Understanding the Aircraft Mass Growth and Reduction Factor

Dieter Scholz  
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EWADE 2020 - 15th European Workshop on Aircraft Design Education
READ 2020 - Research and Education in Aircraft Design
Online, 21st and 22nd October 2020
Initially a short message:

Continuous Special Issue
"Aircraft Design"
of the Open Access Journal
"Aerospace" at MDPI
"Aircraft Design" our "Journal"

Purpose – Establish a "journal" for the aircraft design community.

Approach – Aircraft design is too small as a topic for a journal. Use a continuous Special Issue i.e. only a subset of a journal. Follow the Open Access publishing model that can life already on a low number of papers annually.

Findings – The well established publisher MDPI published already one Special Issue "Aircraft Design" in "Aerospace" (ISSN 2226-4310) and was willing to follow the approach. The journal has a CiteScore (2019) of 2.6 from Scopus and is part of the Emerging Sources Citation Index - Web of Science (Clarivate Analytics).

Tradition – The tradition of the journal Aircraft Design at Elsevier (ISSN: 1369-8869) which was published from 1998 to 2002 with Prof. Egbert Torenbeek and Prof. Dr. Jan Roskam as Editors is continued.

Limitations – Article Processing Charges (APC) have to be paid. A 30% discount is available for authors from EWADE and READ. Further discounts a granted based on reviews.

Originality and Value – This is the only "journal" fully dedicated to aircraft design.
Continuous Special Issue "Aircraft Design" of the Open Access Journal "Aerospace" at MDPI

http://journal.AircraftDesign.org

Dieter Scholz: Aircraft Mass Growth Factor

Special Issue Editors

Dieter Scholz

Egbert Torenbeek
Dieter Scholz:
Publishing in "Aircraft Design" with a Continuous Open Access Special Issue

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Continuous Special Issue "Aircraft Design" of the Open Access Journal "Aerospace" at MDPI

The Publisher: MDPI

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The Publisher: MDPI

MDPI Office in Wuhan, China
Now to my presentation:

Understanding the Aircraft Mass Growth and Reduction Factor
Understanding the Aircraft Mass Growth and Reduction Factor

Abstract

Purpose – This project work shows a literature survey, clearly defines the mass growth factor, shows a mass growth iteration, and derives an equation for a direct calculation of the factor (without iteration). Definite values of the factor seem to be missing in literature. To change this, mass growth factors are being calculated for as many of the prominent passenger aircraft as to cover 90% of the passenger aircraft flying today. The dependence of the mass growth factor on requirements and technology is examined and the relation to Direct Operating Costs (DOC) is pointed out.

Methodology – Calculations start from first principles. Publically available data is used to calculate a list of mass growth factors for many passenger aircraft. Using equations and the resulting relationships, new knowledge and dependencies are gained.

Findings – The mass growth factor is larger for aircraft with larger operating empty mass ratio, smaller payload ratio, larger specific fuel consumption (SFC), and smaller glide ratio. The mass growth factor increases much with increasing range. The factor depends on an increase in the fixed mass, so this is the same for the payload and empty mass. The mass growth factor for subsonic passenger aircraft is on average 4.2, for narrow body aircraft 3.9 and for wide body aircraft (that tend to fly longer distance) 4.9. In contrast supersonic passenger aircraft show a factor of about 14.

Practical implications – The mass growth factor has been revisited in order to fully embrace the concept of mass growth and may lead to a better general understanding of aircraft design.

Social implications – A detailed discussion of aircraft costs as well as aircraft development requires detailed knowledge of the aircraft. By understanding the mass growth factor, consumers can have this discussion with industry at eye level.

Originality/value – The derivation of the equation for the direct calculation of the mass growth factor and the determination of the factor using the method for 90% of current passenger aircraft was not shown.
Understanding the Aircraft Mass Growth and Reduction Factor

Acknowledgment

This presentation is based on the project of

John Singh Cheema

prepared at
Hamburg University of Applied Sciences
Aircraft Design and Systems Group (AERO)

Download from:
http://library.ProfScholz.de

Referenced here as:
Cheema 2020

Project

The Mass Growth Factor – Snowball Effects in Aircraft Design

Author: John Singh Cheema

Supervisor: Prof. Dr.-Ing. Dieter Scholz, MSME
Submitted: 31.03.2020

Faculty of Engineering and Computer Science
Department of Automotive and Aeronautical Engineering
Understanding the Aircraft Mass Growth and Reduction Factor

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  • Obtained from Technology and Range
  • Linked to Direct Operating Costs (DOC)
• Summary
Background

• The aircraft mass growth factor is fundamental to aircraft preliminary design. Due to the fact that mass during some aircraft design phases seems rather to increase than to decrease compared to initial estimates, the factor is called mass growth factor.
• However, a mass reduction factor is mathematically the same. The mass reduction factor can lead to even substantial mass reduction and is as such the secret to efficient aircraft design. For simplicity and tradition we may just talk about mass growth.
• It is usually defined as the ratio of an increase in the total mass (take-off mass) due to an arbitrary increase in local mass (empty mass) determined after a full iteration in aircraft design to achieve the original performance requirements (payload and range).
• The aircraft design iteration sees after each loop another increment in the take-off mass, so that an initial (local) mass increase aggravates the situation like a snow ball transforming into an avalanche. Hence the pseudonym snowball factor.
• The concept of the mass growth factor is probably as old as aviation. It has been discussed heavily from the 1950th to the 1970th and has continued to be mentioned until today. Nevertheless, it seems not to be well enough understood today. Maybe its importance has declined due to modern computing power providing quite accurate mass estimates in each design phase, but detaching the engineer from the feel for the numbers.
Avalanche – Snow Ball Effect
Literature Review (Overview)

Ballhaus 1954 (SAWE Paper)
Saelman 1973 (SAWE Paper)
Fürst 1999 (LTH, Germany)
Sinke 2019 (Lecture Notes, TU Delft)
SAWE 2019 (SAWE Book)

Aircraft Design Books

Torenbeek 1976
Roskam 1989
Jenkinson 1999
Howe 2000
Müller 2003

elaborate the principle
talk about design aspects

See Cheema 2020 for details.

SAWE: Society of Allied Weight Engineers
LTH: Luftfahrttechnisches Handbuch
CLEAR DESIGN THINKING USING THE AIRCRAFT GROWTH FACTOR

By
Wm. F. Ballhaus

TO BE PRESENTED AT THE SAE NATIONAL AERONAUTICAL MEETING
Los Angeles, California
October 8, 1954
Definition

\[ G = \frac{\text{Change in Gross Weight}}{\text{Fixed Weight Added or Change in Fixed Weight}} \]

Ballhaus 1954
Aircraft Design Mass Growth (Overall Picture)

Modified design configuration
(Due to corrected assumptions)

Original design configuration

Design point

Performance reduction as alternative to mass increase

Performance requirement
(Such as range or payload)

New curve may become asymptotic at a mass less than the design value

Howe 2000
Iteration of the Mass Growth Factor (Equations)

\[ \Delta m_L = 1 \text{ kg} \]

\[ m_{MTO,0} = m_{MTO} + \Delta m_G = m_{MPL} + m_{OE} + \Delta m_L + m_F \]

\[ m_{MTO,1} = m_{MPL} + \frac{m_{OE}}{m_{MTO}} \cdot m_{MTO,0} + \Delta m_L + \frac{m_F}{m_{MTO}} \cdot m_{MTO,0} \]

\[ \Delta m_G = m_{MTO,X} - m_{MTO} \]

\[ k_{MGW} = \frac{\Delta m_G}{\Delta m_L} \]
### Iteration of the Mass Growth Factor (Excel Table)

**Dieter Scholz:** Aircraft Mass Growth Factor

**Data and tools uploaded to Harvard Dataverse:** [https://doi.org/10.7910/DVN/6NHDDP](https://doi.org/10.7910/DVN/6NHDDP)

<table>
<thead>
<tr>
<th>Iteration</th>
<th>mMTO,i</th>
<th>mDMG,i</th>
<th>DMG/DmMTO,i</th>
<th>MGW,i</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>78001.000 kg</td>
<td>1.00000 kg</td>
<td></td>
<td>4.00</td>
</tr>
<tr>
<td>1</td>
<td>72001.750 kg</td>
<td>1.75000 kg</td>
<td>75.0000%</td>
<td>1.75000 kg</td>
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<tr>
<td>2</td>
<td>72002.312 kg</td>
<td>2.31200 kg</td>
<td>32.1200%</td>
<td>2.31200 kg</td>
</tr>
<tr>
<td>3</td>
<td>72002.754 kg</td>
<td>2.75400 kg</td>
<td>37.0760%</td>
<td>2.75400 kg</td>
</tr>
<tr>
<td>4</td>
<td>72002.951 kg</td>
<td>2.95100 kg</td>
<td>37.8400%</td>
<td>2.95100 kg</td>
</tr>
<tr>
<td>5</td>
<td>72002.951 kg</td>
<td>2.95100 kg</td>
<td>37.8400%</td>
<td>2.95100 kg</td>
</tr>
<tr>
<td>6</td>
<td>72003.238 kg</td>
<td>3.23800 kg</td>
<td>37.8400%</td>
<td>2.95100 kg</td>
</tr>
<tr>
<td>7</td>
<td>72003.646 kg</td>
<td>3.64600 kg</td>
<td>54.6200%</td>
<td>2.88600 kg</td>
</tr>
<tr>
<td>8</td>
<td>72003.803 kg</td>
<td>3.80300 kg</td>
<td>38.7400%</td>
<td>2.88600 kg</td>
</tr>
<tr>
<td>9</td>
<td>72003.803 kg</td>
<td>3.80300 kg</td>
<td>38.7400%</td>
<td>2.88600 kg</td>
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<tr>
<td>10</td>
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<td>3.93100 kg</td>
<td>38.7400%</td>
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<tr>
<td>11</td>
<td>72003.973 kg</td>
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<td>38.7400%</td>
<td>2.88600 kg</td>
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<tr>
<td>12</td>
<td>72003.906 kg</td>
<td>3.90600 kg</td>
<td>38.7400%</td>
<td>2.88600 kg</td>
</tr>
<tr>
<td>13</td>
<td>72003.829 kg</td>
<td>3.82900 kg</td>
<td>38.7400%</td>
<td>2.88600 kg</td>
</tr>
<tr>
<td>14</td>
<td>72003.947 kg</td>
<td>3.94700 kg</td>
<td>38.7400%</td>
<td>2.88600 kg</td>
</tr>
<tr>
<td>15</td>
<td>72003.930 kg</td>
<td>3.93000 kg</td>
<td>38.7400%</td>
<td>2.88600 kg</td>
</tr>
<tr>
<td>16</td>
<td>72003.970 kg</td>
<td>3.97000 kg</td>
<td>38.7400%</td>
<td>2.88600 kg</td>
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<tr>
<td>17</td>
<td>72003.877 kg</td>
<td>3.87700 kg</td>
<td>38.7400%</td>
<td>2.88600 kg</td>
</tr>
<tr>
<td>18</td>
<td>72003.958 kg</td>
<td>3.95800 kg</td>
<td>38.7400%</td>
<td>2.88600 kg</td>
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<tr>
<td>19</td>
<td>72003.937 kg</td>
<td>3.93700 kg</td>
<td>38.7400%</td>
<td>2.88600 kg</td>
</tr>
<tr>
<td>20</td>
<td>72003.990 kg</td>
<td>3.99000 kg</td>
<td>38.7400%</td>
<td>2.88600 kg</td>
</tr>
</tbody>
</table>
Iteration of the Mass Growth Factor (Convergence)

Convergence of the mass growth factor using the example of the Boeing 767-300
### Considering 90% of Passenger Aircraft

#### World Fleet

**Table A.1** 90% of current aircraft (according to Robson 2019)

<table>
<thead>
<tr>
<th>Aircraft Type</th>
<th>Number in Operation</th>
<th>Percent of total</th>
<th>Sum of most aircraft types in percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boeing 737-800</td>
<td>4804</td>
<td>16.83%</td>
<td>16.83%</td>
</tr>
<tr>
<td>A320</td>
<td>4135</td>
<td>14.48%</td>
<td>31.31%</td>
</tr>
<tr>
<td>A320neo</td>
<td>658</td>
<td>2.30%</td>
<td>33.61%</td>
</tr>
<tr>
<td>A321neo</td>
<td>160</td>
<td>0.56%</td>
<td>34.18%</td>
</tr>
<tr>
<td>A321</td>
<td>1650</td>
<td>5.78%</td>
<td>39.95%</td>
</tr>
<tr>
<td>Boeing 737 Max 8</td>
<td>1249</td>
<td>0.00%</td>
<td>39.95%</td>
</tr>
<tr>
<td>A319</td>
<td>775</td>
<td>2.71%</td>
<td>44.33%</td>
</tr>
<tr>
<td>Boeing 737-700</td>
<td>1005</td>
<td>3.52%</td>
<td>47.85%</td>
</tr>
<tr>
<td>ATR72</td>
<td>599</td>
<td>2.08%</td>
<td>50.56%</td>
</tr>
<tr>
<td>Boeing 777-300(ER)</td>
<td>829</td>
<td>2.90%</td>
<td>53.47%</td>
</tr>
<tr>
<td>Embraer 175</td>
<td>595</td>
<td>2.08%</td>
<td>55.55%</td>
</tr>
<tr>
<td>Boeing 787-9</td>
<td>451</td>
<td>1.58%</td>
<td>57.13%</td>
</tr>
<tr>
<td>A330-300</td>
<td>707</td>
<td>2.48%</td>
<td>59.61%</td>
</tr>
<tr>
<td>Boeing 767-300</td>
<td>622</td>
<td>2.18%</td>
<td>61.79%</td>
</tr>
<tr>
<td>A350-900</td>
<td>261</td>
<td>0.91%</td>
<td>62.70%</td>
</tr>
<tr>
<td>Boeing 757-200</td>
<td>600</td>
<td>2.10%</td>
<td>64.80%</td>
</tr>
<tr>
<td>A330-200</td>
<td>547</td>
<td>1.92%</td>
<td>66.72%</td>
</tr>
<tr>
<td>Boeing 737-900</td>
<td>550</td>
<td>1.93%</td>
<td>68.64%</td>
</tr>
<tr>
<td>De Havilland Canada Dash 8-400</td>
<td>502</td>
<td>1.76%</td>
<td>70.40%</td>
</tr>
<tr>
<td>Embraer 190</td>
<td>495</td>
<td>1.73%</td>
<td>72.14%</td>
</tr>
<tr>
<td>Boeing 737 Max TBD</td>
<td>444</td>
<td>1.56%</td>
<td>73.69%</td>
</tr>
<tr>
<td>Bombardier CRJ900</td>
<td>77</td>
<td>0.27%</td>
<td>73.96%</td>
</tr>
<tr>
<td>A220</td>
<td>431</td>
<td>1.51%</td>
<td>75.47%</td>
</tr>
<tr>
<td>Boeing 777-200</td>
<td>422</td>
<td>1.49%</td>
<td>76.95%</td>
</tr>
<tr>
<td>Embraer ERJ-145</td>
<td>24</td>
<td>0.00%</td>
<td>76.95%</td>
</tr>
<tr>
<td>Boeing 737 Max 10</td>
<td>328</td>
<td>1.15%</td>
<td>78.10%</td>
</tr>
</tbody>
</table>

*Example: 27 aircraft, 78.1% of fleet*
Considering 90% of Passenger Aircraft

\[
\frac{n}{n_{\text{max}}} = 1 - a \cdot e^{b \left( \frac{n}{n_{\text{max}}} \right)_{\text{type}}}
\]

\[
a = 0.7480879, \quad b = -0.047978
\]

with data from Flight 2016
### Mass Growth Factor – Considering 90% of Passenger Aircraft

**Table 3.3**  Evaluation of 90% of all current flying commercial aircraft including two supersonic aircraft

<table>
<thead>
<tr>
<th>Aircraft</th>
<th>(m_{MTO}) [kg]</th>
<th>(m_{OE}) [kg]</th>
<th>(m_{MPL}) [kg]</th>
<th>(k_{MGW})</th>
<th>Sources</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boeing 737-800</td>
<td>78220</td>
<td>41480</td>
<td>14690</td>
<td>5.32</td>
<td>Jenkinson 2019</td>
</tr>
<tr>
<td>A320-200</td>
<td>73500</td>
<td>42100</td>
<td>18633</td>
<td>3.94</td>
<td>Jackson 2011</td>
</tr>
<tr>
<td>A320neo</td>
<td>79000</td>
<td>44300</td>
<td>20000</td>
<td>3.95</td>
<td>Airbus 2005, Wiki 2020</td>
</tr>
<tr>
<td>A321neo</td>
<td>97000</td>
<td>50100</td>
<td>25500</td>
<td>3.80</td>
<td>Airbus 2005a, Wiki 2020</td>
</tr>
<tr>
<td>A321-200</td>
<td>89000</td>
<td>48000</td>
<td>22780</td>
<td>3.90</td>
<td>Jenkinson 2019</td>
</tr>
<tr>
<td>A319-100</td>
<td>64000</td>
<td>39200</td>
<td>17390</td>
<td>3.68</td>
<td>Jenkinson 2019</td>
</tr>
<tr>
<td>Boeing 737-700</td>
<td>69400</td>
<td>37585</td>
<td>11610</td>
<td>5.97</td>
<td>Jenkinson 2019</td>
</tr>
<tr>
<td><strong>ATR 72-500</strong></td>
<td><strong>22500</strong></td>
<td><strong>12950</strong></td>
<td><strong>7350</strong></td>
<td><strong>3.06</strong></td>
<td>Jackson 2011</td>
</tr>
<tr>
<td>Boeing 777-300 ER</td>
<td>299370</td>
<td>155960</td>
<td>68570</td>
<td>4.36</td>
<td>Jackson 2011</td>
</tr>
<tr>
<td>Embraer 175</td>
<td>37500</td>
<td>21810</td>
<td>9890</td>
<td>3.79</td>
<td>Jackson 2011</td>
</tr>
<tr>
<td>Boeing 787-9</td>
<td>244940</td>
<td>128850</td>
<td>52587</td>
<td>4.65</td>
<td>Boeing 2018, Wiki 2020c</td>
</tr>
<tr>
<td>A330-300</td>
<td>217000</td>
<td>118189</td>
<td>48400</td>
<td>4.48</td>
<td>Jenkinson 2019</td>
</tr>
<tr>
<td>Boeing 767-300</td>
<td>156489</td>
<td>87135</td>
<td>39140</td>
<td>3.99</td>
<td>Jenkinson 2019</td>
</tr>
<tr>
<td>A350-900</td>
<td>280000</td>
<td>142400</td>
<td>53300</td>
<td>5.25</td>
<td>Airbus 2005b, Wiki 2020a</td>
</tr>
<tr>
<td>Boeing 757-200</td>
<td>115900</td>
<td>58040</td>
<td>25690</td>
<td>4.51</td>
<td>Jenkinson 2019</td>
</tr>
<tr>
<td>A330-200</td>
<td>230000</td>
<td>120200</td>
<td>36400</td>
<td>6.31</td>
<td>Jenkinson 2019</td>
</tr>
<tr>
<td>Boeing 737-900</td>
<td>74389</td>
<td>42901</td>
<td>19831</td>
<td>3.75</td>
<td>Boeing 2013, Wiki 2020e</td>
</tr>
<tr>
<td>DHC Dash 8-400</td>
<td>24993</td>
<td>14968</td>
<td>7257</td>
<td>3.44</td>
<td>Lambert 1991</td>
</tr>
<tr>
<td>Embraer 190</td>
<td>50300</td>
<td>28080</td>
<td>13530</td>
<td>3.71</td>
<td>Jackson 2011</td>
</tr>
<tr>
<td>Bombardier CRJ900</td>
<td>36500</td>
<td>21430</td>
<td>10320</td>
<td>3.53</td>
<td>AirlinesInform 2020</td>
</tr>
<tr>
<td>A220-100</td>
<td>63049</td>
<td>35221</td>
<td>15127</td>
<td>4.16</td>
<td>Airbus 2019, Wiki 2020d</td>
</tr>
<tr>
<td>Boeing 777-200</td>
<td>242670</td>
<td>135875</td>
<td>54635</td>
<td>4.44</td>
<td>Jenkinson 2019</td>
</tr>
</tbody>
</table>

90% of aircraft considered globally with more than 19 seats by looking at 44 aircraft types.

**ATR 72**: propeller aircraft, short range

Cheema 2020
Wisdom Gained

1.) Larger aircraft do not necessarily have a higher mass growth factor.

2.) It does not matter how large the local mass growth is; the mass growth factor remains unaffected.*

\[ m_{MTO} + \Delta m_G = m_{MPL} + m_{OE} + \Delta m_L + m_F \]

3.) It does not matter whether there is one kg more operating empty mass or one kg more payload on board. The mass growth factor for a growth in the operating empty mass is therefore the same factor as for a growth in the payload.

4.) Old long-range aircraft are more sensitive to local mass growth than new short-range aircraft.

* This as long as the local mass growth is much smaller than the fixed mass. The position of the local mass growth does not matter, if the wing is not yet fixed and is positioned according to the new weight and balance situation.
### Mass Growth Factor – Aircraft Categories

Table 4.1 Mass growth factor for different aircraft categories

<table>
<thead>
<tr>
<th>Aircraft categories</th>
<th>Wide-Body</th>
<th>Narrow-Body</th>
<th>Subsonic</th>
<th>Supersonic</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_{MGW}$</td>
<td>4.91</td>
<td>3.85</td>
<td>4.23</td>
<td>13.82</td>
</tr>
</tbody>
</table>
Mass Growth Factor – Obtained from Payload Fraction

After a longer derivation (Cheema 2020), we find a simple equation:

\[
k_{MGW} = \frac{m_{MTO}}{m_{MPL}} = \frac{1}{1 - \frac{m_F}{m_{MTO}} - \frac{m_{OE}}{m_{MTO}}}
\]

<table>
<thead>
<tr>
<th>Range type</th>
<th>Short-range</th>
<th>Medium-range</th>
<th>Long-range</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m_{OE}/m_{MTO} )</td>
<td>0.60</td>
<td>0.525</td>
<td>0.45</td>
</tr>
<tr>
<td>( m_F/m_{MTO} )</td>
<td>0.15</td>
<td>0.3</td>
<td>0.45</td>
</tr>
<tr>
<td>( k_{MGW} )</td>
<td>4</td>
<td>5.7</td>
<td>10</td>
</tr>
</tbody>
</table>
Mass Growth Factor – Can Go to Infinity

\[ k_{MGW} = \frac{m_{MTO}}{m_{MPL}} = \frac{1}{1 - \frac{m_F}{m_{MTO}} - \frac{m_{OE}}{m_{MTO}}} \]

As soon as

\[ \left( \frac{m_F}{m_{MTO}} + \frac{m_{OE}}{m_{MTO}} \right) \]

approaches 1.0 the mass growth factor, \( k_{MGW} \) goes towards infinity!

This means the design task has no solution!
Mass Growth Factor – Payload Fraction from Statistics

\[ k_{MGW} = \frac{1}{\frac{m_{MPL}}{m_{MTO}}} \]

small A/C; short range

\[ k_{MGW} = 4 \ldots 10 \]

Scholz 2018
Mass Growth Factor – Obtained from Technology and Range

\[
\frac{m_{OE}}{m_{MTO}} = 0.5967 - 0.0000166(1/\text{NM}) \cdot R \quad \text{Lehnert 2018}
\]

\[
k_{MGW} = \frac{1}{1 - (0.5967 - 0.0000166(1/\text{NM}) \cdot R) - \left(1 - e^{-\frac{R}{B}}\right)}
\]

\[
B = \frac{E \cdot V}{c \cdot g}
\]

\[
\frac{m_F}{m_{MTO}} = 1 - e^{-\frac{R}{B}}
\]

\[
\begin{align*}
\text{c} & \quad \text{Specific fuel consumption} \\
\text{R} & \quad \text{Range} \\
\text{B} & \quad \text{Breguet factor} \\
\text{E} & \quad \text{Glide ratio in cruise flight} \\
\text{V} & \quad \text{Cruising speed}
\end{align*}
\]
Mass Growth Factor – Linked to Direct Operating Costs (DOC)

\[ C_{s,m} = \frac{C_a}{c, t} R \cdot n_s = \frac{C_a}{c, a} R \cdot n_s \cdot n_{t,a} \]

\[ DOC = C_{DOC} = C_{a/c, a} \]

depends on cruise speed (given)

\[ DOC \approx m_{MTO} = m_{OE} + m_F + m_{PL} \]

range (given)

fuel costs

payload (given, constant)

depreciation, maintenance costs

\[ \frac{DOC}{n_s} \approx \frac{m_{MTO}}{m_{PL}} = \frac{1}{\frac{m_{PL}}{m_{MTO}}} = k_{MGW} \]

\[ C_{a/c, a} \quad \text{Aircraft annual costs} \]
\[ n_{t,a} \quad \text{Number of Flights per year} \]
\[ C_{s,m} \quad \text{Seat-mile costs} \]
\[ C_{a/c, t} \quad \text{Aircraft trip cost} \]
Summary

• The mass growth (and reduction) factor is well known, but not well understood:
  • Derivation missing => added
  • Numerical values missing => added
• Average value for subsonic passenger aircraft: 4.2
  range from 3.1 (ATR-72) via 6.2 (A380) and 6.5 (B747-400) to 15.6 (Concorde)
• The mass growth factor important for the design phase (snow ball effect) and also a good indicator to quickly understand the aircraft's economy.
• The mass growth factor can be estimated from basic parameters: range, $R$; $E = L/D$; $c = $ SFC; cruise speed, $V$ :

$$k_{MGW} = \frac{1}{1 - (0.5967 - 0.0000166(1/NM) \cdot R) - \left(1 - e^{-\frac{R}{B}}\right)}$$

$$B = \frac{E \cdot V}{c \cdot g}$$
Understanding the Aircraft Mass Growth and Reduction Factor

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References


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