Hochschule für Angewandte Wissenschaften Hamburg
Hamburg University of Applied Sciences

Relative Wing Thickness, Wave Drag, Fuselage Nose Length from Master Thesis

Department of Automotive and Aeronautical Engineering

Turboprop Aircraft Design Optimization - Tool Development

Roberto Segovia García

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Hochschule für Angewandte Wissenschaften Hamburg
Fakultät Technik und Informatik
Department Fahrzeugtechnik und Flugzeugbau
Berliner Tor 9
20099 Hamburg

Verfasser: Roberto Segovia García
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1. Prüfer: Prof. Dr.-Ing. Dieter Scholz, MSME
Tutor: Dipl.-Ing. Andreas Johanning

1 Calculation of the Equation $(t/c) = f(M_{CR})$

As the drag divergence Mach number is unknown for slow aircraft, the problem has one equation and two unknown parameters. Besides, the standard equation for calculation of t/c provides unfeasible results for a very low Mach number. For these reasons, the method which has been developed calculates the thickness ratio as a function of the cruise Mach number for all types of aircraft.

It has been necessary to add more aircraft in the database, with a high cruise Mach number and a very low cruise Mach number. They are necessary to obtain a function in the full range of cruise Mach numbers, because there were only turbofans in the database and if a trend line was calculated the extrapolation for high Mach numbers or for very low Mach numbers was not realistic. The solution is represented with the Equation 1.1.

$$\left(\frac{t}{c}\right) = -0{,}0439 \cdot \tan^{-1}(3.3450 \cdot M_{CR} + -3.0231) \ \ +0{,}0986 \qquad (1.1)$$

The representation of the solution and the trend line is shown Figure 1.1. In this Figure, the solution that has been chosen has a low limit when the cruise Mach number goes to infinite. This limit is represented by 1.2.

$$\left(\frac{t}{c}\right)_{M_{CR}\to\infty} = 0{,}0296 \qquad (1.\ 2)$$

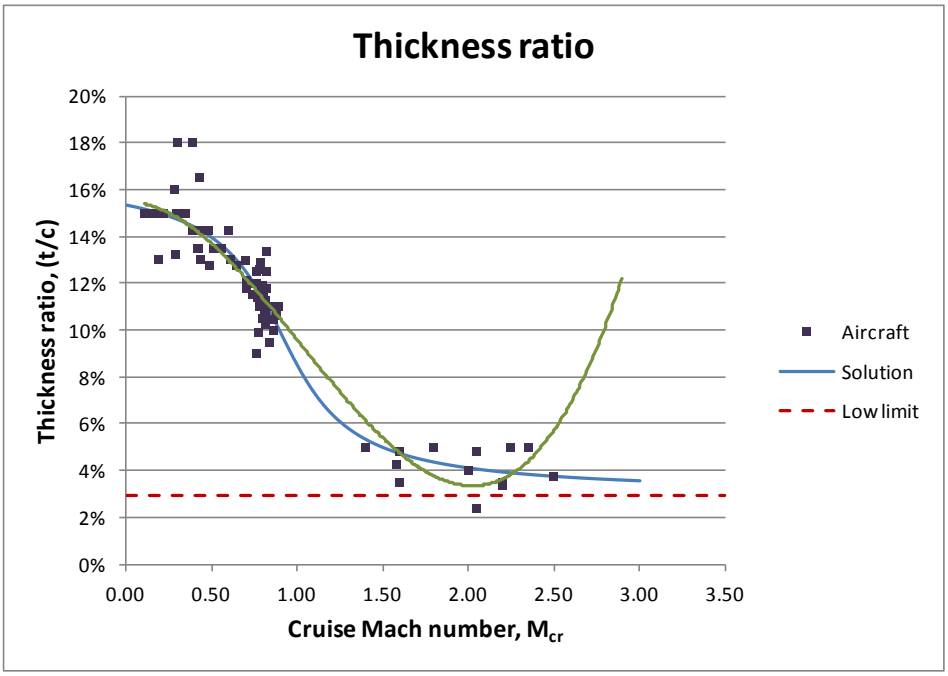

Figure 1.1 Representation of the solution for the thickness ratio as a function of the cruise Mach number

2 Wave Drag

2.1 Introduction

The aim of this Section is the analysis of the wave drag of the aircraft. The initial calculation of the wave drag in PrOPerA was based on the Boeing and Airbus philosophy (**Scholz 1999**), so the tool considers that the cruise Mach number was equal to drag divergence Mach number and the wave drag coefficient was a constant. For turbofans, this concept can be correct, but for the turboprops it is unknown if the aircraft flies at the drag divergence Mach number. For this reason, a method for the calculation of the wave drag is needed. With this implementation, the previous assumption is not needed and the wave drag is calculated for each design.

2.2 Calculation of the Drag Divergence Mach Number M_{DD}

2.2.1 Equation obtained from Shevell

Shevell 1980 describes an analytical method for the calculation of the crest critical Mach number, M_{CC}.

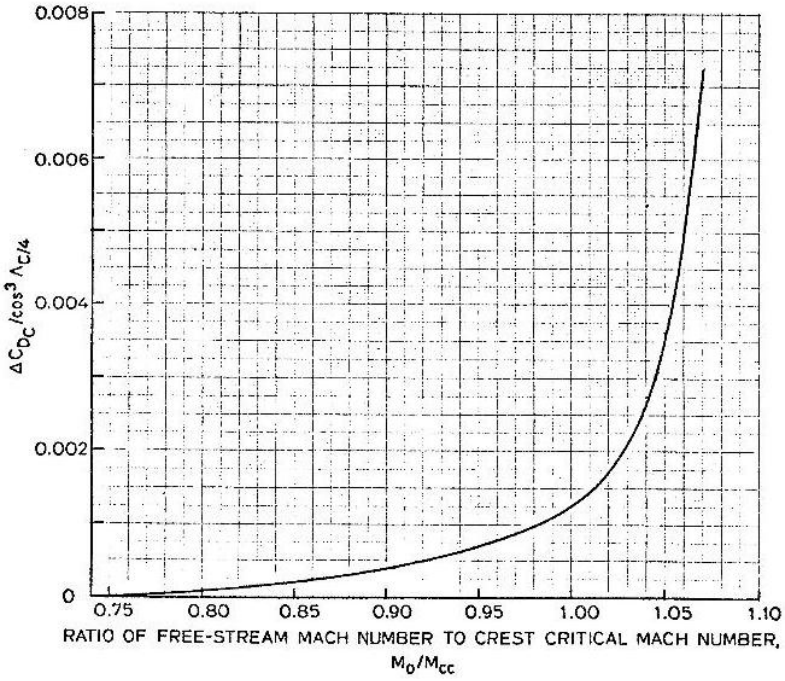

Figure 2.1 Wave drag curve obtained from **Shevell 1980**

From the Figure 2.1, the relation between M_{crit}, which is the critical Mach number, and M_{CC} is represented with

$$\frac{M_{crit}}{M_{CC}} = 0.74 \tag{2. 1}$$

The mathematical approximation for the Shevell's curve that has been realized is the Equation 2.2. The curve is represented in the Figure 2.2.

$$\frac{\Delta c_{D,wave}}{\cos^3(\varphi_{25,w})} = A \cdot \tan\left(B \cdot \left(\frac{M}{M_{crit}}\right) - B\right) \tag{2. 2}$$

Where $A = 0.00057$ and $B = 3.34821$.

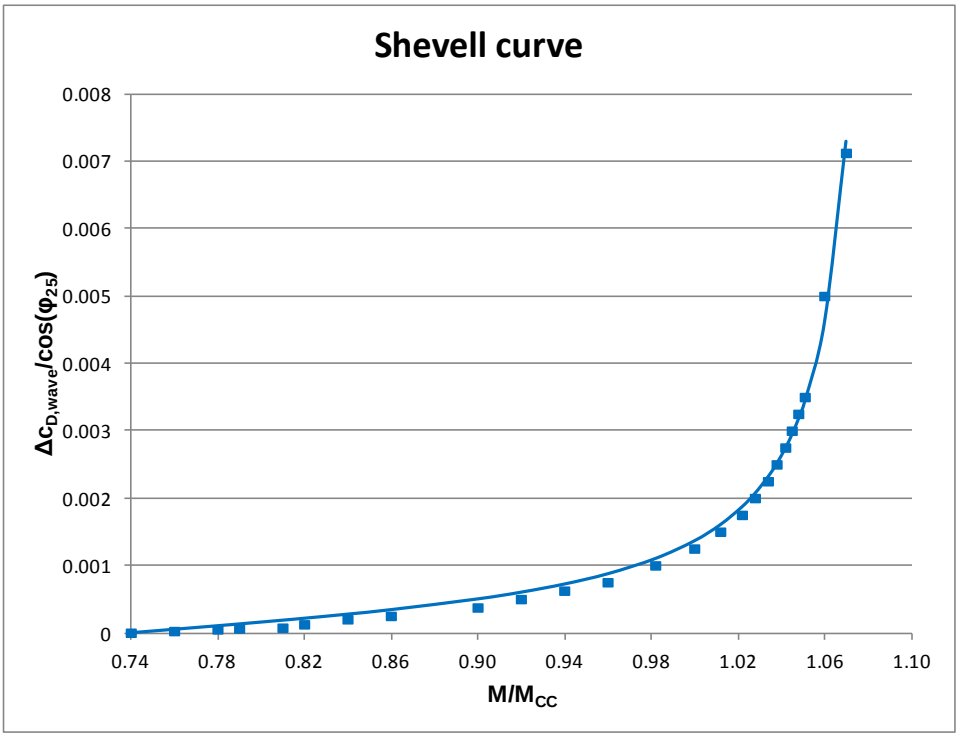

Figure 2.2 Representation of the approximation of the Shevell's curve

The definition that has been taken for the calculation of M_{DD} is given by Boeing and M_{DD} is defined as the Mach number for which the wave drag coefficient has an increment of 0.002, or 20 drag counts [CTS] (2.3). The Equation 2.4 represents the calculation of M_{DD}.

$$\Delta c_{D,wave} = 0.002 \tag{2. 3}$$

$$M_{DD} = \frac{(0.74 \cdot M_{CC})}{B} \cdot \left[\tan^{-1}\left(\frac{0.002}{A \cdot \cos^3(\varphi_{25,w})}\right) + B\right] \tag{2. 4}$$

2.3 Calculation of the Wave Drag Curve

2.3.1 Analysis of the Real Wave Drag Curves

The calculation has been based on experimental data from some aircraft. The aircraft that have been studied are: A320-200, B727-200, B737-800, C-130H and BAe 146-200. The wave drag curves of B727-200, C-130H and BAe 146-200 have been obtained from **Roskam II 1985,** Figure 2.3.

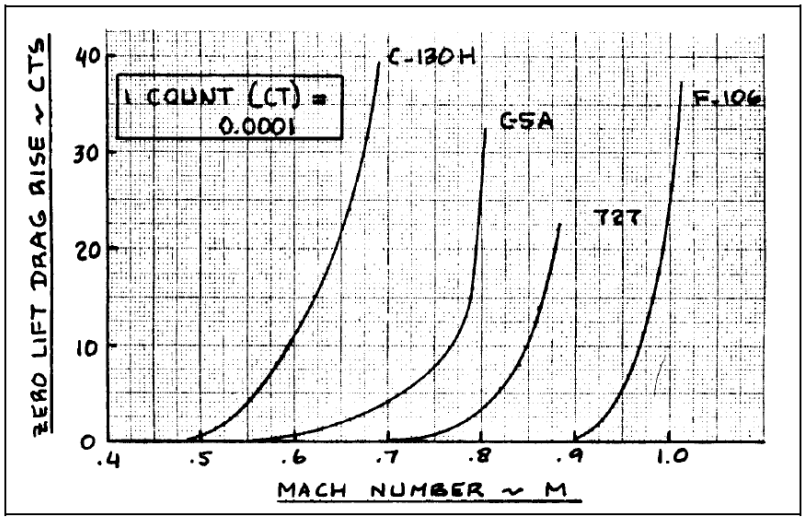

Figure 2.3 Wave drag curves of B727-200, C-130H and BAe 146-200, obtained from **Roskam II 1985**.

With this empirical information, M_{crit} and M_{DD} have been measured. The values of M_{DD} and M_{crit} for the five airplanes are represented in the Table 2.1.

Table 2.1 Values of M_{DD} and M_{crit} for the aircraft that have been analyzed

	A 320-200	B727-200	B737-800	C-130H	BAe 146-200
M_{crit}	0.60	0.70	0.60	0.49	0.53
M_{DD}	0.80	0.88	0.80	0.64	0.67

The wave drag curves have been approximated with equations that have the same structure as 2.5.

$$\frac{\Delta c_{D,w}}{\cos^3(\varphi_{25,w})} = A \cdot \tan\left(B \cdot \left(\frac{M}{M_{crit}}\right) + C\right) + D \qquad (2.5)$$

Due to the conditions of the problem, $\Delta c_{D,w}$ has to be zero at $M = M_{crit}$, so $C = -B$ and $D = 0$. The resultant equation is 2.6.

$$\frac{\Delta c_{D,w}}{\cos^3(\varphi_{25,w})} = A \cdot \tan\left(B \cdot \left(\frac{M}{M_{crit}}\right) - B\right) \qquad (2.\ 6)$$

The values of the constants A and B are given in the Table 2.2 for each aircraft. The curves for the A320-200, B727-200, B737-800, C-130H and BAe 146-200 have been represented in the Figure 2.4. The representation is based on the Equation 2.6 with the constants of the Table 2.2.

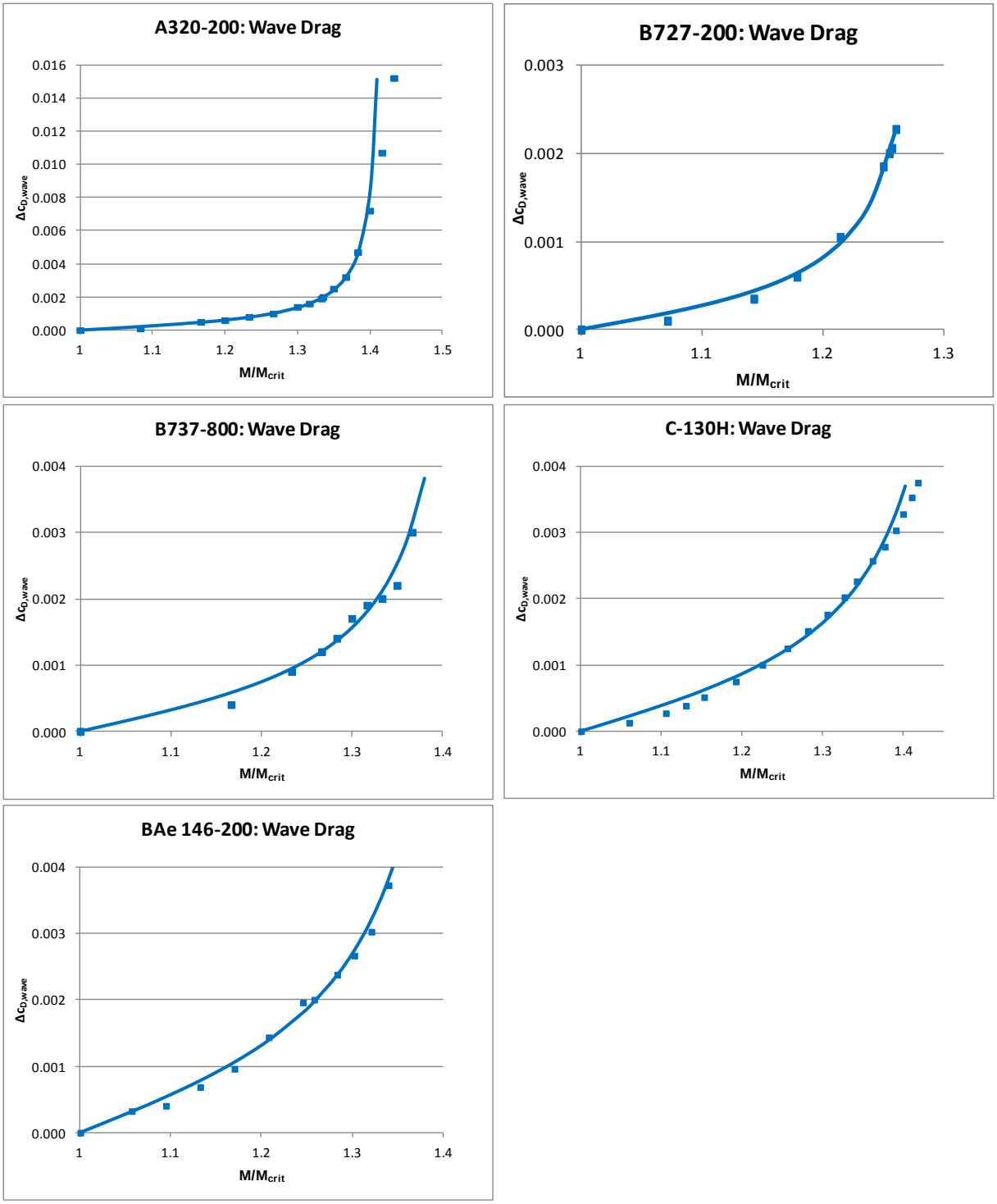

Figure 2.4 Wave drag curves for A320-200, B727-200, B737-800, C-130H and BAe 146-200

Table 2.2 Values of A and B for the approximation of the wave drag curves of the aircraft that have been analyzed

	A 320-200	B727-200	B737-800	C-130H	BAe 146
A	0.001	0.001	0.001	0.001	0.002
B	3.734	5.257	3.543	3.126	3.457

2.3.2 Tangent Equation with the Effect of the Sweep Angle for the Calculation of the Wave Drag Curve

This equation has been developed based on the approximation of the Shevell's curve (2.2). The equation is represented by 2.7.

$$\Delta c_{D,w} = A \cdot \tan\left(B \cdot \left(\frac{M}{M_{crit}}\right) - B\right) \cdot \cos^3\left(\varphi_{25,w}\right) \quad (2.7)$$

The constants A and B are represented in the Table 2.3 and the SEE is represented in the Table 2.4, in [CTS]. The curve defined by 2.7 and the real curves (approximated with the Equation 2.6) are shown in the Figure 2.5.

Table 2.3 Values of A and B for the Tangent Equation based on the Shevell's curve (**Shevell 1980**), Equation 2.7

	Result
A	0.00127
B	3.47663

Table 2.4 SEE obtained for each aircraft, using the Tangent Equation based on the Shevell's curve (**Shevell 1980**), Equation 2.7

	A 320-200	B727-200	B737-800	C-130H	BAe 146-200
SEE [CTS]	9.44	7.41	1.55	23.83	6.99

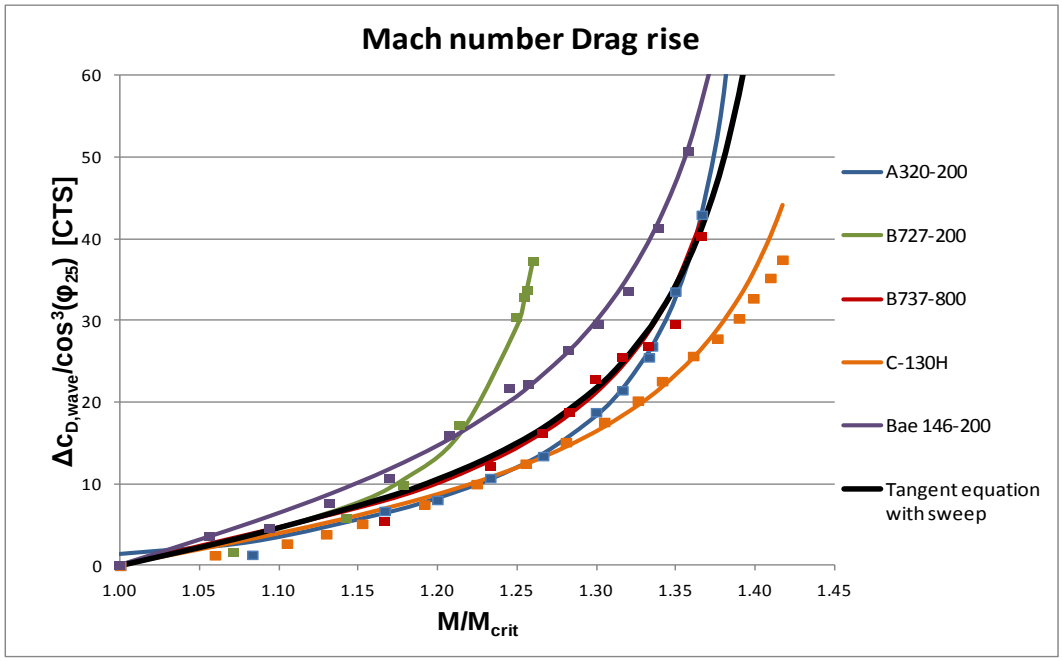

Figure 2.5 Representation of the Tangent Equation based on the Shevell's curve (**Shevell 1980**), Equation 2.7, and the real curves (approximated with the Equation 2.6)

2.4 Calculation of the Critical Mach Number, M_{crit}

2.4.1 Calculation of M_{crit} obtained from the Tangent Equation with the Effect of Sweep Angle

The critical Mach number is obtained from the Equation 2.7. The resultant equation for the calculation of M_{crit} is 2.10.

$$\frac{\Delta c_{D,w}}{\cos^3(\varphi_{25,w})} = A \cdot \tan\left(B \cdot \left(\frac{M}{M_{crit}}\right) - B\right) \tag{2. 8}$$

$$\frac{0.002}{\cos^3(\varphi_{25,w})} = A \cdot \tan\left(B \cdot \left(\frac{M_{DD}}{M_{crit}}\right) - B\right) \tag{2. 9}$$

$$M_{crit} = \frac{B \cdot M_{DD}}{\tan^{-1}\left(\frac{0.002}{A \cdot \cos^3(\varphi_{25,w})}\right) + B} \tag{2. 10}$$

The constants A and B are given in the Table 2.3.

3 Nose length Estimation

3.1 Analysis of the Problem for the Calculation of the Nose Length

The nose length that has been considered is the distance between the nose of the aircraft and the position of the fuselage where the diameter of it begins to be a constant. In PrOPerA, the value of the nose length was based in a statistical calculation. The statistical curve had been obtained from three aircraft (**Trahmer 2009**) and extrapolated for the rest of the aircraft as a function of the maximum operating Mach number.

The problem of this extrapolation was found for the sizing of the turboprops. The maximum operating Mach number of the turboprops aircraft is generally lower than the Mach number for the turbofan airplanes and for this range of Mach, the statistical equation didn't have analyzed and the results weren't feasible.

The statistical equation is represented by 3.1 and the results for completed range of maximum operating Mach number are represented in the Figure 3.1.

$$\frac{l_{nose}}{d_{F,equiv}} = 44.405 \cdot M_{MO}^2 - 71.625 \cdot M_{MO} + 30.441 \quad (3.1)$$

With

l_{nose} The length of the nose in m.

M_{MO} The maximum operating Mach number.

$d_{F,equiv}$ The equivalent diameter, which is defined en 3.2 (**Scholz 1999**, p.104).

$$d_{F,equiv} = \sqrt{h_F \cdot w_F} \quad (3.2)$$

With

h_F The height of the fuselage in m.

w_F The width of the fuselage in m.

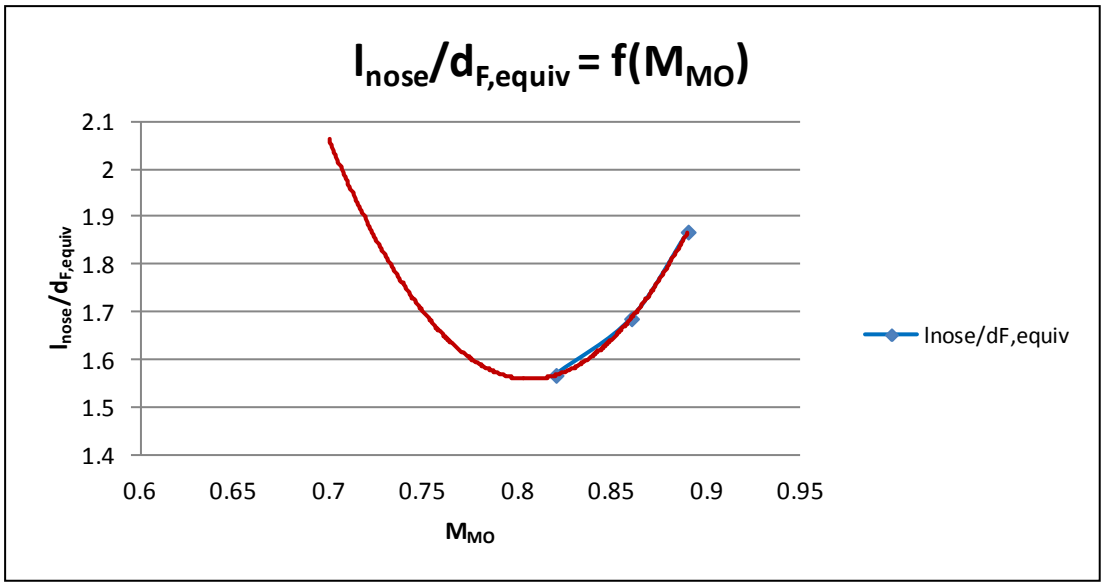

Figure 3.1 Initial statistical approximation for the nose length as a function of M_{MO}

The horizontal range of the plot has been selected from 0.5 to 0.95 to show the result for an aircraft with a low M_{MO}. The solution for airplanes with a M_{MO} around 0.5-0.6 is unfeasible. This is the case of the current turboprops, which would have a not real value of its nose length with this equation. For this reason another method has been analyzed to calculate the nose length.

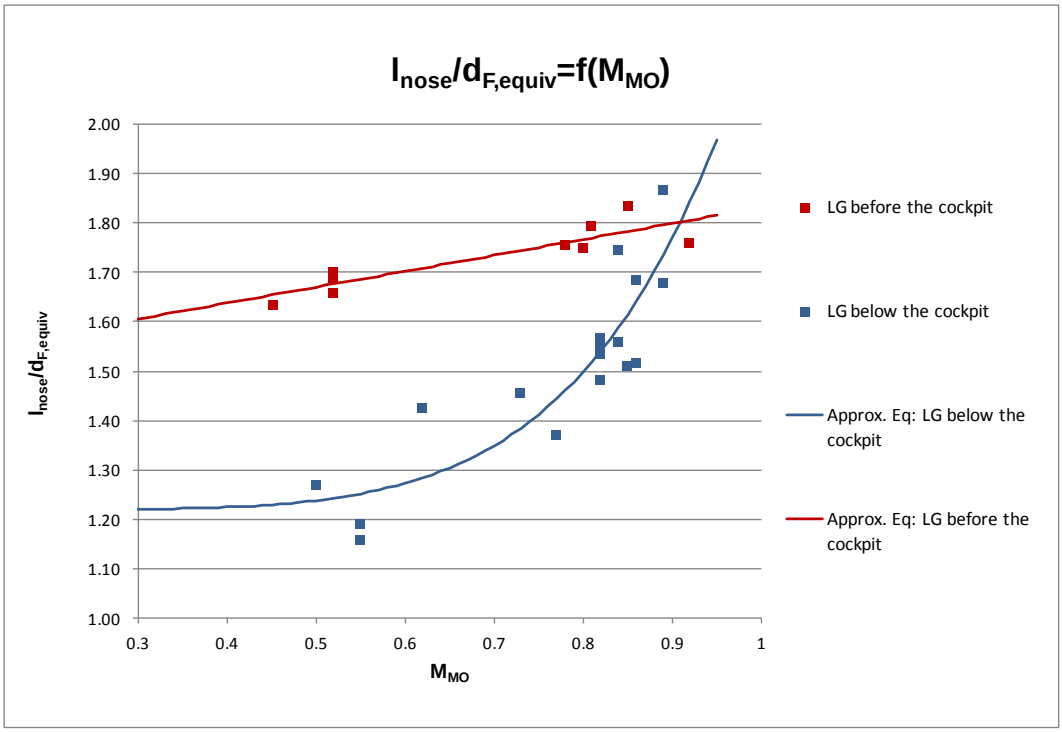

Figure 3.2 Representation of the solutions of $l_{nose}/d_{F,equiv}$ as a function of M_{MO} and the position of the nose landing gear

The solution for the calculation of the nose length are defined by the Equations 3.3 and 3.4 as a function of the position of the nose landing gear and M_{MO}.

- For the nose landing gear before the cockpit:

$$\frac{l_{nose}}{d_{F,equiv}} = 0.323 \cdot M_{MO} + 1.509 \tag{3. 3}$$

- For the nose landing gear below the cockpit:

$$\frac{l_{nose}}{d_{F,equiv}} = 1.007 \cdot (M_{MO})^{5.746} + 1.219 \tag{3. 4}$$