $k = 0$ for infinite h/b ratios. Equation (56), in contrast, allows different values of k to be reached asymptotical depending on k_2/k_4 which seems to be required to represent the box wing.

Another author plotted in FIG. 21 is DeYoung [30]. DeYoung proposes an equation in a different form as (54), however the results are very similar to k from Prandtl in case (a) (TAB. 6).

It is proposed to calculate the induced drag of a box wing configuration with (56) and parameters k_1, k_2, k_3 and k_4 from iDrag according to case (f). It was found that k can reach values as low as about 0.27 for h/b going towards infinity, equivalent to e going to 3.7. For practical values of h/b, k will always be above 0.6 and e staying below 1.7.

TAB. 6 The factors k_1, k_2, k_3, k_4 and factor k from (54) when h/b reaches extremes from different literature sources and own considerations

Case	Configuration	Author / Method	k_1	k_2	k_3	k_4	k for $h/b \to 0$	for $h/b \to \infty$	Reference
(a)	Biplane	Prandtl*	1	-0.66	2.1	7.4	0.976	-0.089	[34]
(b)	Biplane (2)	Prandtl	1	-0.66	1.05	3.7	0.952	-0.178	[34]
(c)	Box wing	Prandtl	1	0.45	1.04	2.81	0.962	0.160	[34]
(d)	Box wing	Rizzo	0.44	0.959	0.44	2.22	1	0.432	[35]
(e)	Box wing	iDrag best fit	1.304	0.372	1.353	1.988	0.964	0.187	-
(f)	Box wing	iDrag $k_1 = k_3$	**1.037**	**0.571**	**1.037**	**2.126**	**1**	**0.269**	-

* here, compared to (54), a different equation is used: $k = 0.5 + \frac{k_1 + k_2 \cdot h/b}{k_3 + k_4 \cdot h/b}$.

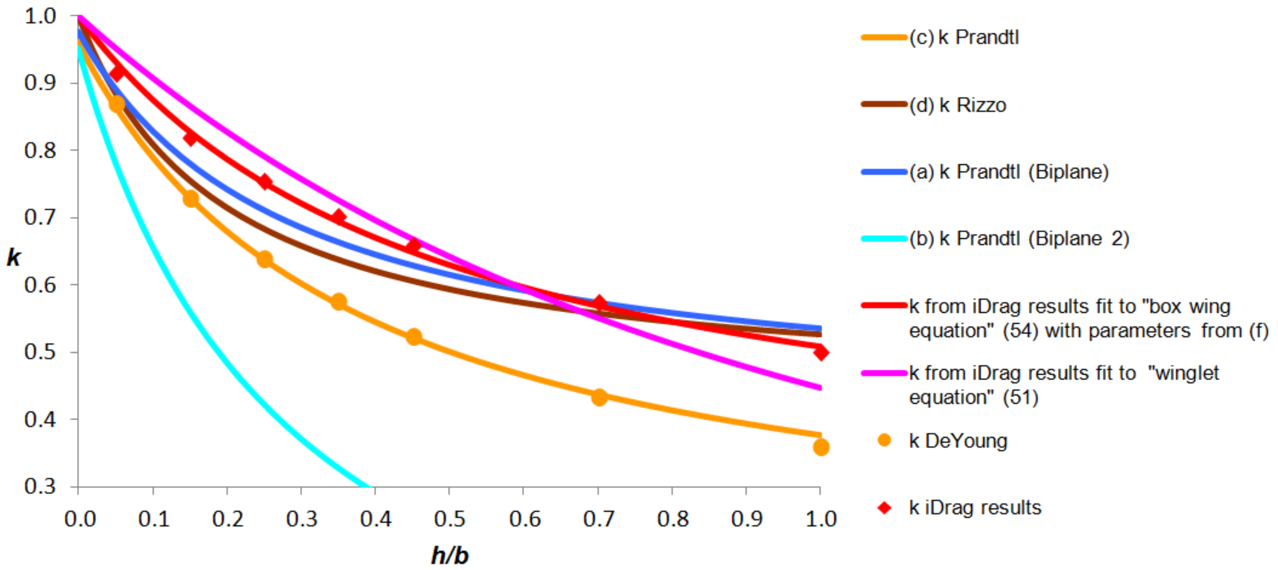


FIG. 21 The factor k as a function of h/b ratio for a box wing from different literature sources and own calculations

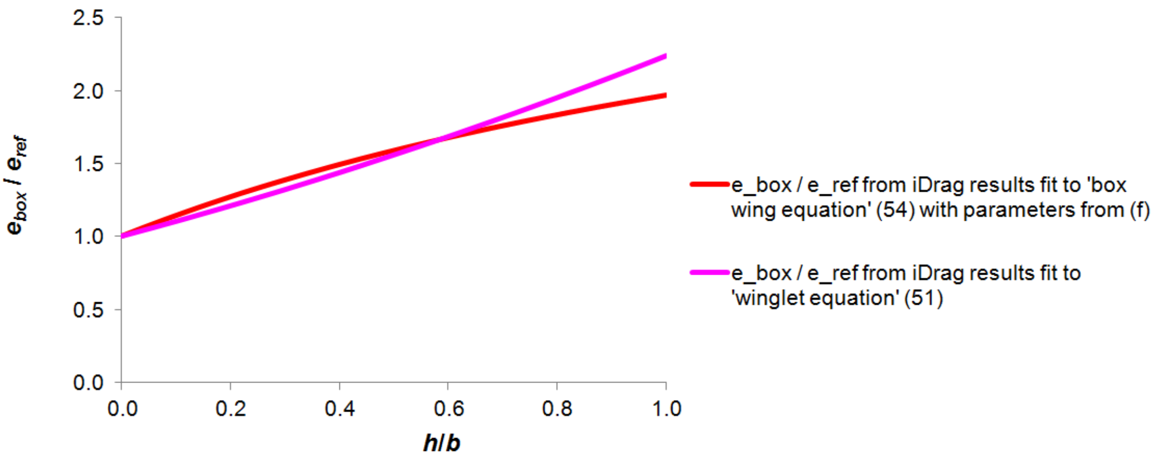


FIG. 22 e_{box} / e_{ref} for a box wing as a function of h/b ratio calculated from iDrag results fit to 'box wing equation' (54) resp. (56) and 'winglet equation' (51)

7. SUMMARY AND CONCLUSIONS

This paper assists aircraft designers in making an accurate estimation of the span efficiency factor *e* during the preliminary stage of the design. The research started by evaluating available methods in literature. The deep understanding of the theoretical background and of the relations between parameters that yielded from this literature study, helped creating the grounds for an own, simple but logical approach, in concordance to the theory. Further on, the paper delivers a method to calculate the Oswald factor for non-planar configurations. More attention was given to the wing with winglets, wing with dihedral and the box wing, for which a second method was adapted.

Most of the authors express the induced drag as being composed of an inviscid term (here noted with Q) and a viscous term (here noted with P).

In most of the cases the Oswald factor is overestimated. Some authors use regressions to sample a virtual design space, yet outside the limits the equations are invalid. Other authors deliver theoretically derived diagrams, which are difficult to manipulate.

To avoid overestimations, besides theoretical corrections, such as for the influence of fuselage or Mach number, a statistical correction is also required. This is why the proposed method started from estimating a theoretical Oswald factor, which was then corrected via a number of factors, including one that matches statistical data to the theoretical results. The average deviation from literature values of *e* was under 4 %.

The paper also investigates the Oswald factor for non-planar configurations. Starting from a simple geometrical approach, an equation was found to assess the correction factor that accounts for the presence of the winglets. A similar derivation is used to assess the effect of the dihedral angle on the aerodynamical efficiency. Based on the resulting equation from this approach, other non-planar configurations were assessed and correction factors were estimated for each case, based on the input from [24]. Special attention was given to the box wing concept, which seems to be the most promising one out of the non-planar configurations (according to [24]). Besides the correction factor obtained from the geometrical derivation of wings with winglets (see TAB. 5), an additional approach was studied and compared with authors like Prandtl [34] and Rizzo [35] (see TAB. 6 and FIG. 21).

To summarize, own contributions were brought in estimating:

- a relation to express the optimal taper ratio for a given sweep angle;
- a theoretical Oswald factor, e;
- a correction factor for e that accounts for the added lift-dependent drag caused by the modification of the span loading due to the presence of the fuselage;
- a statistical correction factor for e that accounts for the worsening of the Oswald factor with each aircraft category, due to the worsening of the zero-lift drag characteristics;
- a correction factor for e that accounts for the effect of the Mach number;
- a correction factor for e that accounts for the presence of winglets;
- a correction factor for e that accounts for the effect of the dihedral angle;
- correction factors for e for basically every non-planar configuration, based on Kroo's chart [24];
- an equation to estimate the Oswald factor for a box wing configuration starting from a conventional reference aircraft of the same lift and span.

The paper covers the estimation of the Oswald factor for both conventional and unconventional configurations. It delivers a reliable method that accounts for all basic aircraft parameters. It is hence suitable for implementation in an automatic design environment, for further aircraft design optimization.

APPENDIX A: DATABASE OF AIRCRAFT AND AIRCRAFT DATA

The following tables summarize the aircraft used in this study and their main characteristics, required for calculations. For all these aircraft the Oswald factor was known from the following literature sources: [1], [16], [17], [36], [37], [38], [39], [40], [41], [42], [43], [44].

TAB. A1 Amount of aircraft used to evaluate existing estimation methods for the Oswald factor

Type	Amount	Total in Group	Aircraft Size	Group Name
General aviation (GA)	9	10	small	GA aircraft
GA aircraft, 2-engines	1		prop	
Propeller aircraft				
2 engines	4	6	medium	propeller
4 engines	1		prop	aircraft
Medium Bomber	1			
Regional jet	2	4	medium	business jet
Business jet	2		jet	
Jet aircraft	11	13	large jet	jet airliner
Military transporter	1			
Long range bomber	1			
Fighter	6	6	fighter	fighter

TAB. A2 List of aircraft and aircraft characteristics

Aircraft / Aircraft category	Type	λ	A	φ_{25}	d_F	b	d_F / b	M_{CR}	M_e	e
Jet airliner										
A 300-600	Twin jet airliner	0.293	7.73	28	-	-	-	0.78	0.78	0.749
A 319	Twin jet airliner	0.240	9.40	25	-	-	-	0.78	0.78	0.753
A320	Twin jet airliner	0.24	9.50	25	4.04	34.1	0.118	0.76	0.76	0.783
B 737-800	Twin jet airliner	0.219	9.45	25	3.88	34.32	0.113	0.78	0.78	0.660
MPC 75	Twin jet airliner	0.260	9.6	23.5	3.45	29.72	0.116	0.77	0.77	0.553
B767-300	Twin jet airliner	0.306	7.99	31.5	-	-	-	0.80	0.80	0.670
MD 90-30	Twin jet airliner	0.193	9.62	24.5	-	-	-	0.76	0.76	0.811*
B 707-320B	Twin jet airliner	0.288	7.05	36	-	-	-	0.82	0.82	0.700
DC 9-30	Twin jet airliner	0.206	6.80	24	-	-	-	0.75	0.30	0.810
TU 154M	Tri-jet airliner	0.267	7.00	35	-	-	-	0.73	0.73	0.665
C 17A Globemaster III	Strategic transport	0.262	7.20	25	-	-	-	0.75	0.30	0.870
B 52-A	Long range bomber	0.044	8.60	36	-	-	-	0.99	0.30	0.924*
A 340-300	Four jet airliner	0.235	9.26	30	-	-	-	0.82	0.82	0.770
Propeller aircraft										
Douglas DC3	Twin propeller aircraft	0.284	9.17	8	3.13	29.98	0.104	0.22	0.30	0.750
Gulfstream GI	Twin propeller aircraft	0.374	10.1	4	2.56	23.93	0.106	0.50	0.30	0.780
Saab SF 340B	Twin propeller aircraft	0.441	11.0	3.5	2.35	21.44	0.109	0.50	0.30	0.800
Boeing 247D	Twin propeller aircraft	0.529	6.55	2	1.95	22.66	0.086	0.26	0.30	0.750
Martin B26F Marauda	Medium bomber twin propeller	0.326	7.67	4	2.40	21.65	0.110	0.31	0.30	0.750
Ilyushin IL 18	Four engines propeller aircraft	0.407	9.99	2	3.60	37.4	0.096	0.56	0.30	0.800
Business jet										
Fokker F28-2000	Regional jet, 2 engines	0.262	7.27	17	3.10	23.58	0.131	0.68	0.30	0.818
Yakolev Yak 40	Regional jet, 3 engines	0.446	8.93	1	2.40	25	0.096	0.49	0.30	0.820
Learjet 35	Business jet	0.566	5.73	13	1.15	12.04	0.095	0.70	0.30	0.827
Learjet M25	Business jet	0.571	5.00	8.7	1.7	10.84	0.156	0.81	0.30	0.820

General aviation aircraft										
Beech 35	GA aircraft	0.562	6.21	2	1.37	10.21	0.134	0.21	0.21	0.820
Beechcraft D 17D	GA aircraft	1	6.84	0	1.16	9.75	0.119	0.27	0.27	0.760
Cessna 177 Cardinal RG	GA aircraft	0.703	7.23	0	1.23	10.82	0.113	0.19	0.19	0.630
Cessna 150	GA aircraft	0.692	7.00	0	1.3	10.21	0.127	0.16	0.16	0.770
Cessna 180	GA aircraft	0.669	7.5	0	1.4	10.98	0.127	0.22	0.22	0.750
Cessna 182S	GA aircraft	0.669	7.5	0	1.4	10.98	0.127	0.22	0.22	0.840
PA 28 Cherokee	GA aircraft	0.669	7.21	0	1.18	10.67	0.110	0.17	0.17	0.760
Cessna 172 Skyhawk	GA aircraft	0.709	7.45	0	1.27	10.97	0.115	0.19	0.19	0.750
Piper J3 Cub	GA aircraft	1	6.96	0	0.84	10.74	0.078	0.10	0.10	0.750
Cessna 310	GA aircraft, 2 engines	0.693	7.62	0	1.5	11.25	0.133	0.27	0.27	0.730
Fighter										
McDonnell F4 Phantom	Fighter (interceptor, bomber)	0.199	2.78	44	-	-	-	0.88	0.30	0.700
Lockheed Martin F22 Raptor	Fighter (stealth air superiority)	0.112	2.36	30	-	-	-	1.58	0.30	0.820
Sukhoi Su 27	Fighter (air superiority)	0.351	3.49	37	-	-	-	2.35	0.30	0.710
Mikoyan-Gurevich MIG 29	Fighter (multirole)	0.210	3.42	35	-	-	-	2.30	0.30	0.850
Mikoyan-Gurevich MIG AT	Fighter (advanced jet trainer)	0.298	5.83	7	-	-	-	0.85	0.30	0.610
Douglas D558-2 Skyrocket	Experimental high speed research	0.558	6.70	35	-	-	-	1.01	0.30	0.820
* questionable value										

NOMENCLATURE

Symbols

a	correction factor for calculating the Oswald factor
A	aspect ratio
b	wing span, correction factor for calculating the Oswald factor
c	chord, correction factor for calculating the Oswald factor
C	coefficient (drag, lift), slope, coefficient for determining e (in [14], [16])
d	diameter
D	drag
e	Oswald factor (span efficiency factor)
h	height of the non-planar element
k	span efficiency factor ($k = 1/e$), correction factor for calculating e, factor for calculating e for aircraft with winglets, with dihedral and for box wing aircraft
K	factor for calculating the viscous term in [2], and term for calculating e in [8]
m	slope of the polar (in [8])
M	Mach number
N	number of engines (in [18])
P	viscous drag factor
q	factor for calculating e (in [15]), $q = 1 - \lambda$
Q	inviscid drag factor
R	leading edge suction parameter
Re	Reynolds number
s	factor for calculating the inviscid term in [2], it corrects the inviscid term for the fuselage influence
S	relative leading edge suction force
t/c	wing thickness ratio
u	factor for calculating the inviscid term in [2], with the significance of a theoretical Oswald factor
v	factor for calculating the twist terms in the drag estimation
w	factor for calculating the twist terms in the drag estimation

Greek symbols

α	angle of attack
β	compressibility factor, factor for calculating e (in [15])
Γ	dihedral angle
δ	span efficiency factor ($\delta = 1/e - 1$)
Δ	deviation, difference
φ	sweep angle
λ	taper ratio
Λ	aspect ratio (in [5])
μ	factor for calculating e (in [15])
η	profile efficiency (in [5]), spanwise position of center of pressure (at Garner, in [14]), kink ratio
π	Pi number
θ	twist angle

Indices

box	box wing
comp	compressibility
CP	center of pressure
D	drag
$D_{,0}$	zero lift drag
$D_{,i}$	induced drag
$D_{,inviscid,twist}$	inviscid drag, that accounts also for twist influence
e	Oswald factor (span efficiency factor), leading edge (suction force), engines, Euler number
eff	effective
F	fuselage
i	induced
inviscid	inviscid
K	kink
L	lift
L_{α}	lift curve slope
LE	leading edge
LER	leading edge radius
max	maximum
NP	non-planar
opt	optimum
r	root
ref	reference
t	tip
theor	theoretical
W	wing
WL	winglet
0,1,2,3,4	identification number

LIST OF REFERENCES

[1] ROSKAM, J.: *Airplane Design*. *Vol. 1: Preliminary Sizing of Airplanes*. Ottawa, Kansas, 1989

[2] KROO, I.: *Aircraft Design : Synthesis and Analysis*. Stanford : Desktop Aeronautics, 2001. URL: http://adg.stanford.edu/aa241/AircraftDesign.html (2012-08-20)

[3] OBERT, E.: *Aerodynamic Design of Transport Aircrcraft*. Delft : IOS Press, 2009

[4] KROO, I.: Drag Due to Lift : Concepts for Prediction and Reduction. In: *Annual Reviews, Fluid Mechanics*, Vol. 33 (2001), pp. 587–617

[5] DUBS, F.: *Hochgeschwindigkeits-Aerodynamik.* Basel : Birkhäuser, 1975

[6] SCHOLZ, D.: *Flugzeugentwurf (Aircraft Design)*. Hamburg University of Applied Sciences, Department of Automotive and Aeronautical Engineering, Lecture Notes, 1999

[7] BÖHNKE, D.; JEPSEN, J.; PFEIER, T.; NAGEL, B.; GOLLNICK, V.; LIERSCH, C.: An Integrated Method for Determination of the Oswald Factor in a Multi-Fidelity Design Environment. In: CEAS: 3rd CEAS Air&Space Conference : Proceedings (CEAS2011, Venice, 24. - 28. October 2011). - ISBN: 978-88-96427-18-7. DocumentID: 313

[8] STINTON, D.: *The Design of the Aeroplane*. Second Edition. Secaucus, NJ : Wiley-Blackwell, 2001

[9] SCHAUFELE, R., D.: *The Elements of Aircraft Preliminary Design*. Santa Ana, CA : Aries, 2000

[10] GROSU, I.: *Calculul şi construcţia avionului, Vol.1*. Bucharest : Editura didactică şi pedagogică, 1965

[11] HÖRNER, S., F.: *Fluid Dynamic Drag*. Dayton Ohio : Otterbein Press, 1951

[12] MCCORMICK, B.: *Aerodynamics, Aeronautics, and Flight Mechanics*. New York : John Wiley and Sons, 1995

[13] Engineering Science Data Unit: ESDU 74035 : *Subsonic lift-dependent drag due to trailing vortex wake for wings without camber or twist*. ESDU International PLC, 1996. – ISBN: 978-0-85679-089-8

[14] TORENBEEK, E.: *Synthesis of Subsonic Airplane Design*. Delft : Delft University Press, 1986

[15] NIŢĂ, M.,M.; PATRAULEA, R.; SÂRBU, A.: *Mecanica Aeronavelor*. University Politehnica of Bucharest, Faculty of Aerospace Engineering, Lecture Notes, 1984

[16] JENKINSON, L.R.; SIMPKIN, P.; ROHDES, D.: *Civil Jet Aircraft Design*. Burlington : Butterworth-Heinemann Publications, 1999

[17] SAMOYLOVITCH, O.; STRELETS, D.: Determination of the Oswald Effciency Factor at the Aeroplane Design Preliminary Stage. In: *Aircraft Design*, Vol. 3 (2000), pp. 167-174

[18] HOWE, D.: *Aircraft Conceptual Design Synthesis*. London : Professional Engeneering Publishing, 2000

[19] URL: http://creativemachines.cornell.edu/eureqa/ (2012-08-20)

[20] BRANDT, S., A.; STILES, R., J.; BERTIN, J., J.; WHITFORD, R.: *Introduction to Aronautics : A Design Perspective*. Reston, VA : AIAA Educational Series, 1997

[21] RAYMER, D., P.: *Aircraft Design : A Conceptual Approach*. Washington : AIAA Education Series : AIAA, 1999

[22] HOAK, D., E.: *USAF Stability and Control Datcom*, Wright-Patterson Air Force Base, Air Force Flight Dynamics Laboratory, Flight Control Division, Ohio, 1978. - Sale: NTIS

[23] DEYOUNG, John; HARPER, Charles W.: *Theoretical Symmetric Span Loading at Subsonic Speeds for Wings Having Arbitrary Plan Form*. NACA Report 921, 1955

[24] KROO, I.: Nonplanar Wing Concepts for Increased Aircraft Efficiency. In: TORENBEEK, E.; DECONINCK, H. (Ed.): *Innovative Configurations and Advanced Concepts for Future Civil Aircraft*. Rhode Saint Genèse : VKI, 2005. – Lecture Series, Von Karman Institute for Fluid Dynamics, VKI-LS-2005-06, ISBN: 2-930389-62-1

[25] ZIMMER, H.: Optimale Mehrdecker- und Einzelflügel-Konfigurationen : Ein Rückblick auf bei Dornier durchgeführte Untersuchungen. Friedrichshafen : Dornier Luftfahrt GmbH, 1991. - Quoted from: MÜLLER, F.: *Flugzeugentwurf : Entwurfssystematik, Aerodynamic, Flugmechanik und Auslegungsparameter für kleinere Flugzeuge*. Fürstenfeldbruck : Dieter Thomas, 2003

[26] BOEING: Blended Winglets for Improved Airplane Performance. In: *AERO Magazin*, No. 17 (2002), pp. 18–31.
URL: http://www.boeing.com/commercial/aeromagazine/aero_17/ (2012-08-20)

[27] AIRBUS: *New “sharklets” and A320neo* . URL: http://www.airbus.com/aircraftfamilies/passengeraircraft/a320family/technology-and-innovation/ (2012-08-20)

[28] WHITCOMB R., T.: *A Design Approach and Selected Wind-Tunnel Results at High Subsonic Speeds for Wing-Tip Mounted Winglets*. NASA Technical Note D-8260. Washington : NASA, 1976

[29] SCHOLZ, D. : *Understanding Winglets*. Hamburg University of Applied Sciences, Aircraft Design Lecture Notes, 2012

[30] DEYOUNG, J.: *Induced Drag Ideal Efficiency Factor of Arbitrary Lateral-Vertical Wing Forms*. NASA Contractor Report 3357, 1980

[31] SCHIKTANZ, D; SCHOLZ, D: Box Wing Fundamentals - An Aircraft Design Perspective. In: DGLR: *Deutscher Luft- und Raumfahrtkongress 2011 : Tagungsband - Manuskripte* (DLRK, Bremen, 27. - 29. September 2011), pp. 601-615. - ISBN: 978-3-932182-74-X. Document ID: 241353. Download: URL: http://Airport2030.ProfScholz.de (2012-08-20)

[32] GRASMEYER, J.: *A Discrete Vortex Method for Calculating the Minimum Induced Drag and Optimum Load Distribution for Aircraft Configurations with Noncoplanar Surfaces*. Department of Aerospace and Ocean Engineering, Virginia Polytechnic Institute and State University, 1997

[33] WAETERSCHOOT, M.: *The Effect of Variations of the Height to Span Ratio of Box Wing Aircraft on Induced Drag and the Spanwise Distribution*. Project, Hamburg University of Applied Sciences, Department of Automotive and Aeronautical Engineering, 2012

[34] PRANDTL, L.: *Induced Drag of Multiplanes*. Hampton : National Advisory Committee for Aeronautics, 1924. – NACA TN 182

[35] RIZZO, E.: *Optimization Methods Applied to the Preliminary Design of Innovative, Non Conventional Aircraft Configurations*. Pisa : Edizioni ETS, 2007. – ISBN 978-884672458-8

[36] ROUX, E: *Influence de l’allongement voilure d’un avion basse vitesse sur ses performances conceptuelles*. Toulouse : SupAero, 2006. URL : http://elodieroux.com/EditionsElodieRouxBonus_Reports.html#AncreALLONGEMENT (2012-08-20)

[37] AIRBUS: *Airplane Characteristics for Airport Planning, AC A320*. Blagnac : Airbus SAS, 2005, Rev. 2011

[38] LAMBERT, M.: *Jane's all the World's Aircraft, 2001*. – Jane's Information Group, 163 Brighton Road, Couldsdon, Surrey CR5 2NH, UK

[39] LAMBERT, M.: *Jane's all the World's Aircraft, 1997*. – Jane's Information Group, 163 Brighton Road, Couldsdon, Surrey CR5 2NH, UK

[40] MPC75: http://MPC75.ProfScholz.de/ (2012-08-20)

[41] SCHLIEMANN, K.; NIEDERKLEINE, M.: Datenbank mit Entwurfsdaten für Passagierflugzeuge. 1999. URL: http://www.fzt.haw-hamburg.de/pers/Scholz/flugzeugdaten/Datenbank.htm (2012-08-20)

[42] URL: http://www.boeing.com/commercial/707family/product.html (2012-08-20)

[43] URL: http://en.wikipedia.org/wiki/Boeing_707 (2012-08-20)

[44] URL: http://www.flugzeuginfo.net (2012-08-20)

ACKNOWLEDGEMENT

The authors would like to thank Allan McDonald for extracting and analyzing aerodynamic data (from literature) for a number of aircraft and Maarten Waeterschoot for calculations using iDrag.